

Exercise session  
(Memory management / Virtual Memory)

Operating Systems – EDA092/DIT400

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# Exercise 1

Jake and Finn have a computer whose memory access time is 150 nanoseconds and whose page-fault service time is 5 milliseconds. A process previously run in an operating system without virtual memory was 2 times faster than the same process run by the actualized operating system (the latter having virtual memory). Every how many page accesses are Jake and Finn experiencing a page fault on average? Would buying a new disk able to bring the page-fault service time to 2 milliseconds make the process no more than 1.5 times slower in the new operating system (compared to the previous one)?

# Exercise 1 - solution

$$\begin{aligned}EAT &= (1 - p) \times (150 \text{ nanoseconds}) + p (5 \text{ milliseconds}) \\&= (150 - p \times 150 + p \times 5,000,000) \\&= 150 + p \times 4,999,850\end{aligned}$$

## Question 1

Without virtual memory  $p = 0 \rightarrow EAT = 150$

With virtual memory  $150 + p \times 4,999,850 = 2 \times 150$

$p = 0.00003$

1 page fault every  $1/p$  (33333) on average

## Question 2

$150 + 0.00005 \times 1,999,850 < 1.5 \times 150 ?$

$250 < 225 ? \rightarrow \text{NO}$

## Exercise 2

- Given the following reference string

7 2 1 2 0 1 1 4 2 3 0 7 0 1 2 7 7 3 0 0 3 0 3

and assuming 4 frames are available, find:

- The minimum number of page faults we can observe
- The extra page faults caused by the FIFO replacement algorithm
- The extra page faults caused by the LRU replacement algorithm

# Exercise 2 - solution

OPTIMAL

7 2 1 2 0 1 1 4 2 3 0 7 0 1 2 7 7 3 0 0 3 0 3

7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	0	0	0	0	0
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
1	1	1	1	1	1	4	4	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1

Page faults

1	1	1	0	1	0	0	1	0	1	0	0	0	1	0	0	0	0	1	0	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

# Exercise 2 - solution

FIFO

7 2 1 2 0 1 1 4 2 3 0 7 0 1 2 7 7 3 0 0 3 0 3

7	7	7	7	7	7	7	4	4	4	4	4	4	4	2	2	2	2	2	2	2	2	2	2
	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
	1	1	1	1	1	1	1	1	1	7	7	7	7	7	7	7	7	7	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Page faults

1 1 1 0 1 0 0 1 0 1 0 1 0 1 1 0 0 1 1 0 0 0 0

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## Exercise 2 - solution

LRU

7 2 1 2 0 1 1 4 2 3 0 7 0 1 2 7 7 3 0 0 3 0 3

## Page faults

1	1	1	0	1	0	0	1	0	1	1	1	0	1	1	0	0	1	1	0	0	0	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

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## Exercise 2 - solution

- Minimum is 8
- FIFO is +3
- LRU is +4

## Exercise 3

- Given processes  $P_1, P_2, P_3$  and  $P_4$  having sizes of 20, 25, 50 and 4 pages and given a total number of frames equal to 80 (10% of which reserved to the OS), compute the number of frames allocated to each process if frames are allocated proportionally to the size of each program

## Exercise 3 - solution

- Frames for  $P_1$ :  $20/99*72 = 14$
- Frames for  $P_2$ :  $25/99*72 = 18$
- Frames for  $P_3$ :  $50/99*72 = 36$
- Frames for  $P_4$ :  $4/99*72 = 2$

Notice that I'm rounding down instead of up otherwise I will violate the assumption that 8 pages should be devoted to the OS!

## Exercise 4

- Suppose a process P has size of 100 bytes. Compute (1) the number of wasted bytes caused by internal fragmentation if the page and frame sizes are set to  $2^4$  bytes and (2) the size in bits of the page table (assume each frame entry requires 1 byte and that dirty bits are also used to speed the swap out of pages).

## Exercise 4 - solution

- Page size = 16 bytes
- Number of pages required =  $\text{ceil}(100/16) = 7$
- Wasted bytes because of internal fragmentation  $16 - 100\%16 = 12$  bytes
- Page table =  $7 * (8 + 1 + 1) = 70$  bits
  - 8 bits → frame entry
  - 1 bit → valid/invalid bit
  - 1 bit → dirty bit