ECE 134 Final

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Definitions

Q (Charge) [C]

$\vec{E}$ (Electric Field) $[\frac{N}{C}]$ or $[\frac{V}{m}]$

$\vec{D}$ (Electric Flux Density) $[\frac{C}{m^2}]$

$\rho_{l,s,v}$ (Charge Density) $[\frac{C}{m}]$ (ρ_l) or $[\frac{C}{m^2}]$ (ρ_s) or $[\frac{C}{m^3}]$ (ρ_v)

Φ (Electric Potential) [V] or $[\frac{J}{C}]$

$\vec{J}$ (Current Density) $[\frac{A}{m^2}]$

C (Capacitance) [F]

U_E (Electric Potential Energy) [J]

$\vec{B}$ (Magnetic Field) [T] $= [\frac{N}{m \cdot A}] = [\frac{kg}{A \cdot s^2}]$ or [G]

↪ *($1T = 10^4\ G$)*

L (Inductance) [H] $= [\frac{V \cdot s}{A}]$

Φ_B (Magnetic Flux) [Wb]

Constants

$\epsilon_o = 8.85 \times 10^{-12}$ $[\frac{F}{m}]$ (Permittivity of Free Space)

$\mu_o = 4\pi \times 10^{-7}$ $[\frac{H}{m}]$ (Permeability of Free Space)

$\sigma_{SB} = 5.6703 \times 10^{-8}$ $[\frac{W}{m^2 K^4}]$ (Boltzmann's Constant)

$Q_{e^-} = -1.60217662 \times 10^{-19}$ [C] (Elementary Charge)

$m_{e^-} = 9.11 \times 10^{-31} [kg]$ (Mass of an electron)

$c = 3 \times 10^8 [\frac{m}{s}]$ (Universal Speed Limit)

$\eta_o = \sqrt{\frac{\mu_o}{\epsilon_o}} = 377 = 120\pi[\Omega]$ (Impedance of Free Space)

Vector Calculus

Gradient: $\nabla\Phi$

Cartesian: $\frac{\partial\Phi}{\partial x}\hat{x} + \frac{\partial\Phi}{\partial y}\hat{y} + \frac{\partial\Phi}{\partial z}\hat{z}$

Cylindrical: $\frac{\partial\Phi}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial\Phi}{\partial\phi}\hat{\phi} + \frac{\partial\Phi}{\partial z}\hat{z}$

Spherical: $\frac{\partial\Phi}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial\Phi}{\partial\theta}\hat{\theta} + \frac{1}{r\sin(\theta)}\frac{\partial\Phi}{\partial z}\hat{\phi}$

Divergence: $\nabla \cdot \vec{A}$

Cartesian: $\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$

Cylindrical: $\frac{1}{r}\frac{\partial(rA_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}$

Spherical: $\frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(A_\theta\sin\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$

Curl: $\nabla \times \vec{A}$

Cartesian: $\hat{x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$

Cylindrical:

$\hat{r}\left(\frac{1}{r}\frac{\partial A_z}{\partial\phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\phi}\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \hat{z}\frac{1}{r}\left(\frac{\partial(rA_\phi)}{\partial r} - \frac{\partial A_r}{\partial\phi}\right)$

Spherical:

$\frac{\hat{r}}{r\sin\theta}\left[\frac{\partial(A_\phi\sin\theta)}{\partial\theta} - \frac{\partial A_\theta}{\partial\phi}\right] + \frac{\hat{\theta}}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial(rA_\phi)}{\partial r}\right] + \frac{\hat{\phi}}{r}\left[\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial\theta}\right]$

Laplacian: $\nabla^2\Phi$

Cartesian: $\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2\Phi}{\partial z^2}$

Cylindrical: $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\Phi}{\partial\phi^2} + \frac{\partial^2\Phi}{\partial z^2}$

Spherical: $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Phi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\Phi}{\partial\phi^2}$

Integrals

$\int_0^c \frac{dx}{a + \frac{b-a}{c}x} = \frac{c\ln(\frac{b}{a})}{b-a}$

$\frac{\partial}{\partial b}\frac{1}{\ln\frac{b}{a}} = -\frac{1}{b(\ln b - \ln a)^2}$

Stupid Stuff I Sometimes Forget

Surface area of a sphere: $4\pi r^2$

Volume of a sphere: $\frac{4}{3}\pi r^3$

Surface area of a cylinder: $2\pi rl$

E field from a point charge: $\vec{E} = \frac{q}{4\pi\epsilon_o r^2}\hat{r}$

Potential from a point charge: $\Phi = \frac{q}{4\pi\epsilon_o r}$

How to Get Basic Stuff

Charge

$Q = \iiint \rho(x,y,z)dV$

Electric Field

$\vec{D} = \epsilon\vec{E}$

Gauss' Law:

$\oiint_S \vec{E} \cdot d\vec{S} = \frac{Q}{\epsilon}$ (Integral Form)

$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon}$ (Differential Form)

$\vec{E} = -\nabla\Phi$

$\vec{E}(x,y,z) = \iiint \frac{\rho(x',y',z')}{4\pi\epsilon_o R^2}dV$

Dielectric Strength: $\vec{E}_{breakdown}$ $[\frac{V}{m}]$

Electric Potential

$\Phi = -\int \vec{E} \cdot d\vec{l}$

$\nabla^2\Phi = -\frac{\rho}{\epsilon}$ (Poisson's Equation)

General Form: ↪ $\nabla \cdot (\epsilon\nabla\Phi) = -\rho$ (works for non-constant ϵ)

Potential Energy

From a charge distribution:

$U_E = \frac{1}{2}\iiint \rho(\vec{r})\Phi(\vec{r})dV$

$U_E = \frac{1}{2}\iiint \epsilon|\vec{E}|^2 dV$

Energy of a sphere of charge:

$U_E = \frac{4\pi\rho^2 b^5}{15\epsilon_o}$

Power

$P_E = \iiint \vec{J} \cdot \vec{E}dV = VI = \frac{V^2}{R} = I^2 R$

Electric Force

$\vec{F_E} = q\vec{E}$

In terms of energy: $\vec{F} = \pm\frac{\partial}{\partial l}(U_E(l))\hat{l}$

Capacitance

$C = \frac{Q}{V}$

$U_c = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$

$C_{coax.} = \frac{2\pi\epsilon L}{\ln\frac{b}{a}}$

Parallel Plate (Special Case)

E $= \frac{\rho_s}{\epsilon} = \frac{V}{d}$

C $= \frac{\epsilon A}{d}$ where $\epsilon = \epsilon_r\epsilon_o$

Boundary Conditions

Surface of a Conductor

$\hat{n} \cdot \vec{E}_{surface} = \frac{\rho_s}{\epsilon}$

$\hat{n} \times \vec{E}_{surface} = 0$

Expressed in terms of potential...

$-\frac{\partial\Phi}{\partial\hat{n}} = \frac{\rho_s}{\epsilon}$

Φ = Constant

Dielectric Boundary

$\hat{n} \cdot \vec{E}_1\epsilon_1 - \hat{n} \cdot \vec{E}_2\epsilon_2 = \rho_s$

$\hat{n} \times \vec{E}_1 = \hat{n} \times \vec{E}_2$

Expressed in terms of potential...

$\epsilon_1\frac{\partial\Phi_1}{\partial n} - \epsilon_2\frac{\partial\Phi_2}{\partial n} = \rho_s$

$\hat{n} \times \nabla\Phi_1\big|_{surface} = \hat{n} \times \nabla\Phi_2\big|_{surface}$

Conductors, Current, and Resistance

Current: $I = \iint \vec{J} \cdot d\vec{S}$

Ohm's Law: $\vec{J} = \sigma\vec{E}$

For Moving Charges: $\vec{J} = \rho\vec{v}$

↪ *ρ is charge density*

Conductivity : σ $[\frac{S}{m}]$

Resistivity : ρ $[\Omega \cdot \mathrm{m}]$

Resistance: $R = \frac{1}{\sigma}\frac{l}{A} = \rho\frac{l}{A}$

↪ *(l is in the direction of current flow)*

↪ *(A is the cross-section which current is flowing through)*

Drift Velocity: $\vec{v}_{drift} = \mu\vec{E}$

↪ *(μ is the electron mobility of a material)*

Sheet Resistors

↪ *Typically have a length (l), width (w) and thickness (t)*

Resistance: $R = \frac{1}{\sigma}\frac{l}{A} = \frac{1}{\sigma}\frac{l}{w \cdot t} = r_{sh}\frac{l}{w}$

↪ $r_{sh} = \frac{1}{\sigma t}$

Series of sheet resistors: R $= r_{sh}(\frac{l}{w} - 0.44N_{corners})$

Heat Transfer

Heat Capacity: $C_p\ [\frac{J}{K}]$

Specific Heat Capacity: $C_{sp} = \frac{C_p}{mass}\ [\frac{J}{gK}]$

$\Delta U_{heat} = C_p \Delta T$

Resistivity w/ Temperature: $\rho(T) = \rho_o[1 + \alpha_{TCR}(T - T_o)]$

$\hookrightarrow$ *ρ_o = resistivity at room temperature*

$\hookrightarrow$ *α_{TCR} = temperature coefficient of resistance*

Methods of Heat Transfer

Energy Balance: $P_{in} = P_{stored} + P_{cond} + P_{conv} + P_{rad}$

$P_{stored} = C_h \frac{dT}{dt}$ (Zero for steady state!!!)

Conduction: $P_{cond} = \frac{T_1 - T_o}{\theta_{th}}$

Convection: $P_{conv} = hA_s(T - T_o)$

$\hookrightarrow$ *h = convection coefficient*

$\hookrightarrow$ *A_s = surface area*

Steady State: $\Delta T_\infty = \frac{I^2 R}{hA_s}$

Radiation: $P_{rad} = e\sigma_{SB}A_s(T^4 - T_o^4)$

$\hookrightarrow$ *e = emissivity ($0 < e < 1$)*

Elementary Magnetostatics

Ampère's Law:

$\int \vec{B} \cdot d\vec{S} = \mu_o I_{inside}$ (Integral form)

$\nabla \times \vec{B} = \mu_o \vec{J}$ (Differential Form)

Magnetic Field Strength (H): $\vec{B} = \mu \vec{H}$

Force on a wire: $\vec{F_B} = I\vec{l} \times \vec{B}$

Lorentz's Force Law: $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

$\hookrightarrow \vec{F}_B = q\vec{v} \times \vec{B}$

Magnetic Fields from Different Objects

Field from a wire: $B = \frac{\mu_o I}{2\pi r}$

Field inside a solenoid: $B = \mu n I$

$\hookrightarrow$ *n = turn density = $\frac{N}{l}$*

Field inside a toroid: $B = \frac{\mu N I}{2\pi r}$

Field from an infinite current sheet: $B = \frac{\mu_o J}{2}$

Vector Potential ($\vec{A}$)

$\nabla^2 \vec{A} = -\mu_o \vec{J}$

$\vec{A}(\vec{r}) = \frac{\mu_o}{4\pi} \iiint \frac{\vec{J}(\vec{r})}{R} dV'$

$\hookrightarrow R = \vec{r} - \vec{r}\,'$

Faraday's Law and Induction

Magnetic Flux: $\Phi_B = \iint \vec{B} \cdot d\vec{S}$

Faraday's Law: $V_{emf} = -\frac{d\Phi_B}{dt}$

$\hookrightarrow$ *For EMF induced in a coil: $V_{emf} = -N\frac{d\Phi_B}{dt}$*

Inductance

In general...

$L = \frac{N\Phi_B}{I}$ [H]

$\hookrightarrow$ *Sanity Check: L should have a factor of N^2*

Magnetic Energy from Inductance: $U_B = \frac{1}{2}LI^2$

Magnetic Force: $F_B = \pm\frac{\partial}{\partial l}(U_B(l))\hat{l}$

For a 2-circuit system (Mutual Inductance):

Flux from Ckt 1 in Ckt 2: $\Phi_{21} = \iint \vec{B}_1 \cdot d\vec{S}_2$

Induced voltage in Ckt 2: $V_{emf} = \frac{-d\Phi_{21}}{dt} = L_{21}\frac{dI_1}{dt}$

Mutual Inductance: $L_{21} = \frac{\Phi_{21}}{I_1}$

Self-Inductance:

Flux from Ckt 1 in Ckt 1: $\Phi_{11} = \iint \vec{B}_1 \cdot d\vec{S}_1$

Self-Inductance: $L_{11} = \frac{\Phi_{11}}{I_1}$

In general...

$L_{21} = L_{12}$, but $L_{11} \neq L_{22}$

We must include both mutual and self-inductance terms!

$V_1 = L_{11}\frac{dI_1}{dt} + L_{12}\frac{dI_2}{dt}$

$V_2 = L_{22}\frac{dI_2}{dt} + L_{21}\frac{dI_1}{dt}$

Magnetic Flux Circuits

Analogous to Resistive Circuits!

For an N-turn Coil On a High-μ Core...

$V = NI$

$R = \mathcal{R} = \mu\frac{l}{A}$ (Reluctance)

$\hookrightarrow$ *(l is in the direction of flux flow)*

$\hookrightarrow$ *(A is the cross-section which flux is flowing through)*

$I = \Phi_B = \frac{NI}{\mathcal{R}}$

Ideal Transformers (Perfect Flux Sharing)

Voltage and Turns: $\frac{V_p}{V_s} = \frac{N_p}{N_s}$

$\hookrightarrow$ *(p = primary, s = secondary)*

Current and Turns: $N_pI_p = N_sI_s$

Phasors

$f(t) = A\cos(\omega t + \phi) \Longrightarrow F = Ae^{j\phi}$

$f(t) = A\sin(\omega t + \phi) \Longrightarrow F = -jAe^{j\phi}$

Euler's Identity: $e^{j\theta} = \cos\theta + j\sin\theta$

$\Re[e^{jx}] = \cos x$

$\Im[e^{jx}] = \sin x$

Plane Waves

Source-Free Wave Equations: $\nabla^2\vec{E} + k_o^2\vec{E} = 0$ & $\nabla^2\vec{H} + k_o^2\vec{H} = 0$

Solutions are linear combinations of:

$\vec{E}/\vec{H} = \vec{E}_o^+/\vec{H}_o^+ e^{-j\vec{k}\cdot\vec{r}}$ (Forward Propagating Wave)

$\vec{E}/\vec{H} = \vec{E}_o^-/\vec{H}_o^- e^{+j\vec{k}\cdot\vec{r}}$ (Reverse Propagating Wave)

$\hookrightarrow$ *$\vec{k}$ points in direction of wave propagation ($k_x\hat{x} + k_y\hat{y} + k_z\hat{z}$)*

$\hookrightarrow$ *$\vec{r}$ is a generic position vector ($x\hat{x} + y\hat{y} + z\hat{z}$)*

$\hookrightarrow$ *e.g. for a wave moving in the $+\hat{z}$ direction, $\vec{k}\cdot\vec{r} = kz$*

General form of an EM Wave: $H_o/E_o\cos/\sin(\omega t \pm k/\beta z + \phi)$

Typical Parameters of Plane Waves

Angular Frequency: $\omega = 2\pi f\ [\frac{rad}{s}]$

Wavenumber: $k/\beta = \omega\sqrt{\mu\epsilon} = \frac{\omega}{v} = \frac{2\pi}{\lambda}$

$\hookrightarrow$ Free Space Wavenumber: $k_o = \omega\sqrt{\mu_o\epsilon_o} = \frac{\omega}{c} = \frac{2\pi}{\lambda_o}$

Impedance: $\eta = \sqrt{\frac{\mu_o}{\epsilon_o\epsilon_r}} = \eta_o\frac{1}{\sqrt{\epsilon_r}}$

$\hookrightarrow$ Impedance of Free Space $= \eta_o = \sqrt{\frac{\mu_o}{\epsilon_o}} = 377\Omega = 120\pi$

To go from H to E: $\vec{E} = -\eta(\hat{a}_n \times \vec{H})$

To go from E to H: $\vec{H} = \frac{1}{\eta}(\hat{a}_n \times \vec{E})$

$\hookrightarrow$ *$\hat{a}_n$ is a unit vector in the direction of propagation*

$\hookrightarrow$ *$\vec{E}$ and $\vec{H}$ point in the direction of polarization*

Propagation Through Lossy Media

General form for an attenuated wave: $E_x = E_oe^{-\alpha z}e^{-j\beta z}$

$\hookrightarrow$ *wave propagating in $+\hat{z}$ direction*

$\hookrightarrow$ *wave polarized in $\hat{x}$ direction*

Attenuation factor: $e^{-\alpha z}$

$\hookrightarrow$ *how much the amplitude has shrunk through distance z*

Phase Constant : β (similar to k)

$\hookrightarrow$ *tells us how much phase changes as wave propagates*

Low-Loss Medium (Dielectric): $\tan\delta = \frac{\sigma}{\omega\epsilon} << 1$

Attenuation Constant: $\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}\ [\frac{Np}{m}]$

$\hookrightarrow 1\frac{Np}{m} = 8.686\frac{dB}{m}$

Phase Constant: $\beta = \omega\sqrt{\mu\epsilon}$

Phase Velocity: $v_p = \frac{\omega}{\beta}$

Intrinsic Impedance: $\eta_c = \sqrt{\frac{\mu}{\epsilon}}(1 + j\frac{\tan\delta}{2})$

Skin Depth: $\delta = \frac{1}{\alpha}$ [m]

Lossy Medium (Good Conductor): $\tan\delta = \frac{\sigma}{\omega\epsilon} >> 1$

Attenuation and Phase Constant: $\alpha = \beta = \sqrt{\pi f\mu\sigma}$

Phase Velocity: $v_p = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{\mu\sigma}}$

Wavelength: $\lambda = \frac{2\pi}{\beta} = \frac{v_p}{f} = 2\sqrt{\frac{\pi}{f\mu\sigma}}$

Intrinsic Impedance: $\eta_c = (1 + j)\frac{\alpha}{\sigma}$

Skin Depth: $\delta = \frac{1}{\alpha} = \frac{1}{\beta} = \frac{\lambda}{2\pi}$ [m]