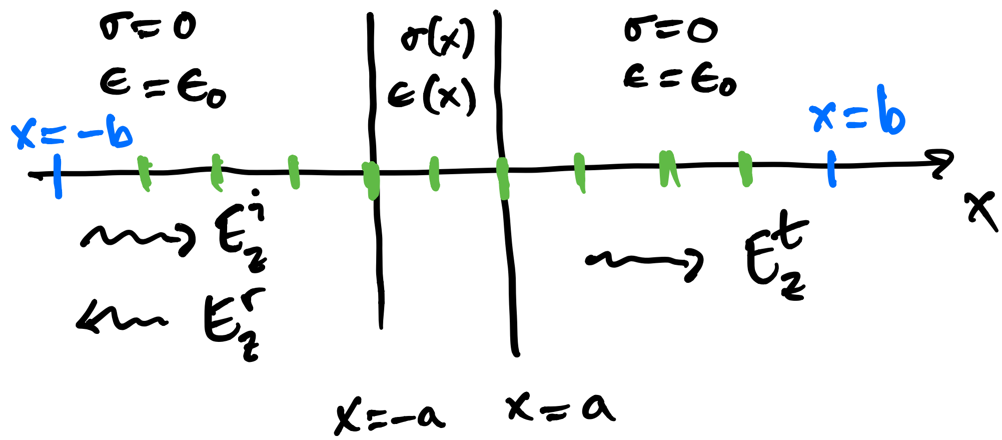

$$-\frac{d^2 E_z(x)}{dx^2} + \mu_0 \left[j\omega\sigma(x) - \omega^2 \epsilon(x) \right] E_z(x) = 0$$

To the left of the slab ($x < -a$), we have $\epsilon = \epsilon_0$ and $\sigma = 0$ and this gives

$k_0^2 = \left(\frac{\omega}{c_0}\right)^2 = \left(\sqrt{\mu_0 \epsilon_0}\,\omega\right)^2$

$$-\frac{d^2 E_z(x)}{dx^2} - \mu_0 \omega^2 \epsilon_0 E_z(x) = 0 \quad (\text{for } x < -a)$$

$$\Rightarrow \frac{d^2 E_z^{(+)}(x)}{dx^2} + k_0^2 E_z^{(+)}(x) = 0$$

This has two solutions

$$E_z^{(+)}(x) = E_0^{(+)} \exp(-jk_0 x) \quad \rightsquigarrow$$

$$E_z^{(-)}(x) = E_0^{(-)} \exp(+jk_0 x) \quad \leftsquigarrow$$

($k_0 = \omega / c_0$ and $c_0 = 1/\sqrt{\epsilon_0 \mu_0}$)

(k_0 = wavenumber = $2\pi/\lambda$

c_0 = speed of light in vacuum)

Check the first solution

$$\frac{d^2 E_z^{(+)}(x)}{dx^2} = \frac{d^2}{dx^2}\left[E_0^{(+)} \exp(-jk_0 x)\right]$$

$$= (-jk_0)^2 \underbrace{E_0^{(+)} \exp(-jk_0 x)}_{= E_z^{(+)}(x)}$$

$$= -k_0^2 E_z^{(+)}(x)$$

which gives

$$\frac{d^2 E_z^{(+)}}{dx^2} + k_0^2 E_z^{(+)} = 0$$

$$\underbrace{-k_0^2 E_z^{(+)}(x) + k_0^2 E_z^{(+)}(x)}_{=0 \text{ (for all } x<-a)} = 0$$

$\Rightarrow E_z^{(+)}(x)$ satisfies the Helmholtz' equation, which corresponds to Maxwell's equation in this problem for $x<-a$.

Do the same check yourself for $E_z^{(-)}(x)$.

" " "

For $x < -a$, we have

$$E_z(x) = E_z^i(x) + E_z^r(x)$$

$$= E_0 \exp(-jk_0 x) + E_r \exp(+jk_0 x)$$

These are given & known

This is not known and must be computed

Construct a boundary condition (to be applied at $x = -b$) by

$$\frac{dE_z(x)}{dx} = \frac{d}{dx}\Bigg[E_0 \exp[-jk_0 x] + E_r \exp[+jk_0 x]\Bigg]$$

$$= -jk_0 \underbrace{E_0 \exp[-jk_0 x]}_{E_z^i(x)} + jk_0 \underbrace{E_r \exp[+jk_0 x]}_{E_z^r(x)}$$

$$= -jk_0 E_z^i(x) + jk_0 E_z^r(x)$$

$$= \left\{ \begin{array}{c} \text{Eliminate } E_z^r(x) \text{ by} \\ E_z^r(x) = E_z(x) - E_z^i(x) \end{array} \right\}$$

$$= -jk_0 E_z^i(x) + jk_0 \left(E_z(x) - E_z^i(x) \right)$$

$$\Rightarrow \frac{dE_z(x)}{dx} - jk_0 E_z(x) = -2jk_0 E_z^i(x)$$

which applies to the left boundary.

To summarize, we have

$$
\begin{cases}
-\dfrac{d^2E_z(x)}{dx^2}+\mu_0\left[j\omega\sigma(x)-\omega^2\epsilon(x)\right]E_z(x)=0 & \text{(fulfilled for } -b<x<b\text{)} \\[2ex]
\dfrac{dE_z(x)}{dx}-jk_0E_z(x)=-2jk_0E_z^i(x) & \text{(fulfilled for } x=-b\text{)} \\[2ex]
\dfrac{dE_z(x)}{dx}+jk_0E_z(x)=0 & \text{(fulfilled for } x=b\text{)}
\end{cases}
$$