

Introduction to on-chip interconnect

Elmore delay model

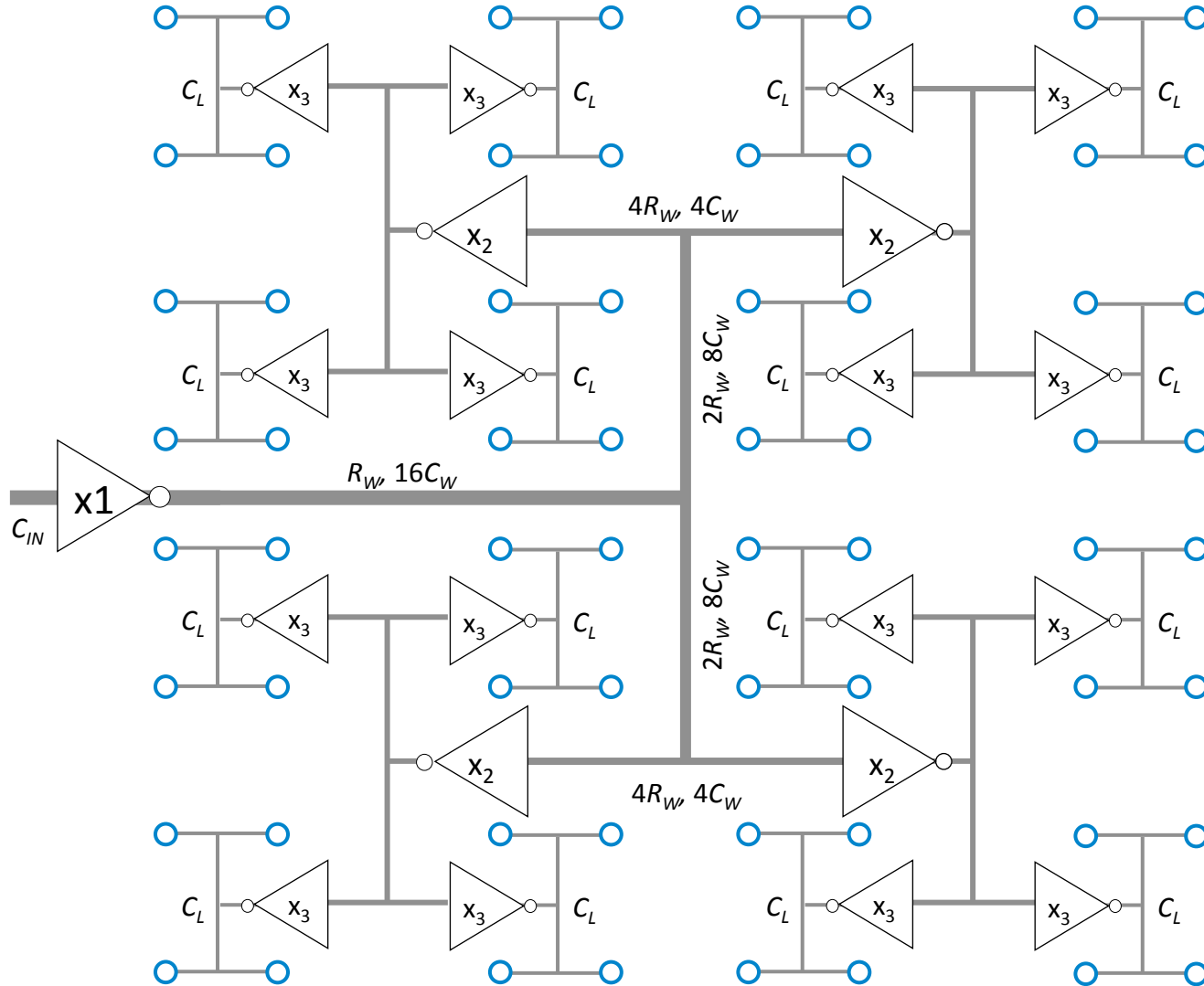
Lecture 7 (or really 9) continued

Tuesday October 2, 2018

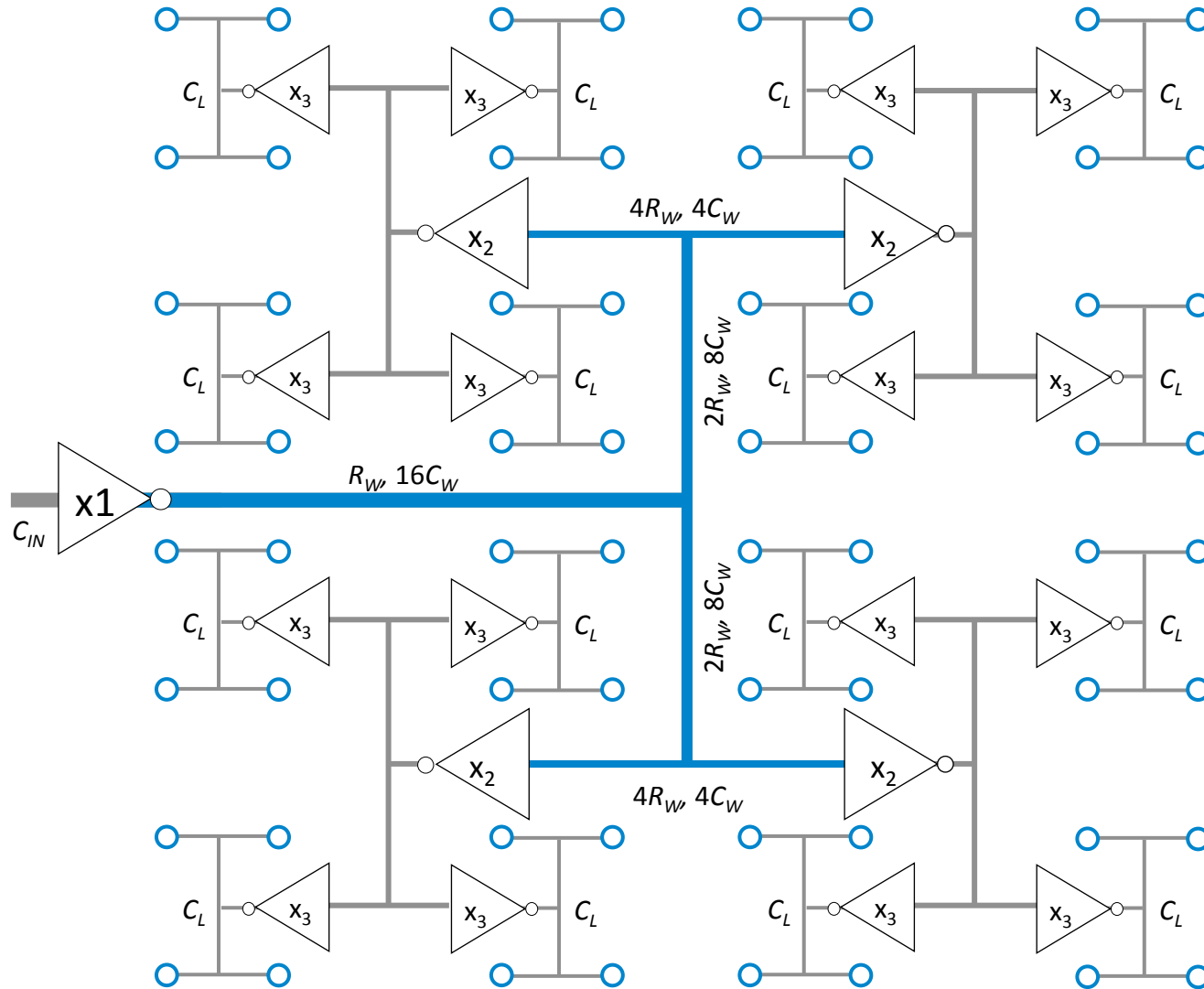
Outline

- Elmore delay model – a generalized model
- How to handle wire branches
- Conclusions

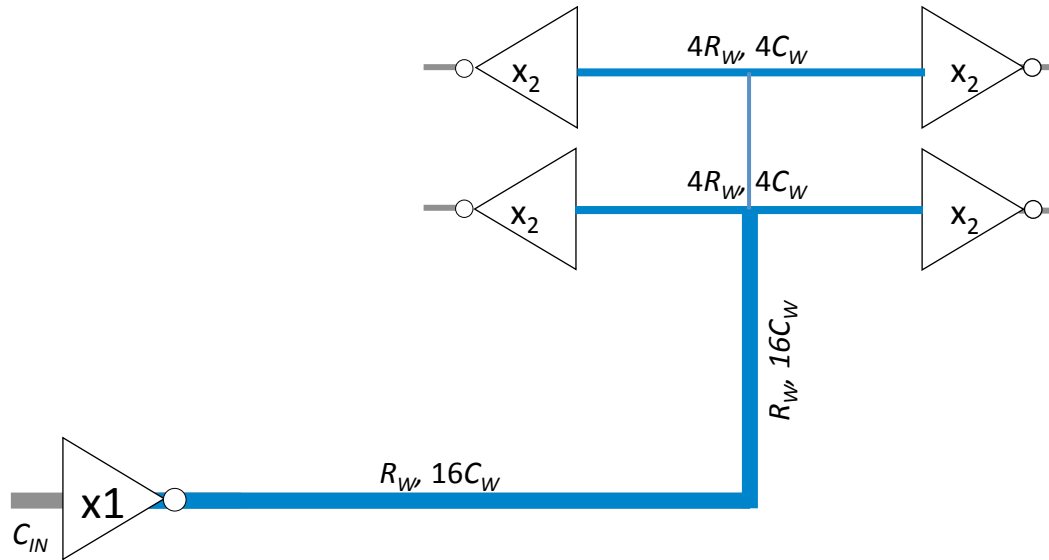
H-tree clock distribution



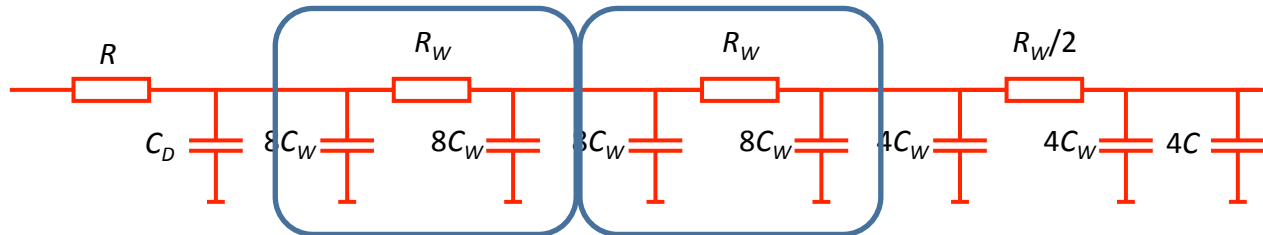
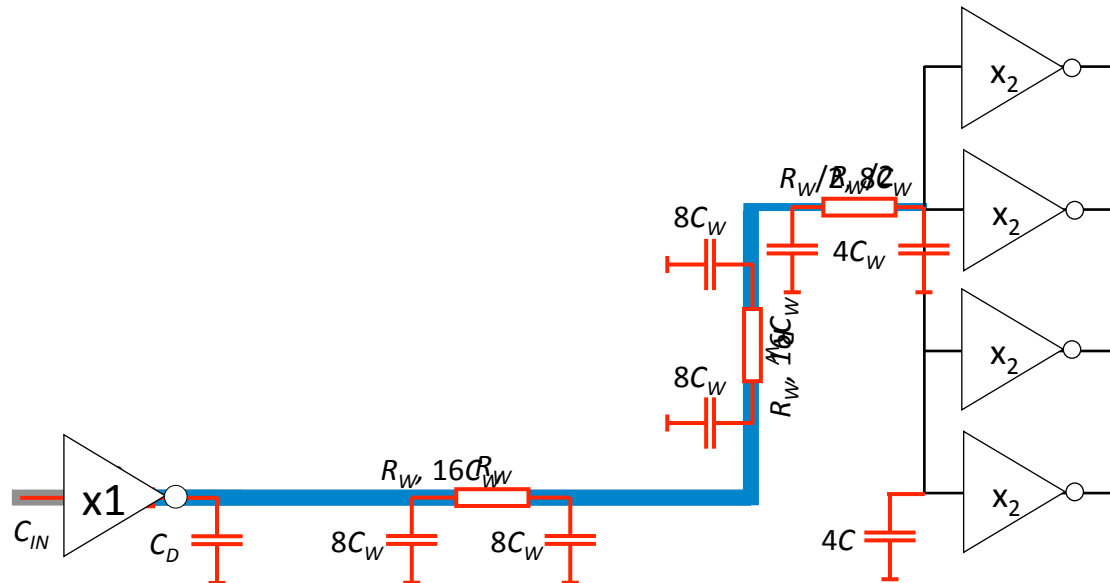
Identify the critical timing path



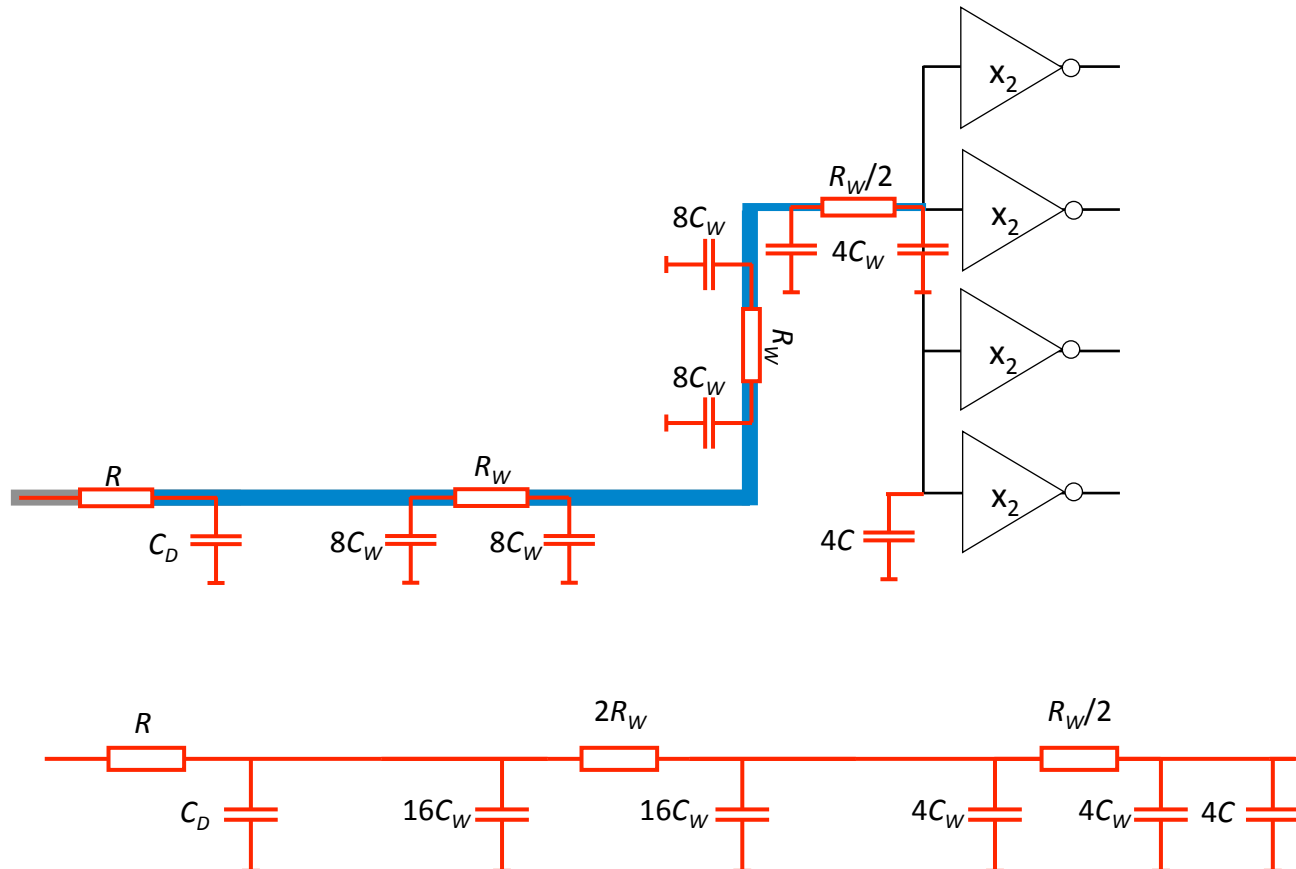
Identify the critical timing path



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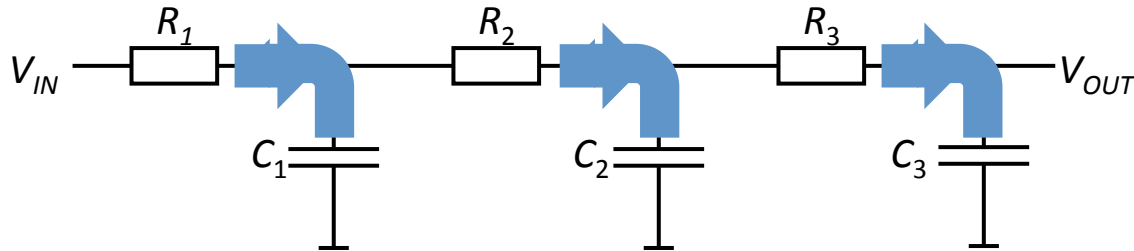
Identify the critical timing path



General solution

Each capacitance is multiplied by its upstream resistance!

$$T_E = R_1(R_1 + R_2 + R_3) + R_2(R_2 + R_3) + R_3R_3$$



Transfer function

$$H(s) = \frac{1}{as^3 + bs^2 + cs + 1}$$

$$a = R_1R_2R_3C_1C_2C_3$$

$$b = R_1R_2C_1C_2 + R_1R_2C_1C_3 + R_1R_3C_2C_3 + R_2R_3C_2C_3$$

$$c = R_1(C_1 + C_2 + C_3) + R_2(C_2 + C_3) + R_3C_3$$

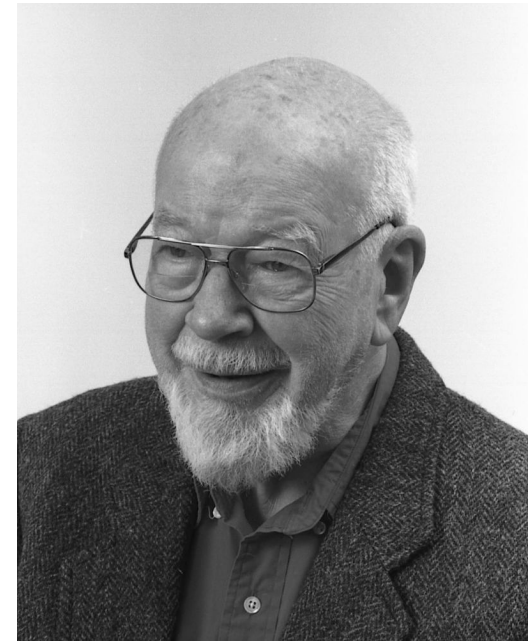
This transfer function corresponds to a third order linear differential equation
The solution is a sum of three exponentials with three different time constants
We cannot solve this analytically.

But if we assume that there is a dominant time constant T_E it is given by c

$$T_E = R_1(C_1 + C_2 + C_3) + R_2(C_2 + C_3) + R_3C_3$$

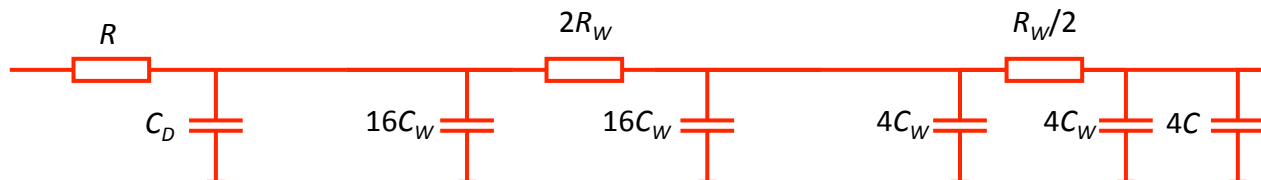
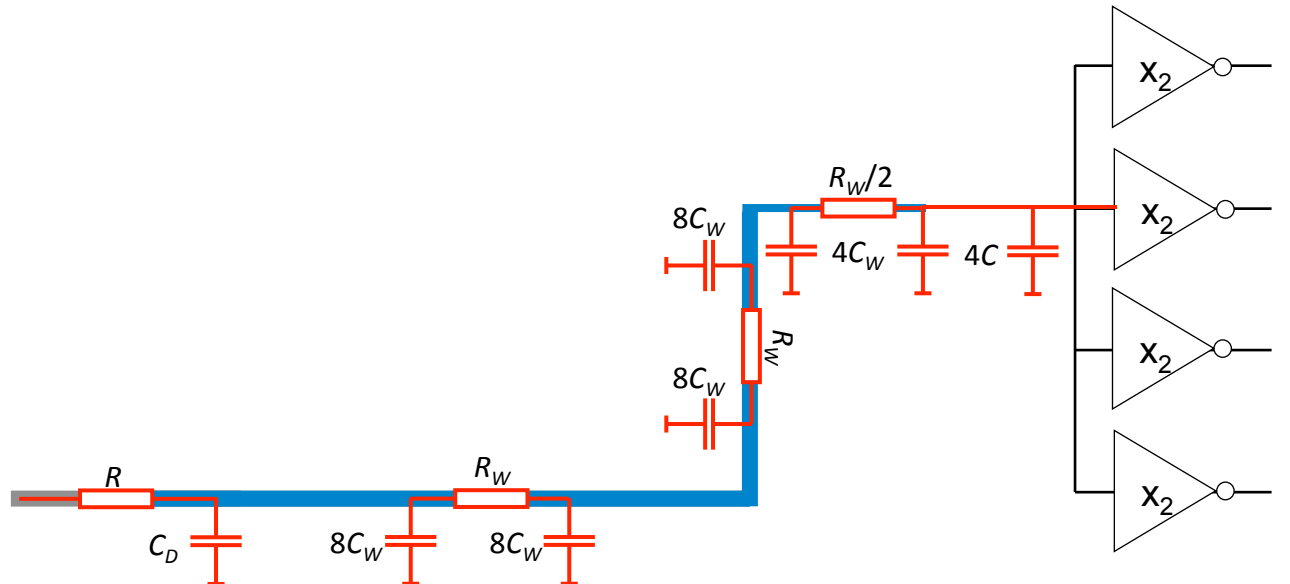
The man behind Elmore delay

- William Cronk Elmore (1909 - 2003) was an American physicist, educator, and author.
- He is best known for his work on and related to the Manhattan project during World War II.
- Professor of Physics at Swarthmore College, Pennsylvania, from 1938 to 1974.
- Authored two influential books during his life,
 - Electronics-Experimental Techniques with M. Sands
 - Physics of Waves with Mark Heald.
- He is also known for deriving a simple approximation for the delay through an RC network, known as the Elmore delay.
- Despite his clear potential for advancing theoretical and experimental physics, Elmore was known for developing (and publishing) laboratory experiments that effectively taught students the fundamentals of physics.



W. C. Elmore

Identify the critical timing path

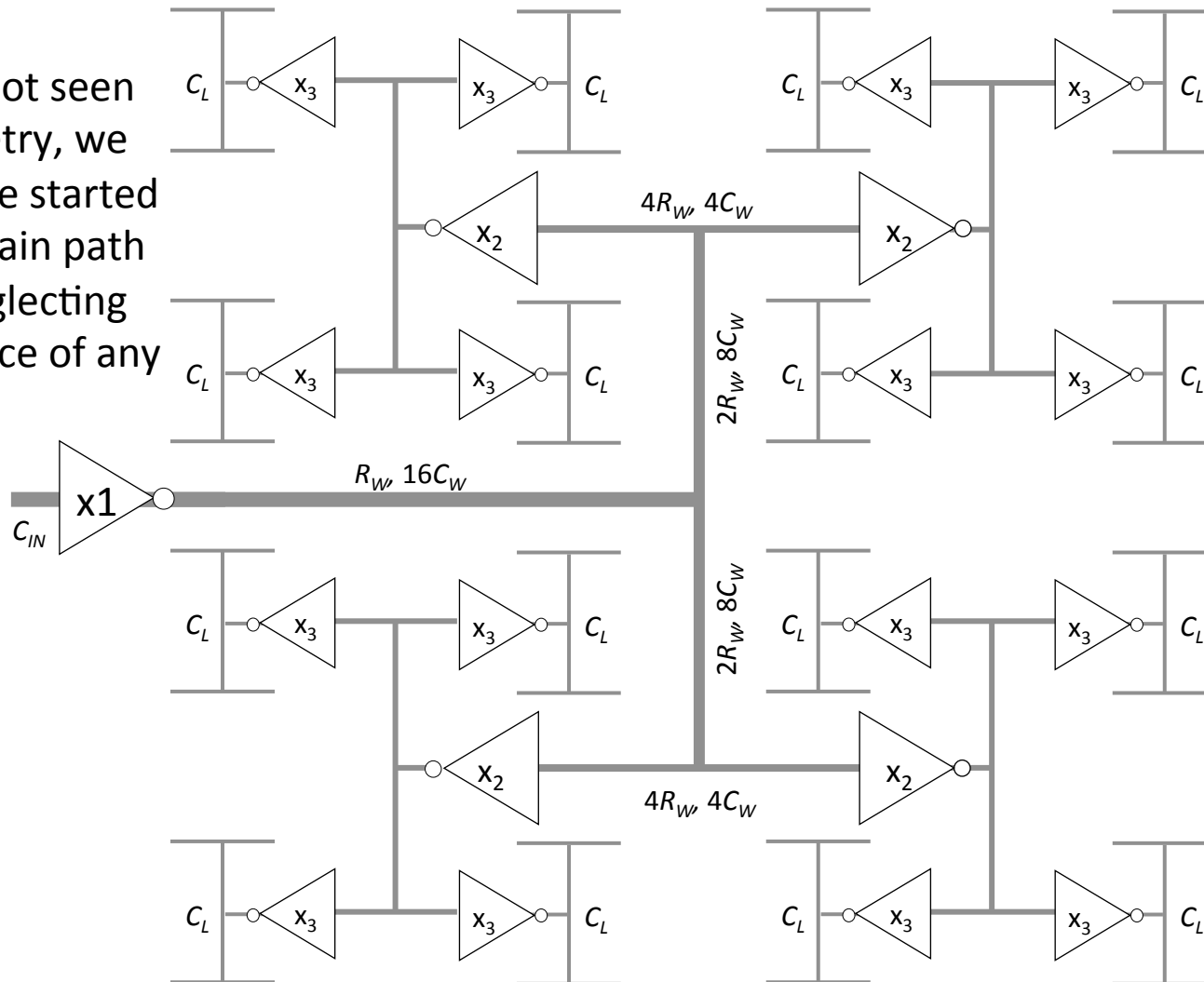


$$T_E = R(C_D + 4C) + R \times 40C_W + 2R_W(4C + 24C_W) + \frac{R_W}{2}(4C + 4C_W)$$

$$T_E = R(C_D + 4C) + 40RC_W + 10R_W C + 50R_W C_W$$

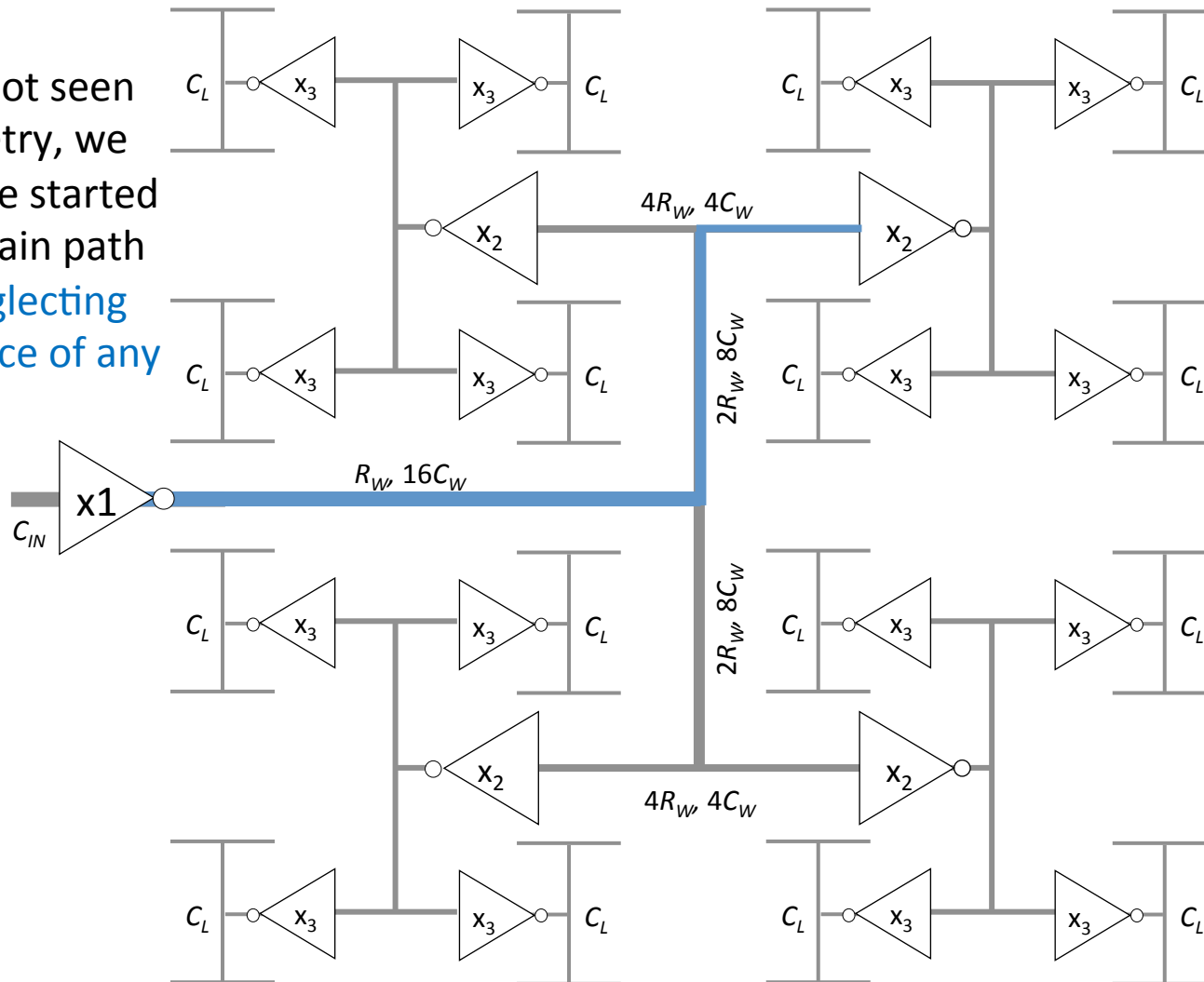
H-tree clock distribution

If we had not seen the symmetry, we should have started with the main path delay – neglecting the influence of any branches



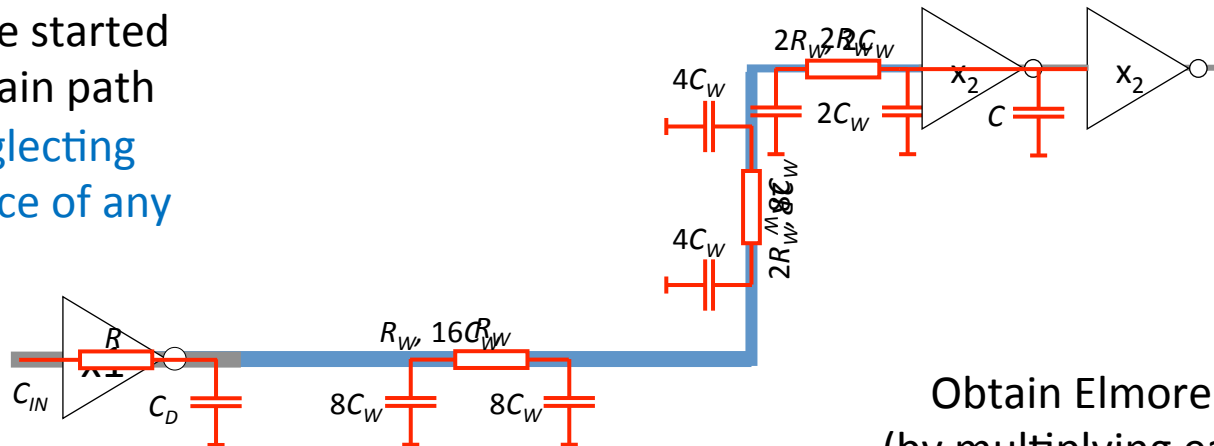
Always start by identifying main path

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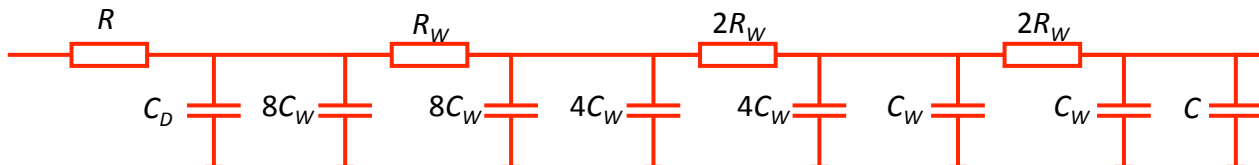
Always start by identifying main path

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Electrical RC wire model (neglecting branches)

Obtain Elmore time constant (by multiplying each resistance by all downstream capacitances)



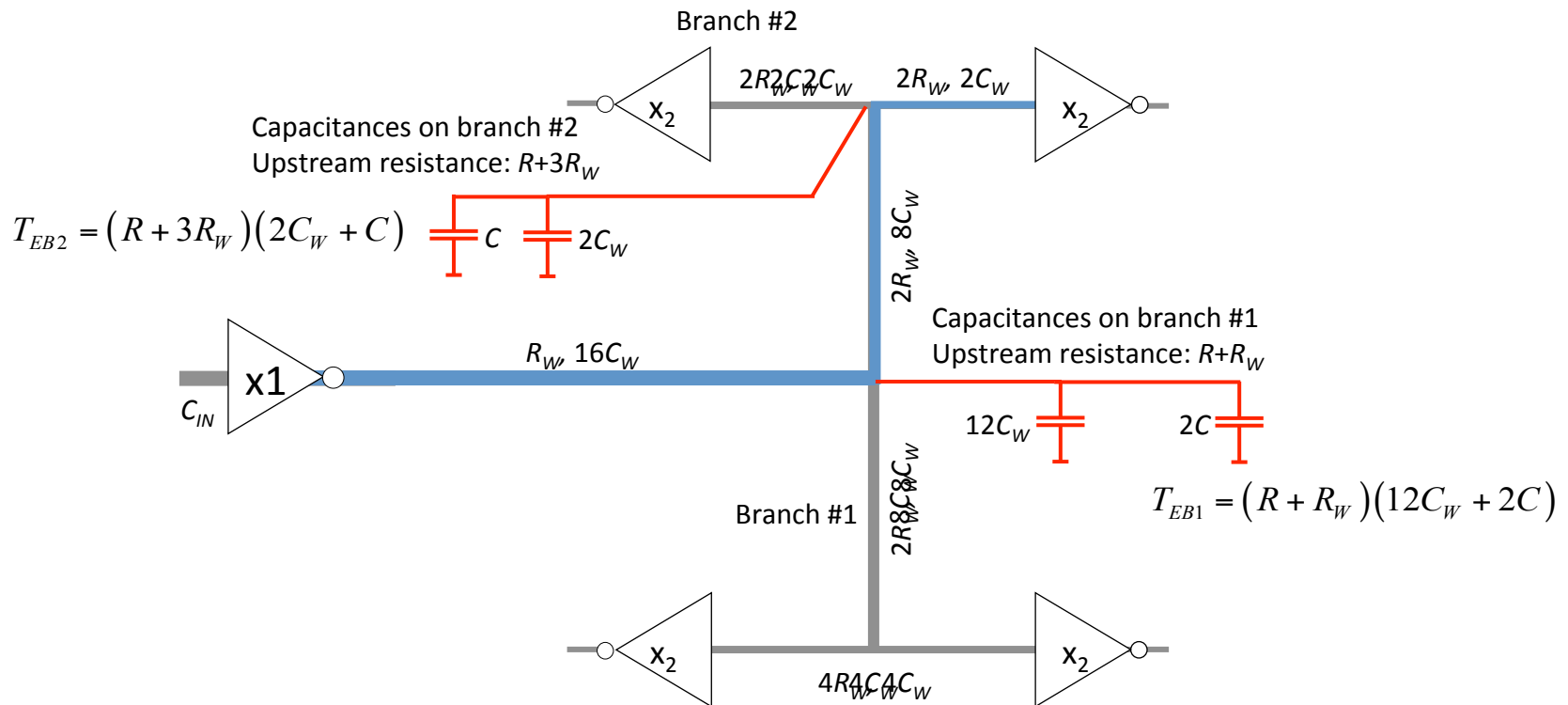
$$T_{E,main} = R(C_D + C) + R \times 26C_W + R_W(C + 18C_W) + 2R_W(C + 6C_W) + 2R_W(C + C_W)$$

$$T_{E,main} = R(C_D + C) + 26RC_W + 5R_W C + 32R_W C_W$$

Then consider neglected branches

We have two neglected branches: #1 and #2

Rule of thumb: Forget about the branch resistances, only consider branch capacitances



Branch contribution to Elmore time constant: $T_{E,branches} = 3RC + 14RC_W + 5R_W C + 18R_W C_W$

Total Elmore time constant: $T_E = R(C_D + 4C) + 40RC_W + 10R_W C + 50R_W C_W$

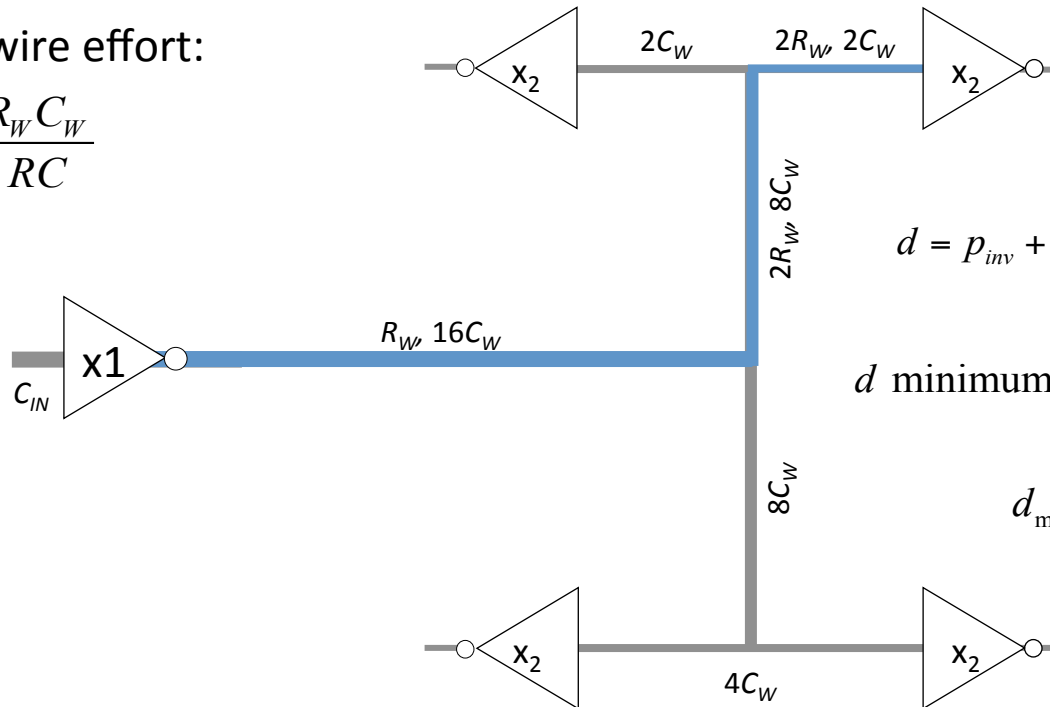
Inverter sizing

How to size driver inverter to minimize the time constant, and hence the wire delay?

Normalize Elmore time constant: $d = \frac{T_E}{RC} = p_{inv} + 4 + 40 \frac{R_W C_W}{RC} \frac{R}{R_W} + 10 \frac{R_W}{R} + 50 \frac{R_W C_W}{RC}$

Define wire effort:

$$W_E = \frac{R_W C_W}{RC}$$



$$d = p_{inv} + 4 + 40W_E \frac{R}{R_W} + 10 \frac{R_W}{R} + 50W_E$$

$$d \text{ minimum when } \frac{\partial d}{\partial R} = 4 \frac{W_E}{R_W} - 1 \frac{R_W}{R^2} = 0$$

$$d_{\min} \text{ for } R = \frac{R_W}{2\sqrt{W_E}}$$

Total Elmore time constant: $T_E = R(C_D + 4C) + 40RC_W + 10R_W C + 50R_W C_W$

Conclusion

We have learnt two things:

- The Elmore model – a generalized delay model
 - Relying on the existence of a dominating time constant
- How to handle the influence on delay of branches
 - Main timing path first, neglecting branches
 - Then consider branches neglecting any branch resistances only considering branch capacitances!

Thanks a lot for listening!