Computer lab – part B
Capacitance per unit length of a shielded transmission line

Computer lab in Electromagnetic fields for E2 – EEM 015

November 22, 2019

Do the following:	1)	Solve theory problems 1-8 in part A with pen and paper and present your solutions to one of the teaching assistants. Lists with time slots for the presentation are provided at the lectures.
	2)	Solve computer problems 9-11 in part A during the teacher-supervised computer session, and take careful notes of your results! Present your results to the teaching assistant during the computer session.
	3)	Solve problems for part B, after you have been approved on part A. Take careful notes!
	4)	Write a report on part B based on the notes you have taken!
	5)	Upload your report to Canvas. Reports uploaded no later than 2019-12-06 will be corrected by 2019-12-20.

1 Introduction

A transmission line consists of two or more conductors and, here, we consider the case of two conductors shielded by a third conductor. Shielded transmission lines are used to guide electromagnetic waves. As an example, we can mention applications where electromagnetic energy needs to be transported to small active components that operate at microwave frequencies. An important parameter that characterizes such a transmission line is its capacitance per unit length, i.e. C/l_z. Here, l_z denotes the length of the transmission line along the z-axis, which is perpendicular to the cross section of the transmission line.

The cross section for the shielded transmission line is shown in Fig. 1. It consists of a shield of rectangular cross-section with the width w and the height h. Inside the shield, we have two conductors of circular cross-section, where the radii are r_1 and r_2 as shown in Fig. 1. Conductor i is placed with its center at the point (x_i, y_i) and it has a dielectric insulator of thickness d_i with the permittivity ϵ_i, where the index i is 1 or 2. In the following, the shield is considered to be the third conductor and it is indexed by $i = 3$.

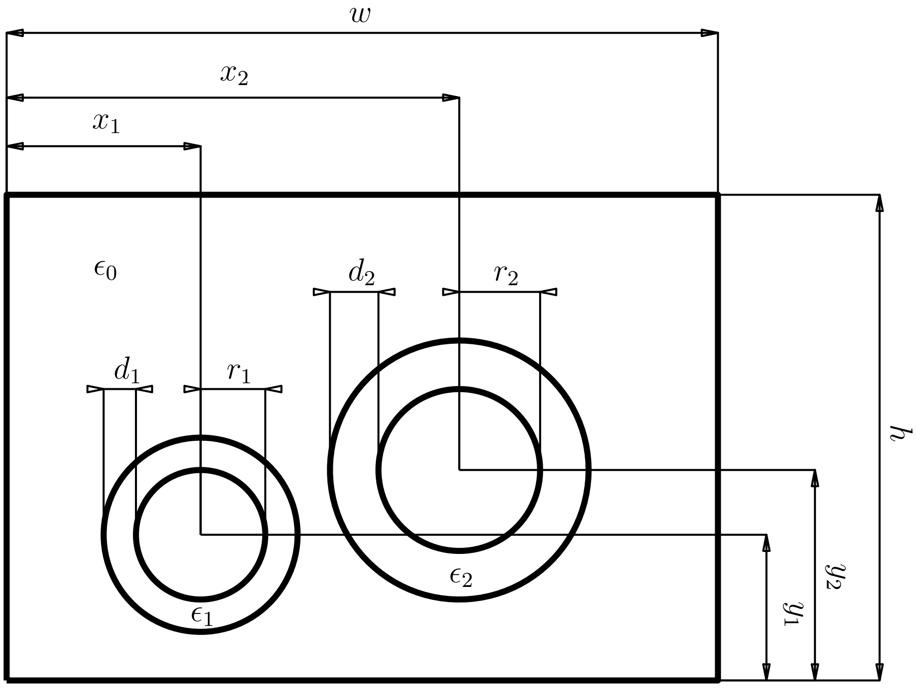

Figure 1: Design of the shielded transmission line. The geometrical dimensions and the permittivity values in the figure are individual for each group, as described in Sec. 1.1 in this lab-PM.

1.1 Parameter values

The parameter values shown in Fig. 1 are given as follows. Remember to write down the parameter values that you have used in the report.

$$
\begin{aligned}
w &= 44\,\text{mm}\\
h &= 30\,\text{mm}\\
x_1 &= 12\,\text{mm}\\
y_1 &= 9\,\text{mm}\\
r_1 &= 4\,\text{mm}\\
d_1 &= 2\,\text{mm}\\
x_2 &= 28 + p_4/2\,\text{mm}\\
y_2 &= 13 + p_6/2\,\text{mm}\\
r_2 &= 5 + p_9/10\,\text{mm}\\
d_2 &= 3 - p_9/10\,\text{mm}\\
\epsilon_{\text{r},1} &= 3 + p_{10}/2\\
\epsilon_{\text{r},2} &= 8 - p_{10}/2
\end{aligned}
$$

Here, p_4, p_6, p_9 and p_{10} is the 4-th, 6-th, 9-th and 10-th digit in your personal identification number. If you work in groups of two or three students: replace p_4 by $\lfloor \bar{p}_4 \rfloor$. Here, $\bar{p}_4$ is the average of the 4-th digit in the personal identification number of all the group members. Further, $\lfloor k \rfloor$ is the nearest integer towards zero from k, which means that $\lfloor 4.8 \rfloor = 4$ and

$\lfloor 7 \rfloor = 7$ as two examples. Then use the same method to obtain the corresponding values for p_6, p_9 and p_{10}.

1.2 Coefficients of capacitance

The total charge q_i on the three conductors are related to their potentials v_j by

$$q_1 = c_{11}v_1 + c_{12}v_2 + c_{13}v_3 \quad (1)$$
$$q_2 = c_{21}v_1 + c_{22}v_2 + c_{23}v_3 \quad (2)$$
$$q_3 = c_{31}v_1 + c_{32}v_2 + c_{33}v_3 \quad (3)$$

where the constants c_{ij} are called the coefficients of capacitance and they depend only on the geometry and the material of the problem. Note that `COMSOL` solves a *two-dimensional* problem formulated for the cross section of an infinitely long transmission line. Thus, `COMSOL` actually computes the charge per unit length q_i/l_z and, then, we work with the coefficients of capacitance per unit length c_{ij}/l_z. It is simple to divide Eqs. (1)-(3) by l_z on both sides to get the corresponding system that relates q_i/l_z to c_{ij}/l_z and v_j, where i and j take the values 1, 2 and 3.

1.3 Coefficients of capacitance computed by `COMSOL`

The coefficients of capacitance c_{11}, c_{21} and c_{31} can be determined by `COMSOL`. Follow this procedure:

1. Set $v_1 = 1$ and $v_2 = v_3 = 0$ in `COMSOL`.
2. Compute q_1, q_2 and q_3 by solving the electrostatic problem in `COMSOL`.
3. Determine c_{11}, c_{21} and c_{31} from Eqs. (1)-(3), i.e. $c_{i1} = q_i/v_1 = q_i$ for $i = 1, 2, 3$.

The other coefficients of capacitance can be determined in a similar manner by solving two additional problems in `COMSOL`: (i) the boundary conditions $v_2 = 1$ and $v_1 = v_3 = 0$ give the coefficients of capacitance c_{12}, c_{22} and c_{32}; and (ii) the boundary conditions $v_3 = 1$ and $v_1 = v_2 = 0$ give the coefficients of capacitance c_{13}, c_{23} and c_{33}.

2 Assignment

The tasks listed below must be addressed in the report.

2.1 Analytical calculations

Solve the following problems by analytical calculations, i.e. use pen and paper to derive formulas.

- Let the shield be connected to ground, i.e. set $v_3 = 0$ in Eqs. (1)-(3). Charge the capacitor that consists of the two circular conductors with the total charge Q, i.e. set $q_1 = +Q$ and $q_2 = -Q$ in Eqs. (1)-(2).

- Use Eqs. (1)-(2) to analytically calculate v_1 and v_2 for this situation, i.e. use pen and paper to derive formulas for v_1 and v_2. Note that the potentials v_1 and v_2 should be linear functions of Q.
- Calculate analytically the potential difference between conductor 1 and 2, i.e. $U = v_1 - v_2$. Note that the potential difference U should be a linear function of Q.
- Calculate analytically the capacitance between conductor 1 and 2 by means of

  $$C = \frac{Q}{U} \tag{4}$$

  Note that the final expression for C should only involve the coefficients of capacitance c_{11}, c_{12}, c_{21} and c_{22}.

2.2 Numerical computations

Solve the following problems by numerical computations, i.e. use `COMSOL` to compute numerical values.

2.2.1 Shield with only air

Use your personal parameter values as described in Sec. 1.1.

- Compute the coefficients of capacitance per unit length c_{ij}/l_z by `COMSOL`, as described in Sec 1.3.
- Compute the potentials v_1 and v_2 with the aid of the analytical formulas derived in Sec. 2.1, where you need to choose some appropriate value for Q/l_z. Use these potential values (that give $q_1/l_\mathrm{z} = +Q/l_\mathrm{z}$ and $q_2/l_\mathrm{z} = -Q/l_\mathrm{z}$ on the two conductors) to visualize the fields.

  1. Visualize the electric potential in color. Comment on the field distribution, where you can use the following questions as a starting point.
     - Where is the maximum value of the electric potential?
     - Where is the minimum value of the electric potential?
     - How does the electric potential vary on metal surfaces? How can this be related to the boundary conditions for a metal surface?
     - How does the electric potential vary on interfaces between different dielectric media? How can this be related to the boundary conditions for an interface between two dielectric media?
     - What is the value of the electric potential on the shield?
  2. Visualize the norm of the electric field in color together with its field lines in black. Comment on the field distribution, where you can use the following questions as a starting point.
     - Where is the maximum value of the electric field?
     - Where is the electric field small?
     - Where can you find abrupt changes in the electric field? How can these abrupt changes be explained?

- How does the electric field vary on metal surfaces? How can this be related to the boundary conditions for a metal surface?
- How does the electric field vary on interfaces between different dielectric media? How can this be related to the boundary conditions for an interface between two dielectric media?
- How is the electric field related to the electric potential?

3. Visualize the norm of the electric flux density in color together with its field lines in black. Comment on the field distribution, where you can use the following questions as a starting point.
   - Where is the maximum value of the electric flux density?
   - Where is the electric flux density small?
   - Where can you find abrupt changes in the electric flux density? How can these abrupt changes be explained?
   - How does the electric flux density vary on metal surfaces? How can this be related to the boundary conditions for a metal surface?
   - Where is the free charge located and how is it distributed?
   - How does the electric flux density vary on interfaces between different dielectric media? How can this be related to the boundary conditions for an interface between two dielectric media?
   - How is the electric flux density related to the electric field?

- Compute the capacitance per unit length C/l_z with the aid of the analytical formula derived in Sec. 2.1.

2.2.2 Shield partially filled with water

Now analyze the situation when water has partially filled the shield where the conductors are located. The depth of the water is 10 mm and its relative permittivity is 80.

- Compute the coefficients of capacitance per unit length c_{ij}/l_z by `COMSOL`, as described in Sec 1.3.

- Compute the potentials v_1 and v_2 with the aid of the analytical formulas derived in Sec. 2.1, where you need to choose some appropriate value for Q/l_z. Use these potential values (that give $q_1/l_\mathrm{z} = +Q/l_\mathrm{z}$ and $q_2/l_\mathrm{z} = -Q/l_\mathrm{z}$ on the two conductors) to visualize the fields.

1. Visualize the electric potential in color. Comment on the field distribution, where you can use the following questions as a starting point.
   - Where is the maximum value of the electric potential?
   - Where is the minimum value of the electric potential?
   - How does the electric potential vary on metal surfaces? How can this be related to the boundary conditions for a metal surface?
   - How does the electric potential vary on interfaces between different dielectric media? How can this be related to the boundary conditions for an interface between two dielectric media?
   - What is the value of the electric potential on the shield?

- How does the situation with water differ from the situation with only air?

2. Visualize the norm of the electric field in color together with its field lines in black. Comment on the field distribution, where you can use the following questions as a starting point.
   - Where is the maximum value of the electric field?
   - Where is the electric field small?
   - Where can you find abrupt changes in the electric field? How can these abrupt changes be explained?
   - How does the electric field vary on metal surfaces? How can this be related to the boundary conditions for a metal surface?
   - How does the electric field vary on interfaces between different dielectric media? How can this be related to the boundary conditions for an interface between two dielectric media?
   - How is the electric field related to the electric potential?
   - How does the situation with water differ from the situation with only air?
3. Visualize the norm of the electric flux density in color together with its field lines in black. Comment on the field distribution, where you can use the following questions as a starting point.
   - Where is the maximum value of the electric flux density?
   - Where is the electric flux density small?
   - Where can you find abrupt changes in the electric flux density? How can these abrupt changes be explained?
   - How does the electric flux density vary on metal surfaces? How can this be related to the boundary conditions for a metal surface?
   - Where is the free charge located and how is it distributed?
   - How does the electric flux density vary on interfaces between different dielectric media? How can this be related to the boundary conditions for an interface between two dielectric media?
   - How is the electric flux density related to the electric field?
   - How does the situation with water differ from the situation with only air?

- Compute the capacitance per unit length C/l_{z} with the aid of the analytical formula derived in Sec. 2.1. How does the situation with water differ from the situation with only air? Did the capacitance per unit length increase or decrease? Why?

3 Report

You are asked to document your work on "Computer lab – part B" in a written report, which should not exceed 5 pages including figures. Note that this is not so much space so you need to carefully choose what include and, at the same time, make sure that you do not miss any important information. Your solutions and your argumentation should be precise, well motivated and well structured. The reports are uploaded to Canvas.