$$
\left.\begin{array}{ll}
\text{FL} & \nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \\[2ex]
\text{AL} & \nabla \times \vec{H} = \vec{J}_v + \dfrac{\partial \vec{D}}{\partial t} \\[2ex]
\text{GL} & \nabla \cdot \vec{D} = \rho_v \\[2ex]
\text{NMM} & \nabla \cdot \vec{B} = 0
\end{array}\right.
$$

$\nabla \cdot \vec{D} = \rho_v$, $\nabla \cdot \vec{B} = 0$: initial conditions to be fulfilled at $t=0$

Constitutive relations

$$
\left\{\begin{array}{l}
\vec{D} = \epsilon \vec{E} \\
\vec{B} = \mu \vec{H} \\
\left(\vec{J}_v = \vec{J}_v^{\,imp} + \vec{J}_v^{\,eddy}\right) \\
\vec{J}_v^{\,eddy} = \sigma \vec{E} \quad \text{(Ohm's law)}
\end{array}\right.
$$

$(U = RI \Leftrightarrow I = GU)$

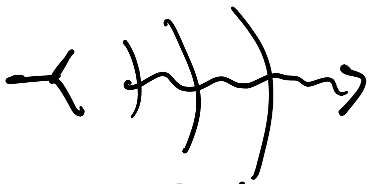
$$
\left\{\begin{array}{l}
\vec{E} = \text{electric field } [V/m] \\
\vec{H} = \text{magnetic field } [A/m] \\
\vec{D} = \text{electric flux density } [C/m^2] \\
\vec{B} = \text{magnetic flux density } [T] \\
\vec{J}_v = \text{volume current density } [A/m^2] \\
\rho_v = \text{volume charge density } [C/m^3]
\end{array}\right.
$$

$$
\left\{\begin{array}{l}
\epsilon = \text{permittivity } [F/m] \\
\mu = \text{permeability } [H/m] \\
\sigma = \text{conductivity } [S/m]
\end{array}\right.
\quad \text{(material parameters)}
$$

$$
\left(\frac{\partial \vec{D}}{\partial t} = \text{displacement current density } [A/m^2]\right)
$$

$$\begin{aligned} \text{FL} \quad & \nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \\ \text{AL} \quad & \nabla \times \vec{H} = \epsilon \frac{\partial \vec{E}}{\partial t} \end{aligned}$$

(Special case:
ϵ & μ are const wrt t
$\vec{J}_v = \vec{0}$ (i.e. no sources or losses))



$(\vec{E}, \vec{H})$

$$\nabla \times \left(\frac{1}{\mu} \nabla \times \vec{E} \right) = -\nabla \times \left(\frac{\partial \vec{H}}{\partial t} \right) = -\frac{\partial}{\partial t} \underbrace{(\nabla \times \vec{H})}_{= \epsilon \frac{\partial \vec{E}}{\partial t}}$$

$$\Rightarrow \nabla \times \left(\frac{1}{\mu} \nabla \times \vec{E} \right) + \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} = \vec{0}$$

(vector wave eq.)

$\frac{\partial^2}{\partial t^2} \to (j\omega)^2$

$$\Rightarrow \nabla \times \left(\frac{1}{\mu} \nabla \times \vec{\mathbb{E}} \right) - \omega^2 \epsilon \vec{\mathbb{E}} = \vec{0}$$

(vector Helmholtz eq.)

Non-magnetic materials $\Rightarrow \mu = \mu_0$
(with $\mu_0 = 4\pi \cdot 10^{-7}$ H/m)

$$\nabla \times \nabla \times \vec{E} + \mu_0 \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} = \vec{0}$$

$$\nabla \times \nabla \times \vec{\mathbb{E}} - \omega^2 \mu_0 \epsilon \vec{\mathbb{E}} = \vec{0}$$

Compare with hand-in assign. #2:
(should $\sigma = 0$)

$$\vec{\mathbb{E}} = \hat{z}\, \mathbb{E}_z(x)$$

$$\Rightarrow \nabla \times \vec{\mathbb{E}} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & 0 & \mathbb{E}_z(x) \end{vmatrix} = -\hat{y} \frac{\partial \mathbb{E}_z(x)}{\partial x}$$

$$\Rightarrow \nabla \times \nabla \times \vec{\mathbb{E}} = -\hat{z} \frac{\partial^2 \mathbb{E}_z(x)}{\partial x^2}$$

$$\Rightarrow -\frac{\partial^2 \mathbb{E}_z(x)}{\partial x^2} - \omega^2 \mu_0 \epsilon\, \mathbb{E}_z(x) = 0$$

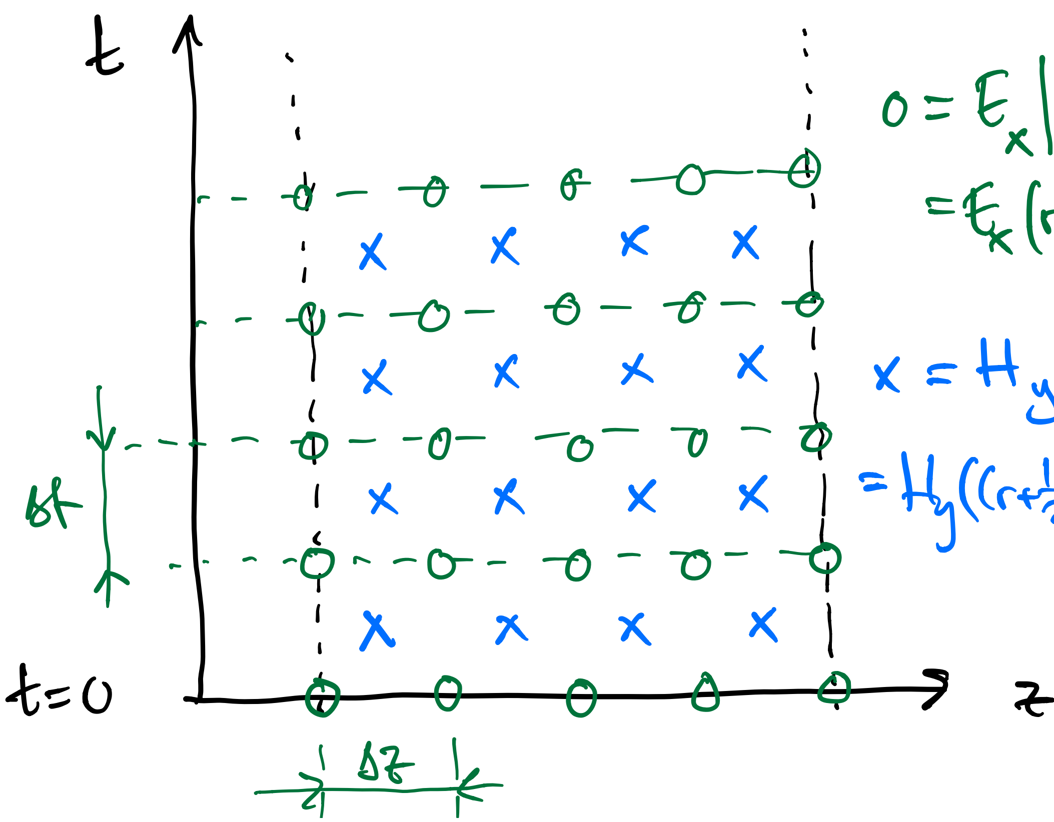
$$\left(E_z(x,t) = \mathrm{Re}\left\{ \mathbb{E}_z(x) e^{j\omega t} \right\} \right)$$

$$\begin{cases} \nabla \times \vec{E} = -\mu \dfrac{\partial \vec{H}}{\partial t} \\ \nabla \times \vec{H} = \epsilon \dfrac{\partial \vec{E}}{\partial t} \end{cases}$$

Assume:

$$\vec{E} = \hat{x} E_x(z,t)$$
$$\vec{H} = \hat{y} H_y(z,t)$$

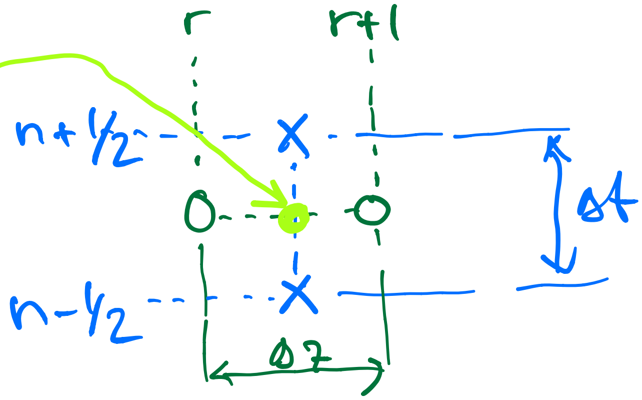
$$\begin{cases} \dfrac{\partial E_x}{\partial z} = -\mu \dfrac{\partial H_y}{\partial t} \\ -\dfrac{\partial H_y}{\partial z} = \epsilon \dfrac{\partial E_x}{\partial t} \end{cases} \Rightarrow \begin{cases} \dfrac{\partial H_y}{\partial t} = -\dfrac{1}{\mu} \dfrac{\partial E_x}{\partial z} \\ \dfrac{\partial E_x}{\partial t} = -\dfrac{1}{\epsilon} \dfrac{\partial H_y}{\partial z} \end{cases}$$



$$o = E_x\Big|_r^n = E_x(r\Delta z, n\Delta t)$$

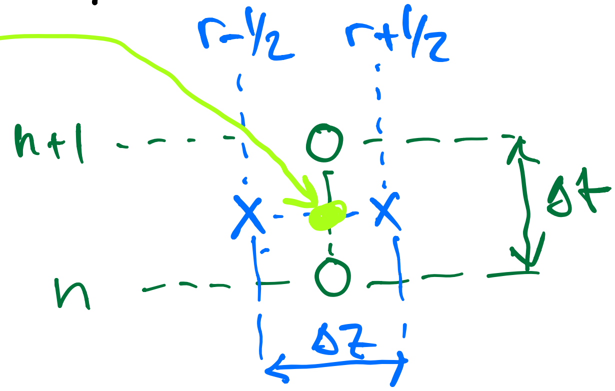
$$\times = H_y\Big|_{r+1/2}^{n+1/2} = H_y\left((r+\tfrac{1}{2})\Delta z, (n+\tfrac{1}{2})\Delta t\right)$$

$$\frac{\partial H_y}{\partial t} = -\frac{1}{\mu}\frac{\partial E_x}{\partial z}$$



$$\Rightarrow \frac{H_y\big|_{r+1/2}^{n+1/2} - H_y\big|_{r+1/2}^{n-1/2}}{\Delta t} = -\frac{1}{\mu}\frac{E_x\big|_{r+1}^{n} - E_x\big|_{r}^{n}}{\Delta z}$$

$$\frac{\partial E_x}{\partial t} = -\frac{1}{\epsilon}\frac{\partial H_y}{\partial z}$$



$$\Rightarrow \frac{E_x\big|_{r}^{n+1} - E_x\big|_{r}^{n}}{\Delta t} = -\frac{1}{\epsilon}\frac{H_y\big|_{r+1/2}^{n+1/2} - H_y\big|_{r-1/2}^{n+1/2}}{\Delta z}$$

Yee-cell