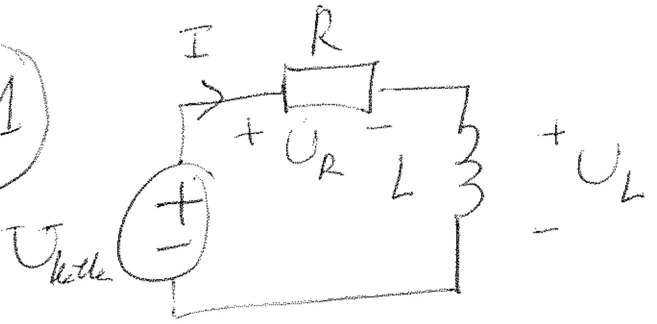
AC 1



$U_{källa} = 10\,V$
$R = 100\,\Omega$
$L = 0{,}050\,H$
$\omega = 2000\,rad/s$

a) Komplexa spänningen över induktorn:

$$U_L = U_{källa} \frac{j\omega L}{j\omega L + R} = 10 \cdot \frac{j\, 2000 \cdot 50 \cdot 10^{-3}}{j\, 2000 \cdot 50 \cdot 10^{-3} + 100} = 10 \frac{j}{j+1}$$

$$= \frac{10\, e^{j90^\circ}}{\sqrt{2}\, e^{j45^\circ}} \approx \underline{\underline{7{,}07 \cdot e^{j45^\circ}}} \text{ volt}$$

Tidsfunktion:

$$u_L(t) = \text{Im}\left\{U_L \cdot e^{j\omega t}\right\} = 7{,}07 \cdot \sin(2000t + 45^\circ)$$

$$\approx \underline{\underline{7{,}1 \cdot \sin(2000t + 45^\circ)}} \text{ volt}$$

b) $P = \frac{1}{2} R |I|^2$ där $|I|^2 = \left|\frac{U_{källa}}{R + j\omega L}\right|^2 = \frac{U_{källa}^2}{R^2 + \omega^2 L^2}$

$= 0{,}005\,A^2$

$\Rightarrow P = \underline{\underline{0{,}25\,W}}$

c) $U_R = U_{källa} \dfrac{R}{R + j\omega L} = \dfrac{10}{1+j} \simeq 7{,}07 \cdot e^{-j\,45°}$ volt

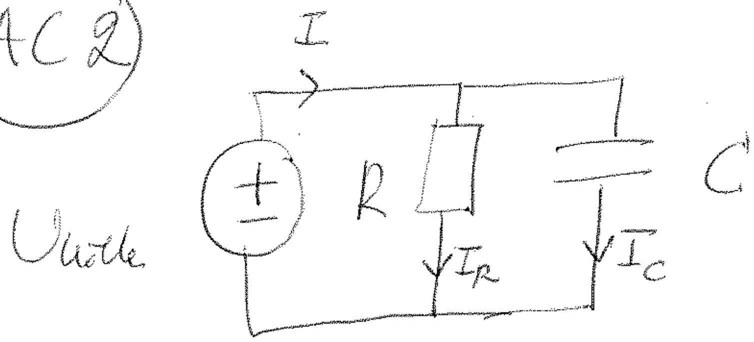
$\Rightarrow \quad \hat{u}_R \simeq 7{,}07$ V dvs. samma som $\hat{u}_L$.

[Alt. lösn. $U_R = I \cdot R$ ...]

$$\hat{u}_R^2 + \hat{u}_L^2 = \hat{u}_{källa}^2$$

d) —

AC 2



$U_{källa} = 10$ V

$R = 50\ \Omega$

$C = 5{,}0\ \mu F$

$\omega = 2000$ rad/s

a) $Y = \frac{1}{R} + j\omega C = \frac{1}{50} + j2\cdot 10^3\cdot 5\cdot 10^{-6} = \frac{2+j}{100} \approx$

$\approx \underline{0{,}0224\cdot e^{j26{,}6^\circ}}$ S

$I = U_{källa}\cdot Y \approx 0{,}224\, e^{j26{,}6^\circ}$ ampere

$i(t) = \text{Im}(I\, e^{j\omega t}) \approx 0{,}224\cdot \sin(2000t + 26{,}6^\circ)$ ampere

$\approx \underline{0{,}22\cdot \sin(2000t + 26{,}6^\circ)}$ ampere

b) $I_R = \frac{U_{källa}}{R} = 0{,}2$ A $\Rightarrow$ $\hat{i}_R = 0{,}2$ A

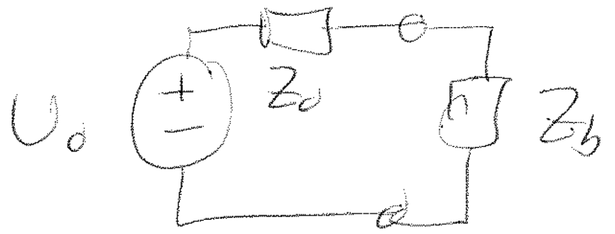
$I_c = U_{källa}\cdot j\omega C = j\, 10\cdot 2\cdot 10^3\cdot 5\cdot 10^{-6} = 0{,}1\cdot e^{j90^\circ}$ A

$\Rightarrow$ $\hat{i}_c = 0{,}1$ A

$\hat{i}_R^{\,2} + \hat{i}_C^{\,2} = \hat{i}^{\,2}$

c) —

AC4



$U_0 = 5{,}0$ V
$\omega = 5000$ rad/s
$Z_0 = 50 - j20\ \Omega$

a) $Z_b = \bar{Z}_0 = 50 + j20\ \Omega$

$\Rightarrow Z_{tot} = Z_0 + Z_b = 100\ \Omega$; $P_b = \frac{1}{2} Re\{\overbrace{Z_b I}^{\text{Spänning över last}} \bar{I}\}$

$\Rightarrow P = \frac{1}{2} R_b |I|^2 = \frac{1}{2} R_b \left|\frac{U_0}{Z_{tot}}\right|^2 = \frac{1}{2} 50 \left(\frac{5}{100}\right)^2$

R_b: realdelen av Z_b

$= \underline{\underline{0{,}0625\ W}}$

b) $Z_b = |Z_0| = \sqrt{50^2 + (-20)^2} \simeq 53{,}85 \simeq \underline{\underline{54\ \Omega}}$

$P_b = \frac{1}{2} R_b |I|^2 = \frac{1}{2} R_b \left|\frac{U_0}{Z_0 + Z_b}\right|^2 = \frac{53{,}85}{2} \frac{5^2}{((50+53{,}85)^2 + (20)^2)}$

R_b: nu $= Z_b$

$\simeq 0{,}0602 \simeq \underline{\underline{0{,}06\ W}}$

c) $\left.\begin{array}{l} Z_b = |Z_b| e^{j45°} \\ |Z_b| = |Z_0| \simeq 53{,}85\ \Omega \end{array}\right\} \Rightarrow Z_b \simeq 53{,}85\, e^{j45°} =$

$= \underline{\frac{53{,}85 (1+j)}{\sqrt{2}}}\ \Omega$

$\Rightarrow \quad Z_b \simeq 38{,}08\,(1+j) \simeq \underline{\underline{38\,(1+j)}}\ \Omega$

$$P_b = \frac{1}{2} R_b |I|^2 = \frac{38{,}08}{2} \cdot \frac{5^2}{(50+38{,}08)^2 + (-20+38{,}08)^2}$$

$$\simeq 0{,}0589 \simeq \underline{\underline{0{,}059\ W}}$$

d) $Z_b = R_b + j40\ \Omega$ (ej fix fas $\Rightarrow |Z_b| = |Z_0|$ kan ej anv.)

Se $j40\,\Omega$ som del av $Z_0 \Rightarrow$

$R_b = |50 - j20 + j40| \simeq 53{,}85\ \Omega \simeq \underline{\underline{54\ \Omega}}$

$$P_b = \frac{1}{2} R_b \left|\frac{U_{källa}}{Z_0 + Z_b}\right|^2 \simeq 0{,}0602\ W \simeq \underline{\underline{0{,}06\ W}}$$

e) $Z_b = 100 + jX_b\ \Omega$

P_b maximeras om strömmen i kretsen maximeras

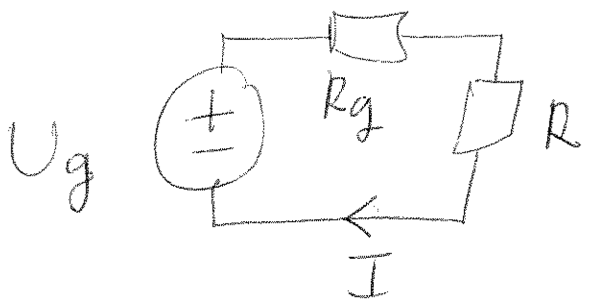
dvs Z_{total} = reell i vårt fall

$\Rightarrow$ Välj $jX_b = -jX_0 = -(-j20) = \underline{\underline{j20}}\ \Omega$

$$P_b = \frac{1}{2} R_b \left(\frac{U_{källa}}{R_0 + R_b}\right)^2 \simeq 0{,}0556 \simeq \underline{\underline{0{,}056\ W}}$$

AC15

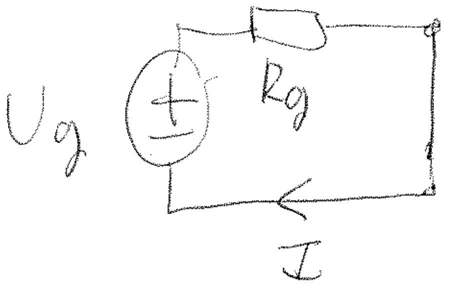
$\omega \to 0 \Rightarrow$ kretsen kan approximeras med:



("kondensator avbrott")
("induktor kortslutn.")

$$\Rightarrow \quad |I| = \frac{|U_g|}{R_g + R}$$

$\omega \to \infty \Rightarrow$ kretsen kan approximeras med:

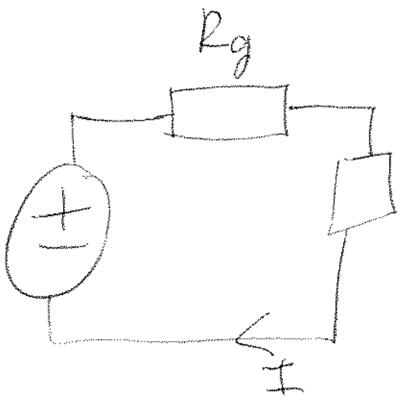


$$|I| = \frac{|U_g|}{R_g}$$

("kondensator kortsluten")
("induktor avbrott")

Med dessa uttryck kan deluppg. a, b, d, e lösas

Vid resonans (Z_{total} = reell) av kretsen:



← realdelen av parallellkopplingen av $\frac{1}{j\omega C}$ & $R + j\omega L$

$$Z_{tot} = R_g + \frac{(R + j\omega L)\frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}} = \ldots =$$

$$= R_g + \frac{R\cdot(1-\omega^2 LC) + \omega^2 LCR + j\left[\omega L(1-\omega^2 LC) - \omega CR^2\right]}{(1-\omega^2 LC)^2 + (\omega RC)^2}$$

$$\mathrm{Im}\, Z_{tot} = 0 \Rightarrow \omega_r L(1-\omega_r^2 LC) - \omega_r CR^2 = 0$$

$$\Rightarrow \begin{cases} \omega_r = 0 & \text{(ingen oscillation)} \\ L(1-\omega_r^2 LC) = CR^2 \end{cases}$$

$$\Rightarrow \omega_r^2 = \frac{1}{LC}\left(1 - \frac{R^2 C}{L}\right) = \omega_0^2\left(1 - \frac{R^2 C}{L}\right)$$

$$\Rightarrow \boxed{\omega_r = \omega_0\sqrt{1 - \frac{R^2 C}{L}}}$$

(om $R > \sqrt{\frac{L}{C}}$ så blir ω_r imaginär ⇒ dämpning, ingen oscillation)

Nu har vi fått fram resonansvinkelfrekv. ω_r

$$\omega = \omega_r \Rightarrow Z_{tot} = R_g + \frac{L}{C} \cdot \frac{1}{R}$$

$$\Rightarrow |I| = \frac{|U_g|}{R_g + \frac{L}{CR}}$$ ⇒ löser deluppg. c, g, i

Kvarstår att bestämma strömmen då $\omega = \omega_0 = \frac{1}{\sqrt{LC}}$.

Ins. av $\omega = \omega_0$ i Z_{tot} på föregående sida ger:

$$Z_{tot} = R_g + \frac{1 - jR\sqrt{\frac{C}{L}}}{\frac{RC}{L}}$$

$$\Rightarrow |I| = \frac{\cancel{|U_g|}\ |U_g|}{\left|R_g + \frac{1-jR\sqrt{\frac{C}{L}}}{\frac{RC}{L}}\right|} = \frac{|U_g|}{\sqrt{\left(R_g + \frac{L}{RC}\right)^2 + \left(-\sqrt{\frac{L}{C}}\right)^2}}$$

⇒ löser deluppg. c, f, h.