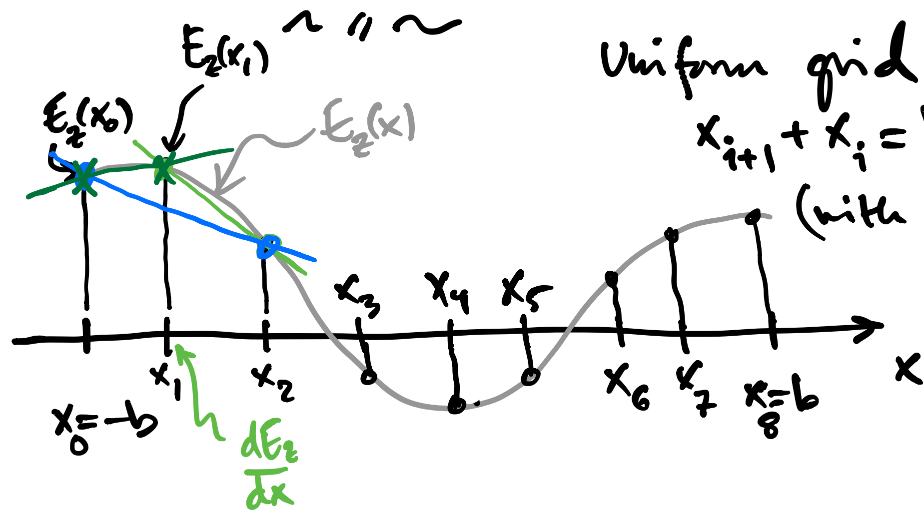
$$\begin{cases} -\dfrac{d^2E_z(x)}{dx^2} + \mu_0\left[j\omega\sigma(x) - \omega^2\epsilon(x)\right]E_z(x) = 0 & \text{(fulfilled for } -b<x<b) \\[2ex] \dfrac{dE_z(x)}{dx} - jk_0 E_z(x) = -2jk_0 E_z^i(x) & \text{(fulfilled for } x=-b) \\[2ex] \dfrac{dE_z(x)}{dx} + jk_0 E_z(x) = 0 & \text{(fulfilled for } x=b) \end{cases}$$



Uniform grid

$x_{i+1} + x_i = h$

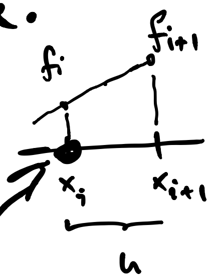
(with $i = 0, \ldots, 7$)

$$\zeta_i = E_z\big|_i = E_z(x_i)$$

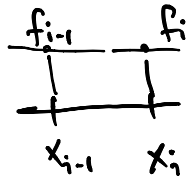
$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

Finite-difference approx.

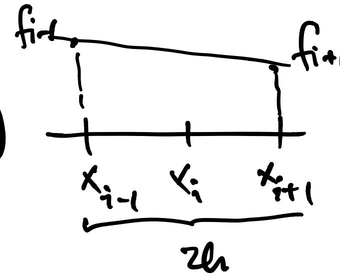
$$f'_i = \left.\frac{df}{dx}\right|_i \simeq \frac{f_{i+1} - f_i}{h} + \mathcal{O}(h)$$

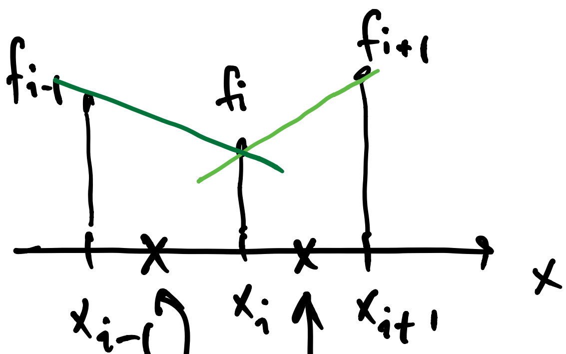


$$f'_i = \left.\frac{df}{dx}\right|_i \simeq \frac{f_i - f_{i-1}}{h} + \mathcal{O}(h)$$



$$f'_i = \left.\frac{df}{dx}\right|_i \simeq \frac{f_{i+1} - f_{i-1}}{2h} + \mathcal{O}(h^2)$$





$$f'_{i+1/2} = \left.\frac{df}{dx}\right|_{i+1/2} \simeq \frac{f_{i+1} - f_i}{h}$$

$$f'_{i-1/2} = \left.\frac{df}{dx}\right|_{i-1/2} \simeq \frac{f_i - f_{i-1}}{h} + \mathcal{O}(h^2)$$

$$f''_i = \frac{d}{dx}\left(\underbrace{\frac{df}{dx}}_{g}\right)\Bigg|_i \simeq \frac{\underbrace{f'_{i+1/2}}_{g_{i+1/2}} - \underbrace{f'_{i-1/2}}_{g_{i-1/2}}}{h}$$

$$= \frac{\frac{f_{i+1}-f_i}{h} - \frac{f_i - f_{i-1}}{h}}{h}$$

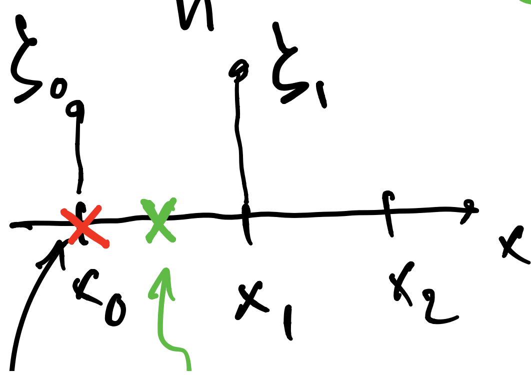
$$= \frac{f_{i+1} - 2f_i + f_{i-1}}{h^2} + O(h^2)$$

$$
\begin{cases}
-\dfrac{d^2 E_z(x)}{dx^2} + \mu_0\left[j\omega\sigma(x) - \omega^2\epsilon(x)\right]E_z(x) = 0 & \text{(fulfilled for } -b<x<b\text{)} \\[2ex]
\dfrac{dE_z(x)}{dx} - jk_0 E_z(x) = -2jk_0 E_z^i(x) & \text{(fulfilled for } x=-b\text{)} \\[2ex]
\dfrac{dE_z(x)}{dx} + jk_0 E_z(x) = 0 & \text{(fulfilled for } x=b\text{)}
\end{cases}
$$

$$
-\frac{\zeta_{i+1} - 2\zeta_i + \zeta_{i-1}}{h^2} + \mu_0\left[j\omega\sigma(x_i) - \omega^2\epsilon(x_i)\right]\zeta_i = 0
$$

$$
i = 1, 2, \dots, 7
$$

$$
\frac{\zeta_1 - \zeta_0}{h} - jk_0\zeta_0 = -2jk_0 E_z^i(x_0)
$$

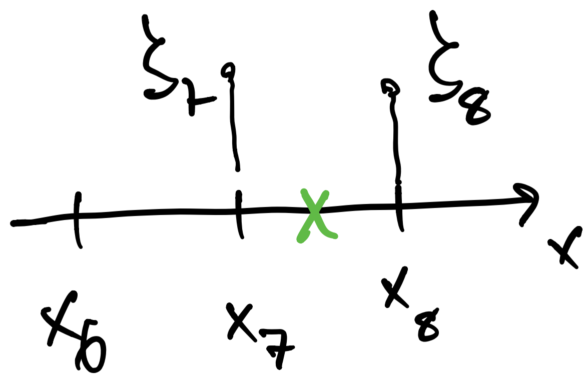


$$
\zeta_0 \rightarrow \frac{\zeta_1 + \zeta_0}{2}
$$

$$
E_z^i(x_0) \rightarrow E_z^i(x_{1/2})
$$

The boundary condition is evaluated at $x = x_0$ $x_{1/2}$

$$\frac{x_1 + x_0}{2}$$



$$\frac{\zeta_8 - \zeta_7}{h} + jk_0 \frac{\zeta_8 + \zeta_7}{2} = 0$$

(MATLAB: "$z = A \backslash b$")

$$\Rightarrow \quad A z = b \quad \Rightarrow \quad z = A^{-1} b$$

$$z = \begin{bmatrix} \zeta_0 \\ \zeta_1 \\ \vdots \\ \zeta_8 \end{bmatrix} \qquad b = \begin{bmatrix} -2jk_0 E_z^i\left(\frac{x_1 + x_0}{2}\right) \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$A = \left[\begin{array}{cccc} -\frac{1}{h} - \frac{jk_0}{2} & \frac{1}{h} - \frac{jk_0}{2} & 0 & \cdots \\ -\frac{1}{h^2} & \frac{2}{h^2} + \mu_0 [z]_1 & -\frac{1}{h^2} & \cdots \\ 0 & -\frac{1}{h^2} & \frac{2}{h^2} + \mu_0 [z]_2 & \end{array}\right.$$