Introduction to on-chip interconnect

Elmore delay model

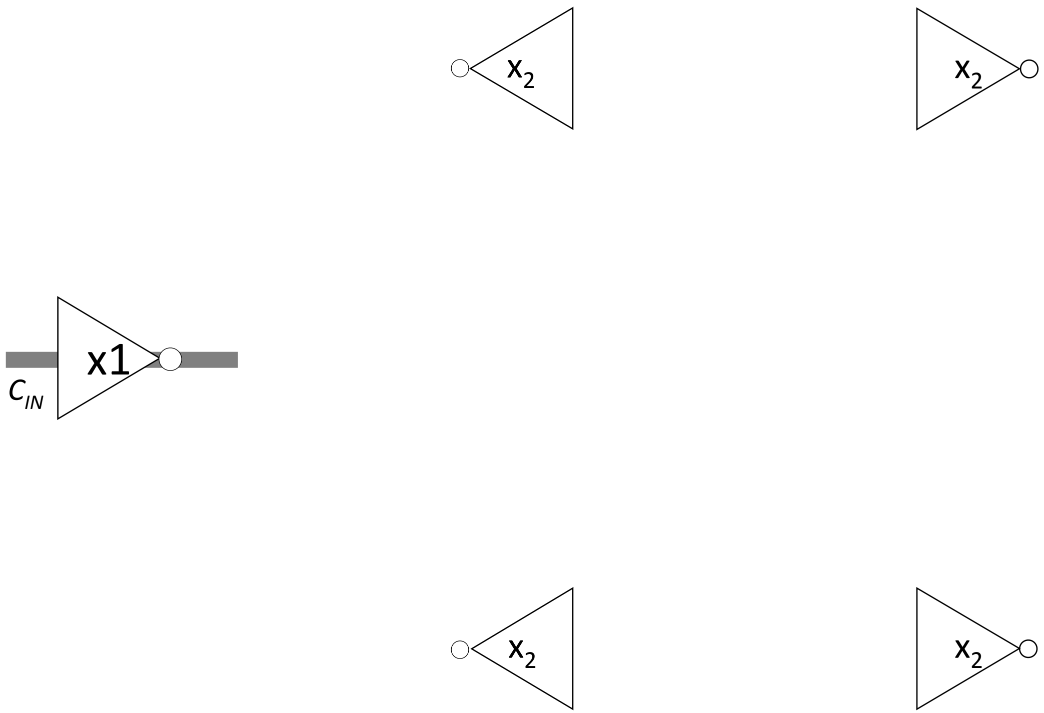
Lecture 7 (or really 9) continued

Tuesday October 2, 2018

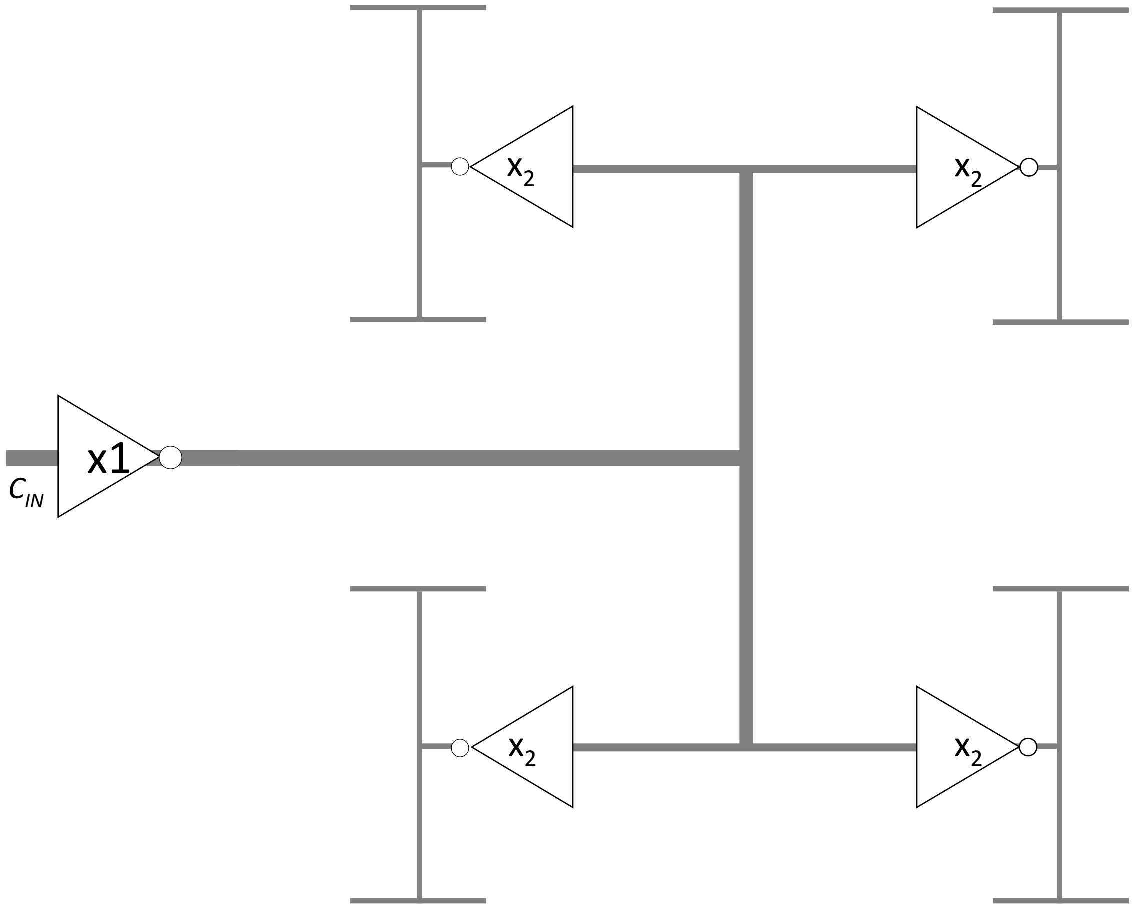
Outline

- Elmore delay model – a generalized model
- How to handle wire branches
- Conclusions

H-tree clock distribution



H-tree clock distribution



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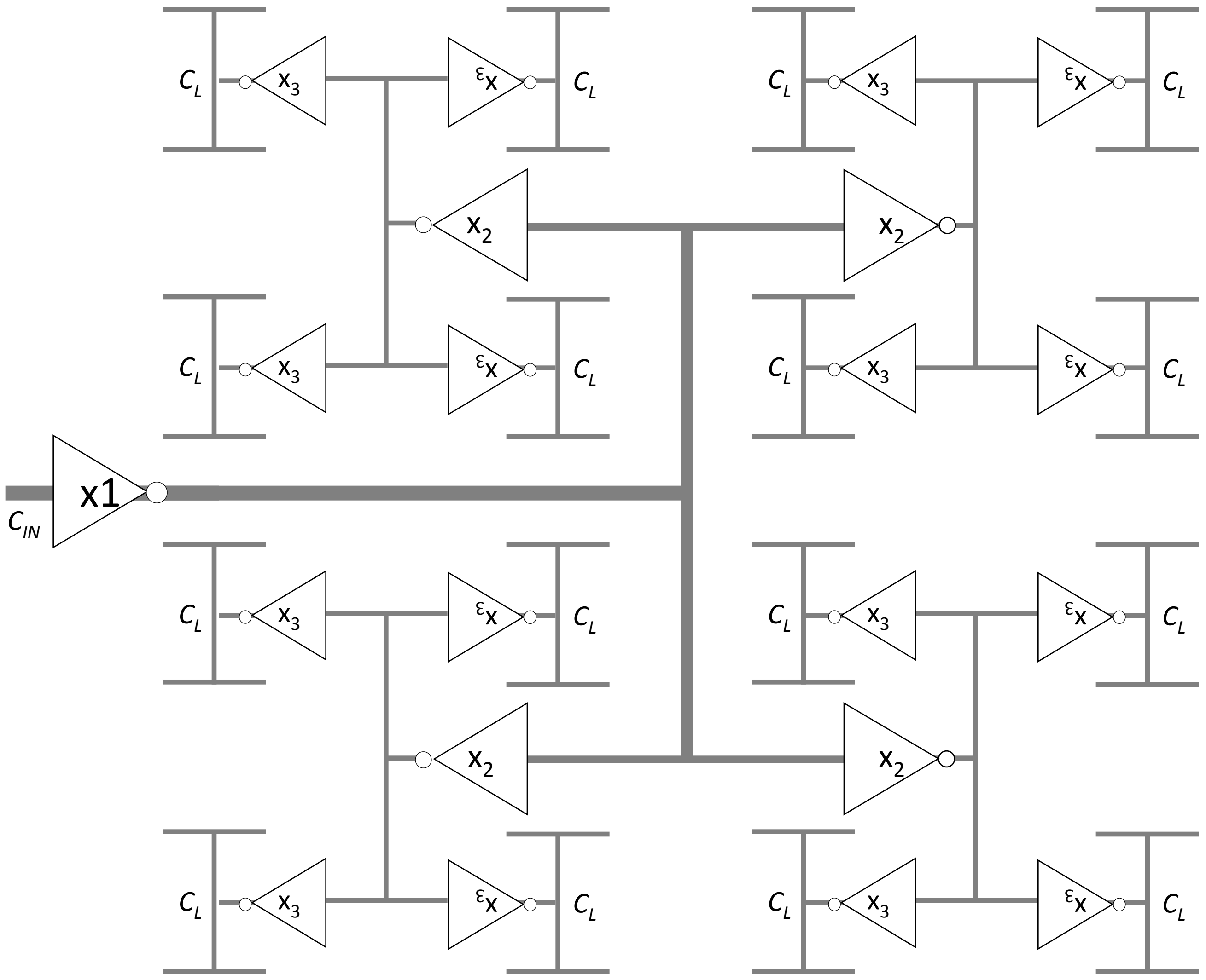

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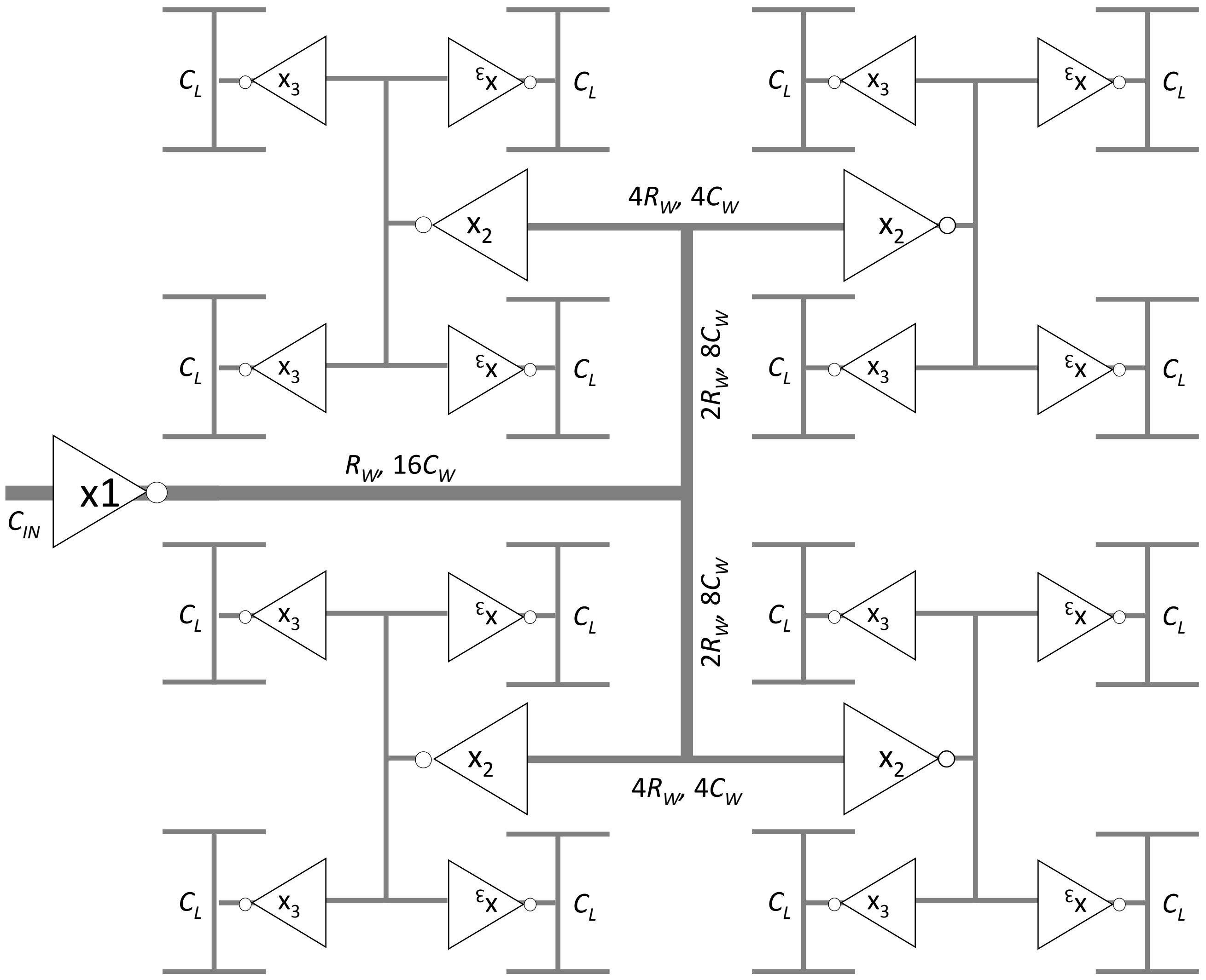

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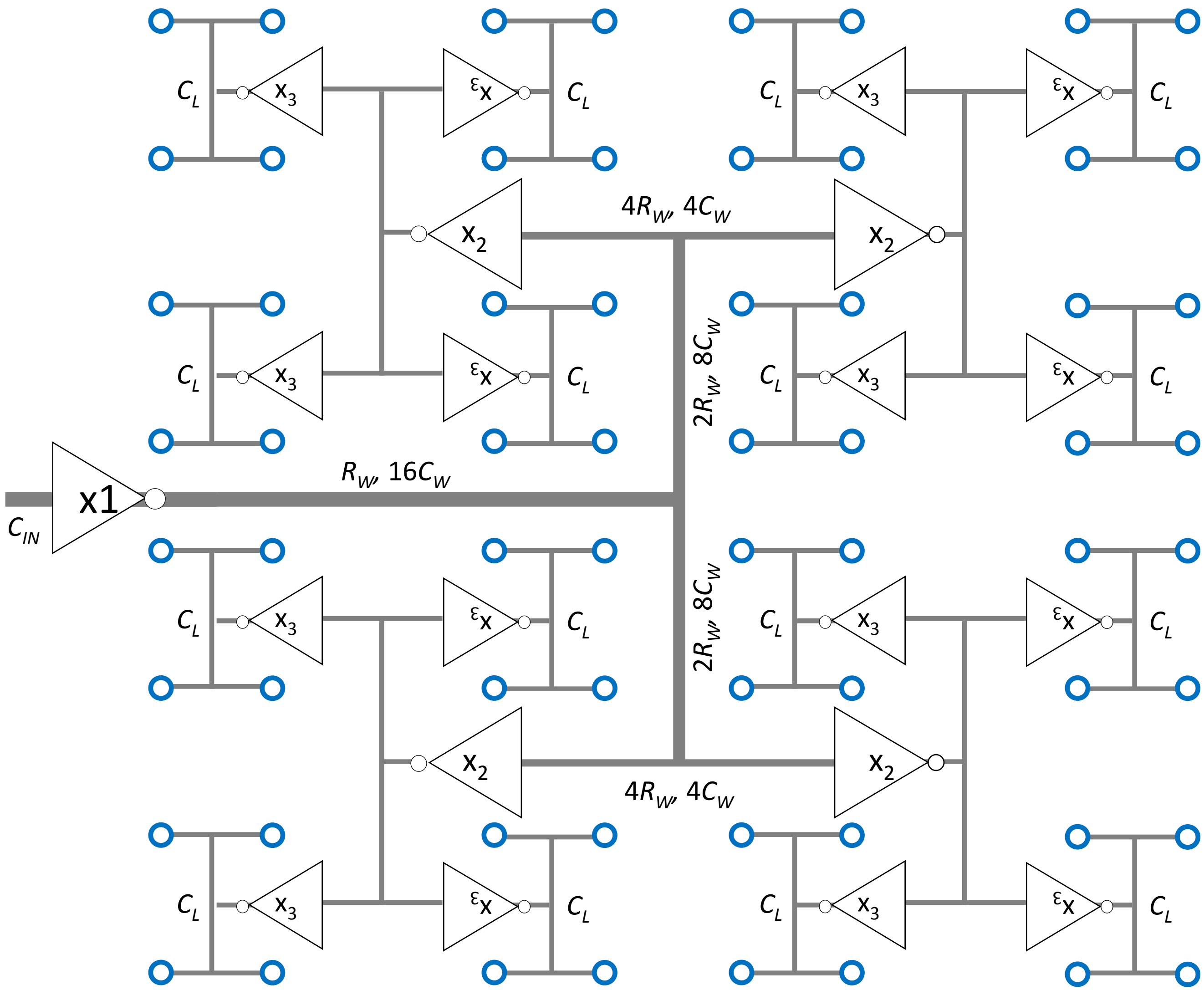
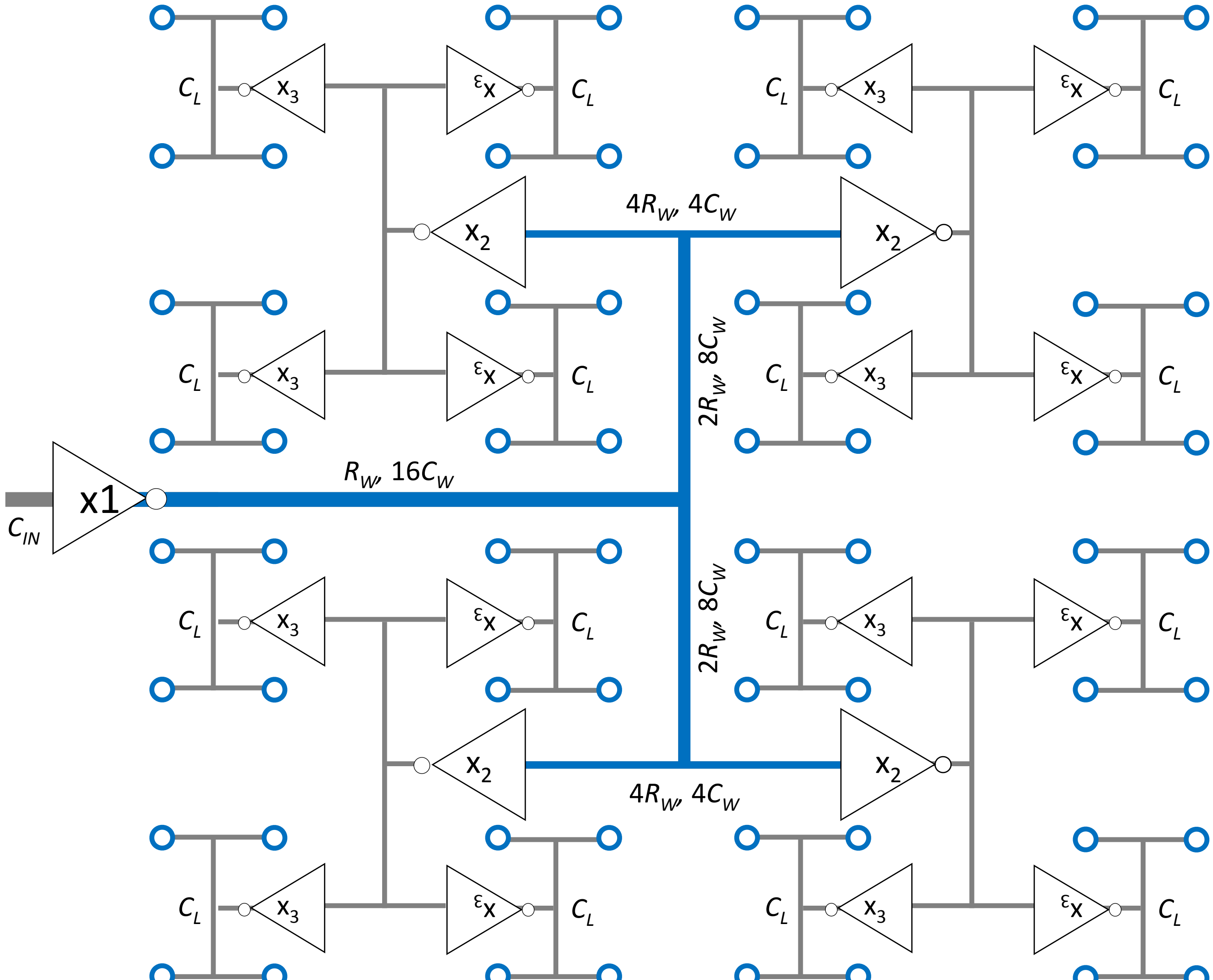


Identify the critical timing path



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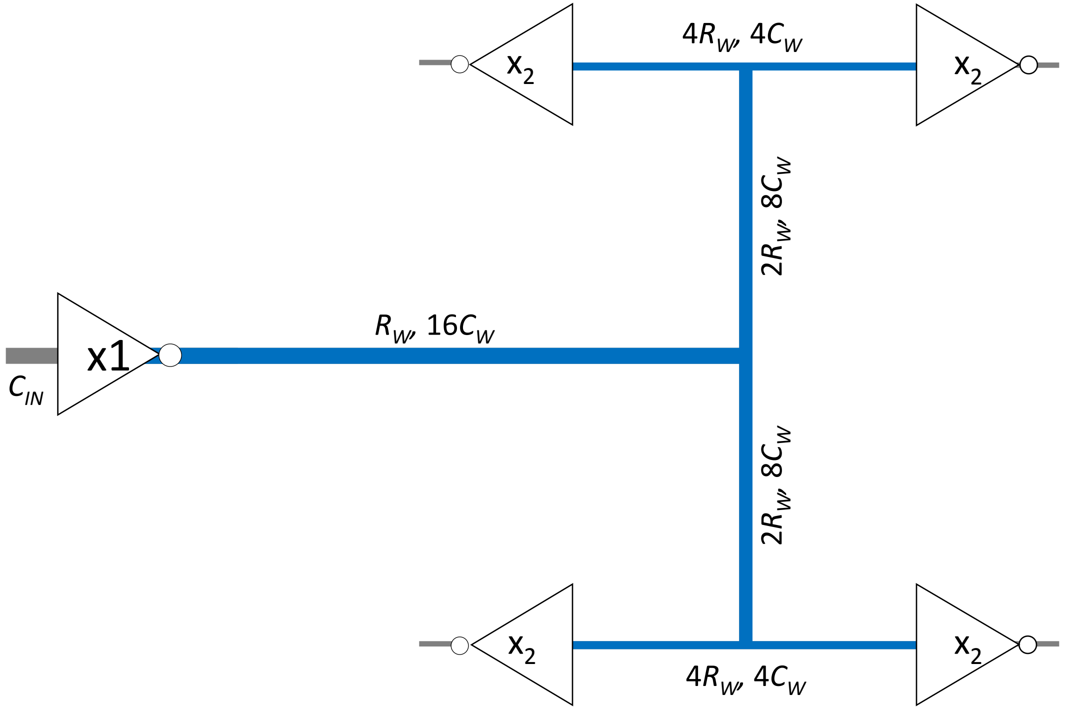

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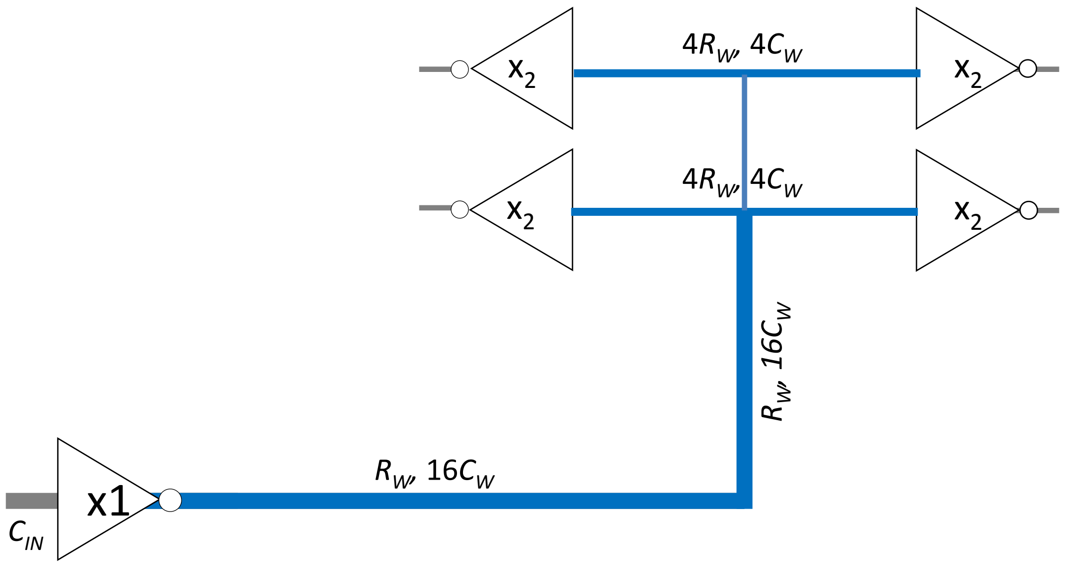

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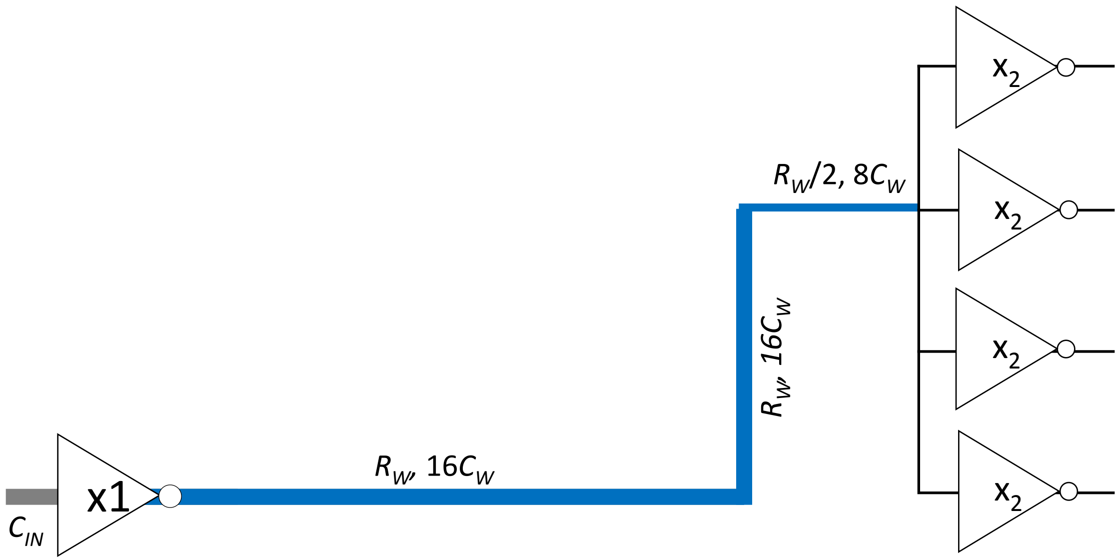

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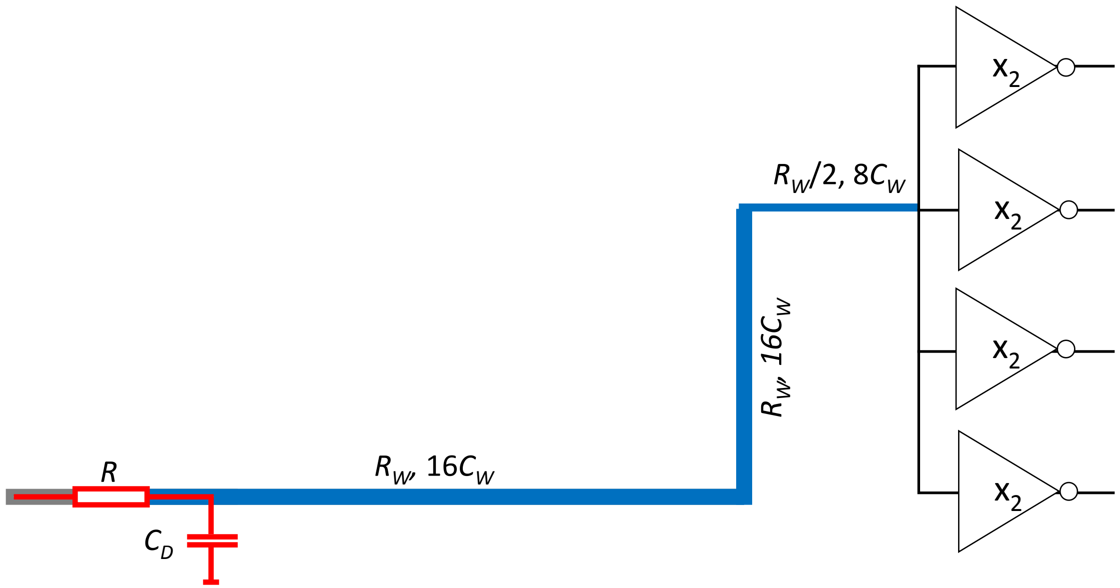

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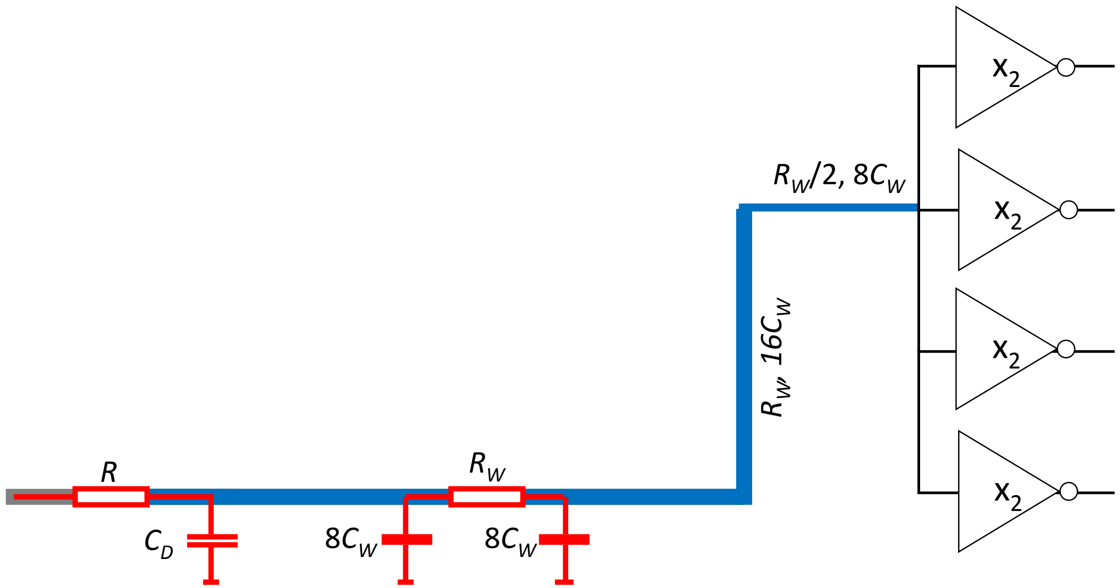

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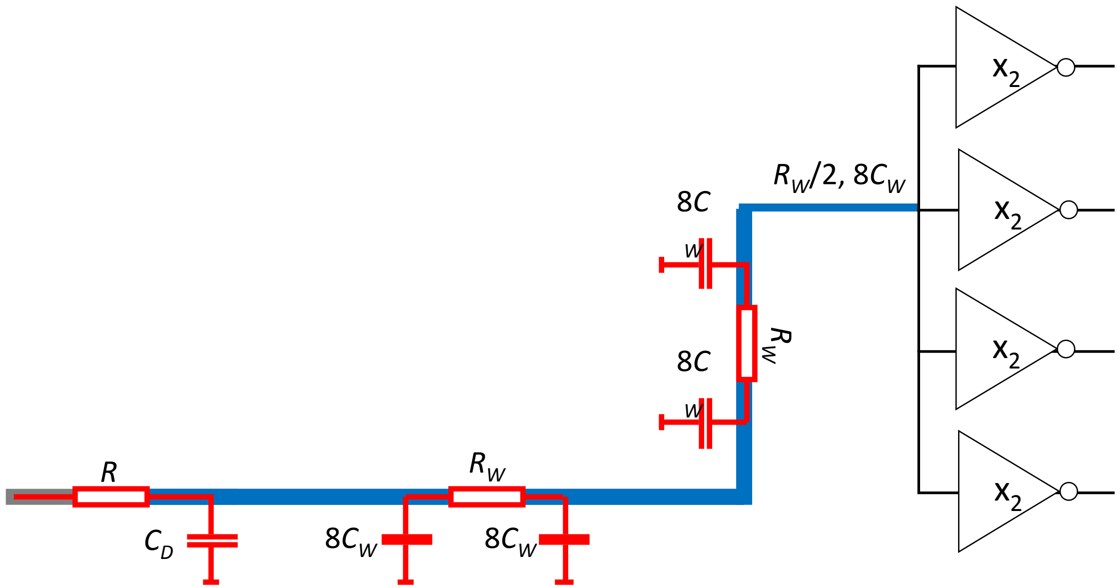

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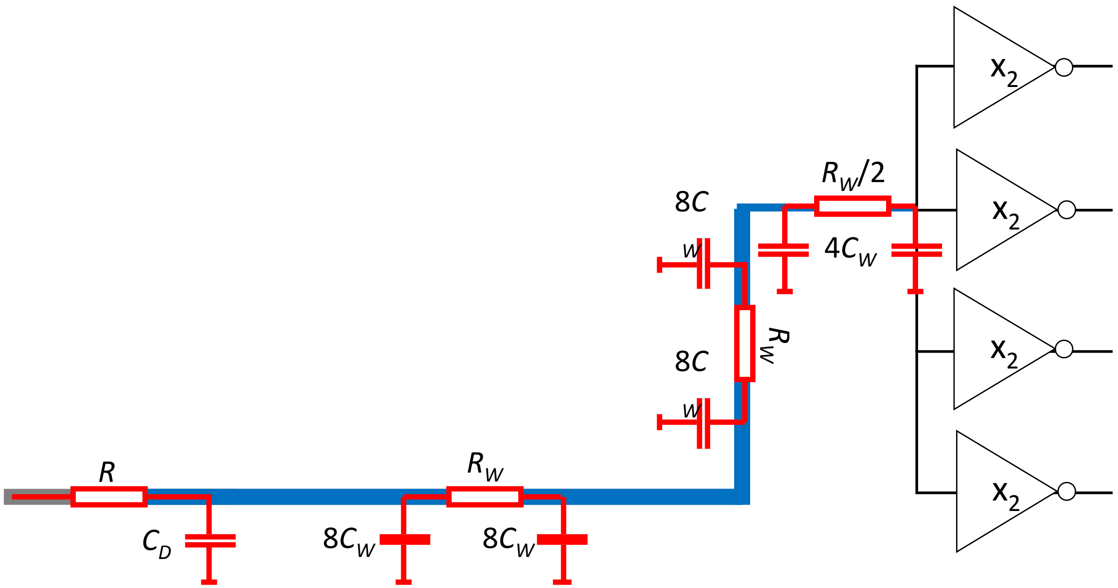

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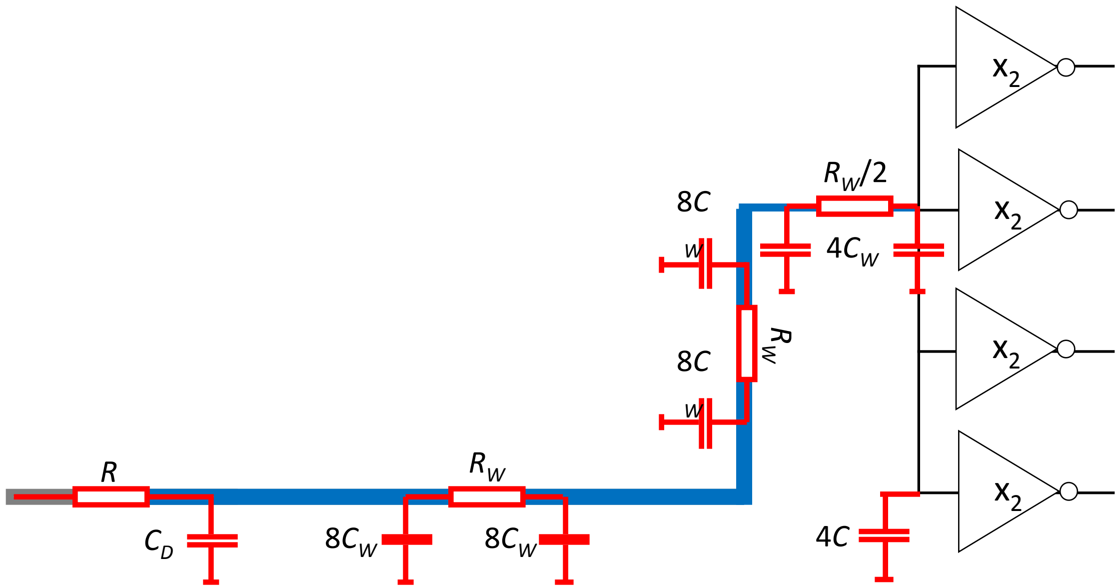

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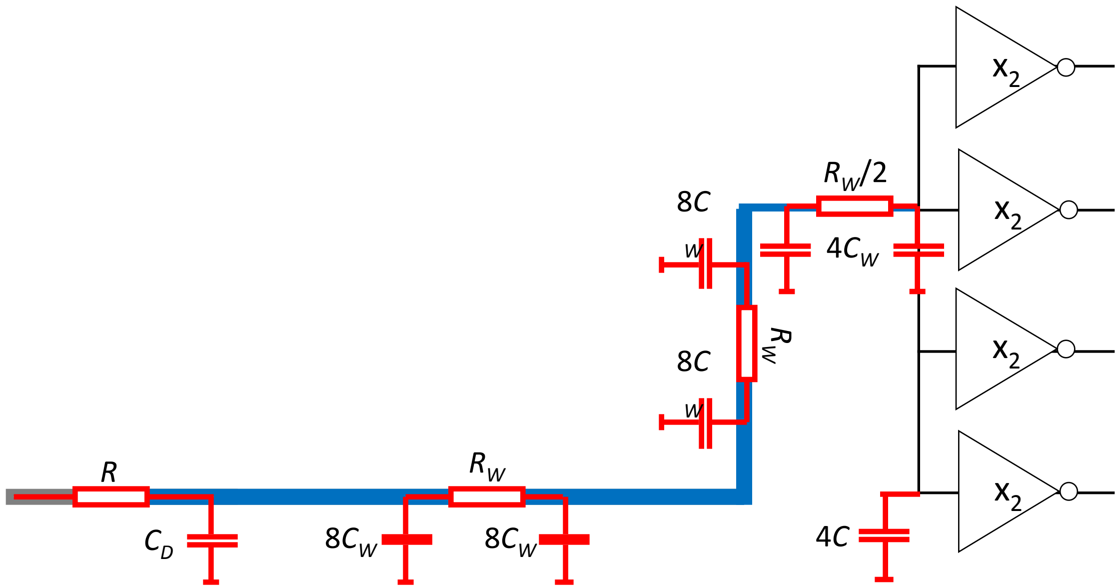

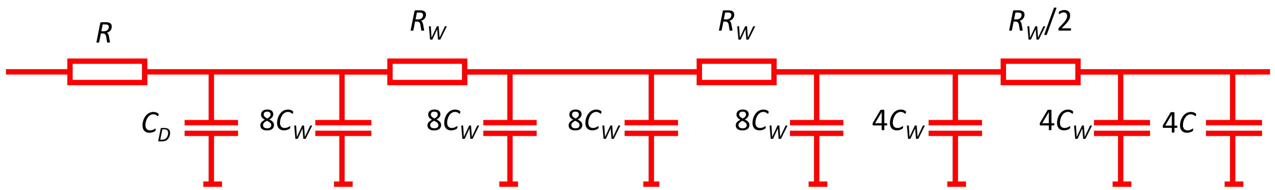

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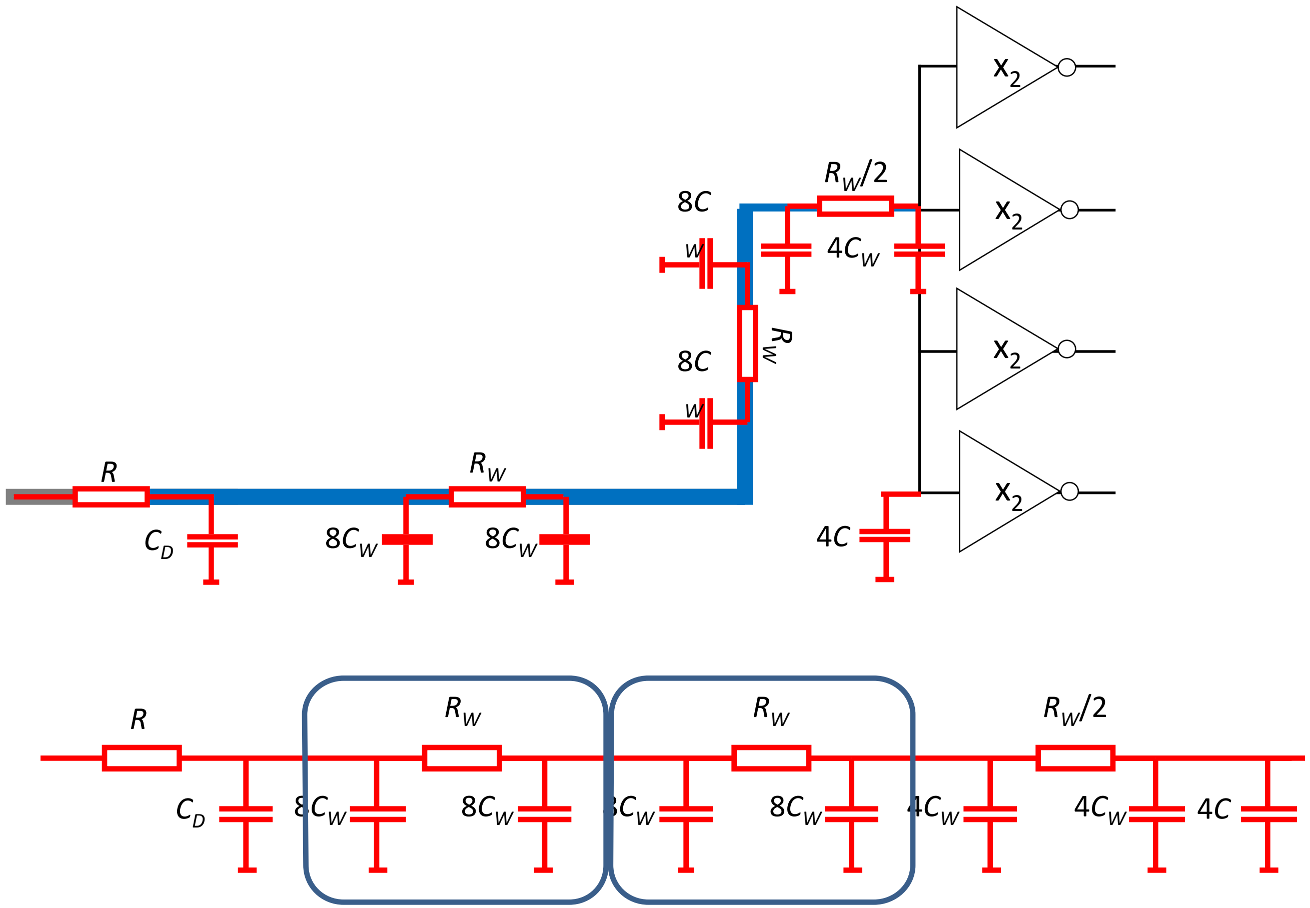

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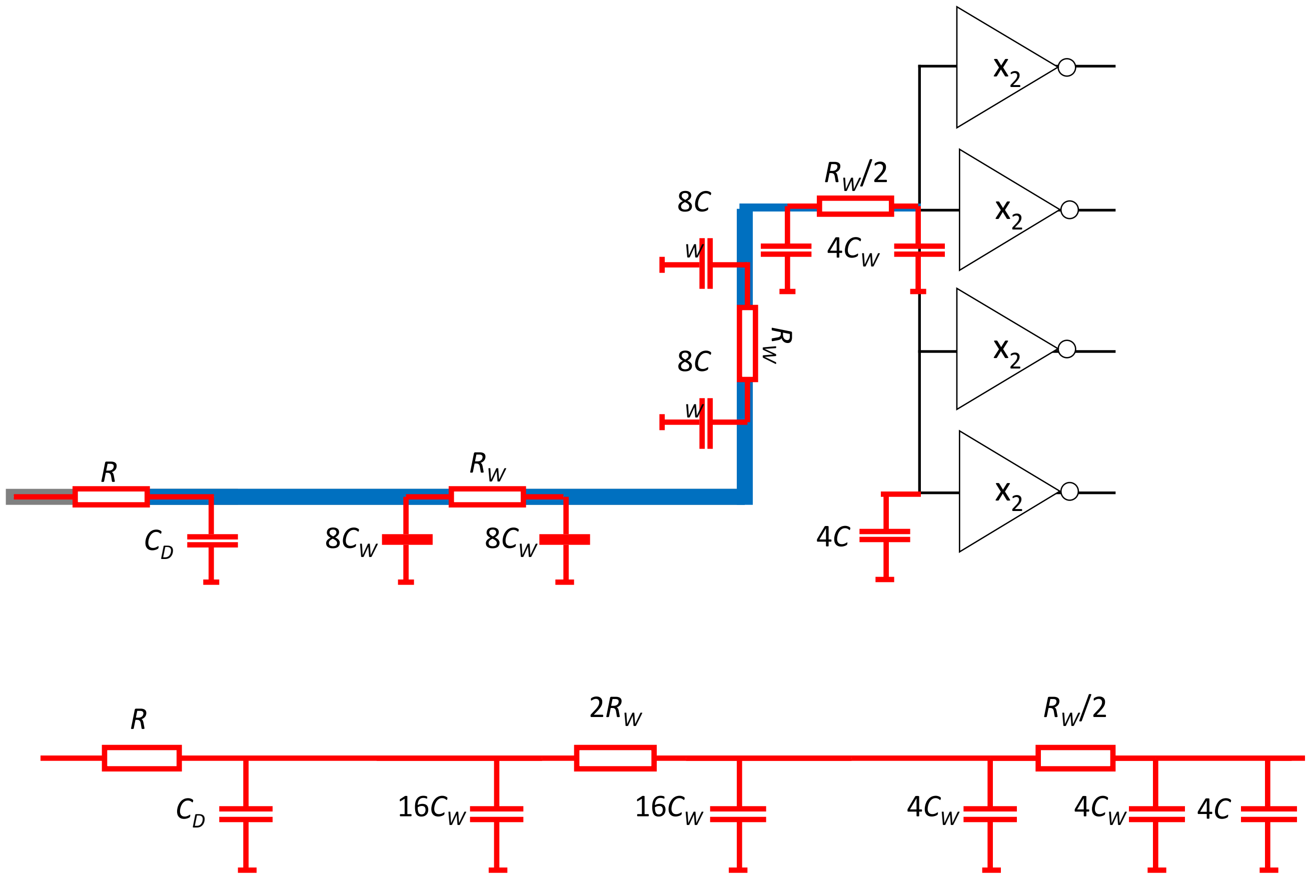
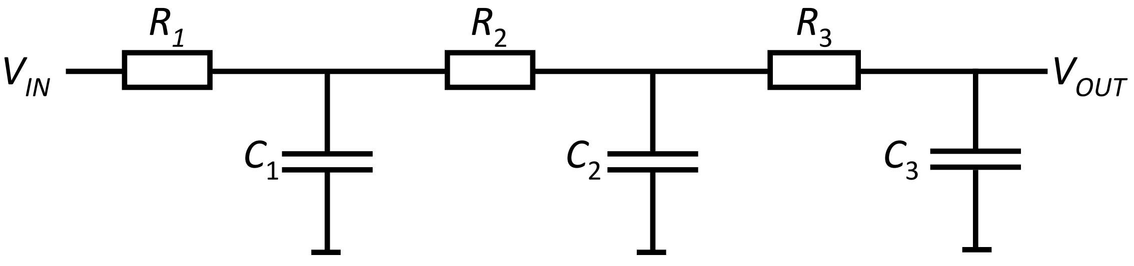
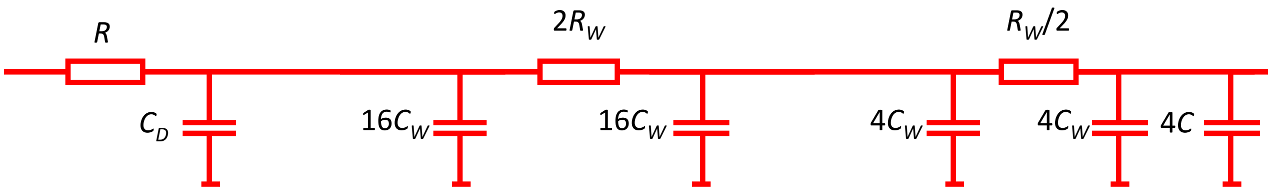


General solution





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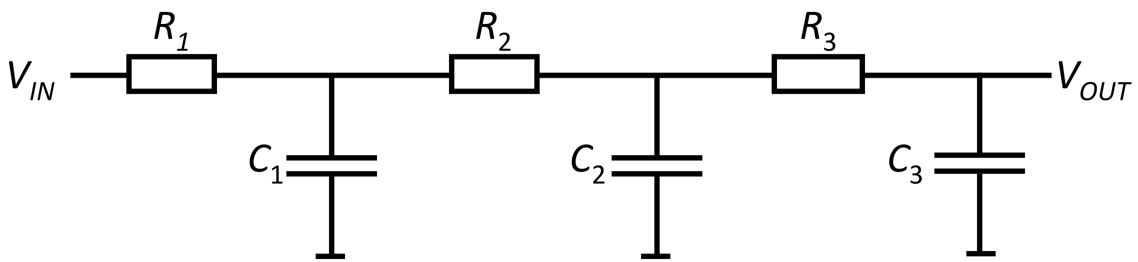
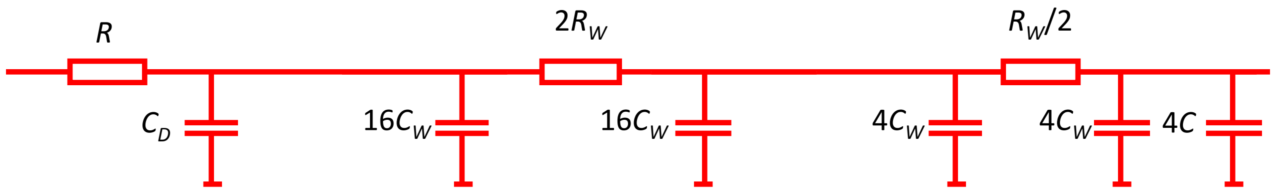


Transfer function

$$H(s) = \frac{1}{as^3 + bs^2 + cs + 1}$$

$$a = R_1 R_2 R_3 C_1 C_2 C_3$$
$$b = R_1 R_2 C_1 C_2 + R_1 R_2 C_1 C_3 + R_1 R_3 C_2 C_3 + R_2 R_3 C_2 C_3$$
$$c = R_1 (C_1 + C_2 + C_3) + R_2 (C_2 + C_3) + R_3 C_3$$



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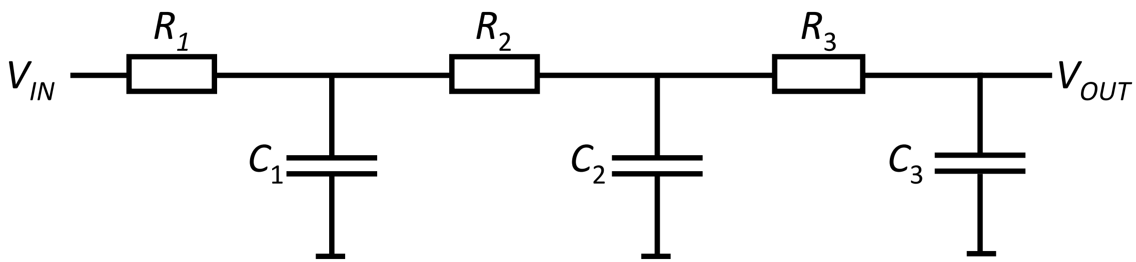

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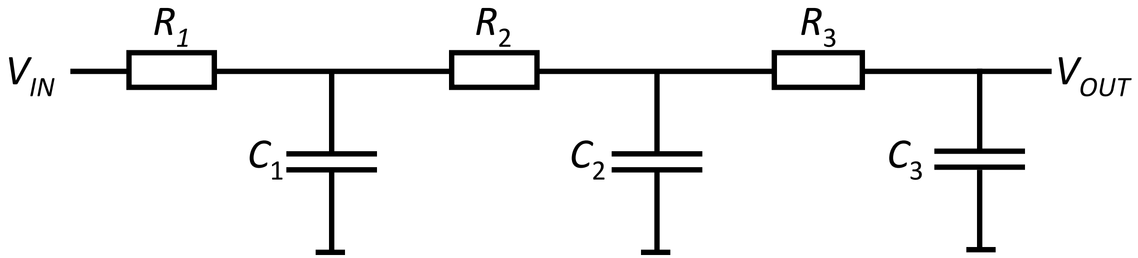
$$c = R_1 (C_1 + C_2 + C_3) + R_2 (C_2 + C_3) + R_3 C_3$$

This transfer function corresponds to a third order linear differential equation
The solution is a sum of three exponentials with three different time constants
We cannot solve this analytically.
But if we assume that there is a dominant time constant T_E it is given by c

General solution



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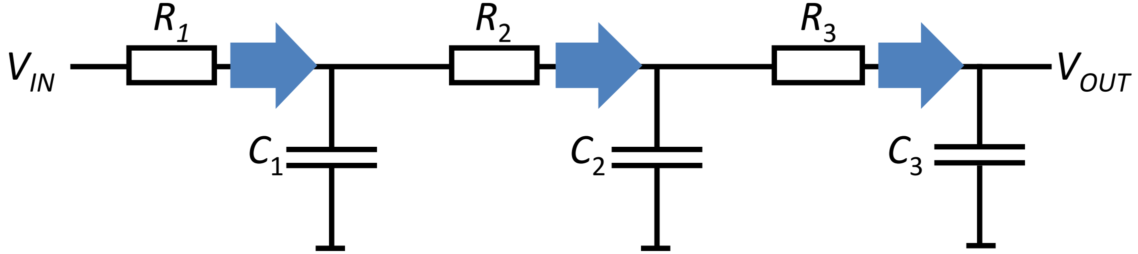
$$T_E = R_1(C_1 + C_2 + C_3) + R_2(C_2 + C_3) + R_3C_3$$

General solution

Each resistance is multiplied by its downstream capacitance!

$$T_E = R_1(C_1 + C_2 + C_3) + R_2(C_2 + C_3) + R_3C_3$$



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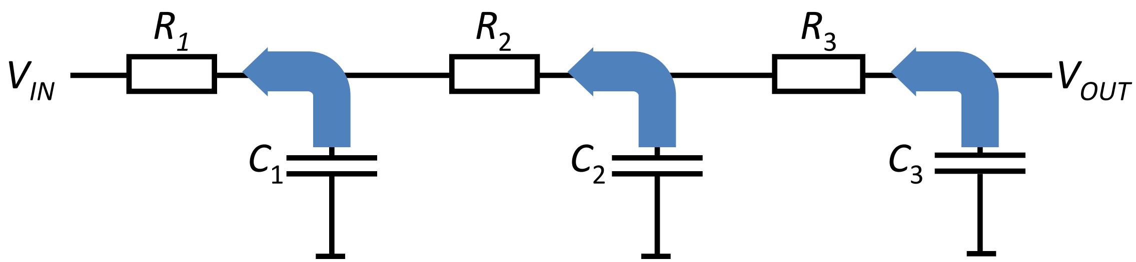
$$T_E = R_1(C_1 + C_2 + C_3) + R_2(C_2 + C_3) + R_3C_3$$

General solution

Each capacitance is multiplied by its upstream resistance!

$$T_E = C_3\left(R_1 + R_2 + R_3\right) + C_2\left(R_1 + R_2\right) + C_1 R_1$$



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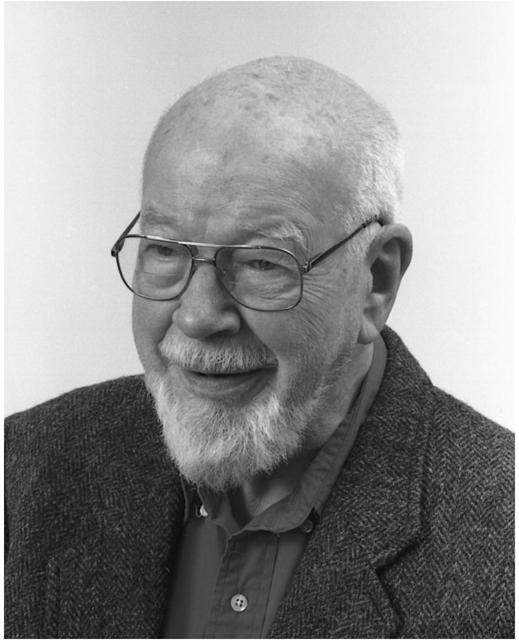
The man behind Elmore delay

- William Cronk Elmore (1909 - 2003) was an American physicist, educator, and author.
- He is best known for his work on and related to the Manhattan project during World War II.
- Professor of Physics at Swarthmore College, Pennsylvania, from 1938 to 1974.
- Authored two influential books during his life,
  - Electronics-Experimental Techniques with M. Sands
  - Physics of Waves with Mark Heald.
- He is also known for deriving a simple approximation for the delay through an RC network, known as the Elmore delay.
- Despite his clear potential for advancing theoretical and experimental physics, Elmore was known for developing (and publishing) laboratory experiments that effectively taught students the fundamentals of physics.



W. C. Elmore

W.C. Elmore. The Transient Analysis of Damped Linear Networks with Particular Regard to Wideband Amplifiers, J. Applied Physics, vol. 19(1), 1948

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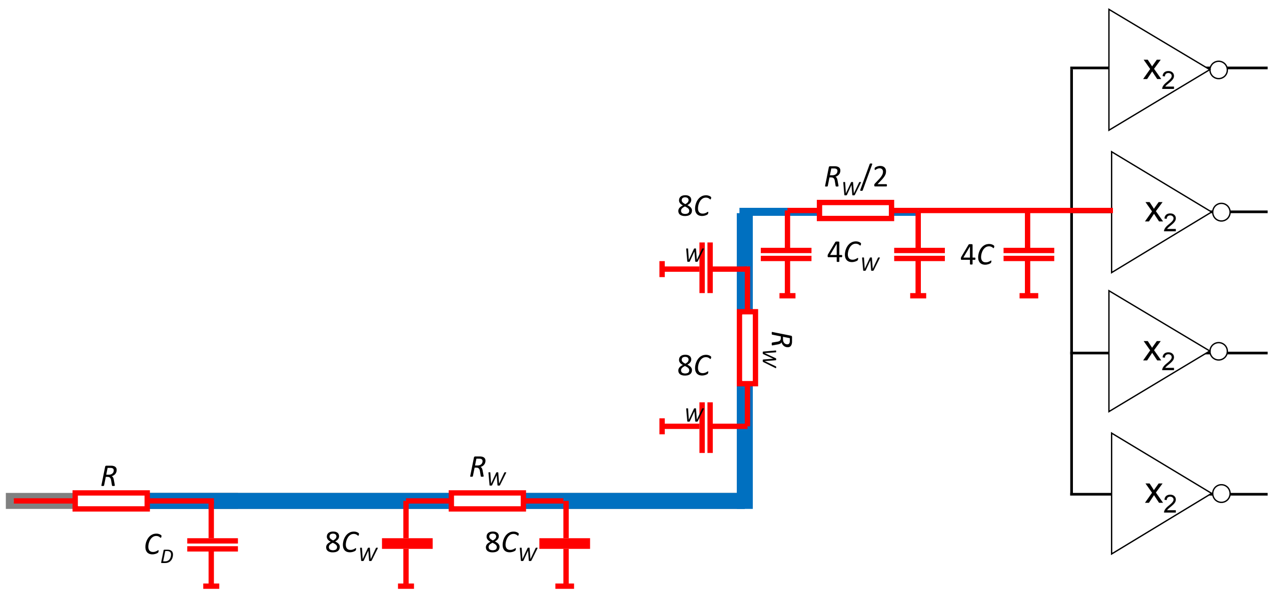
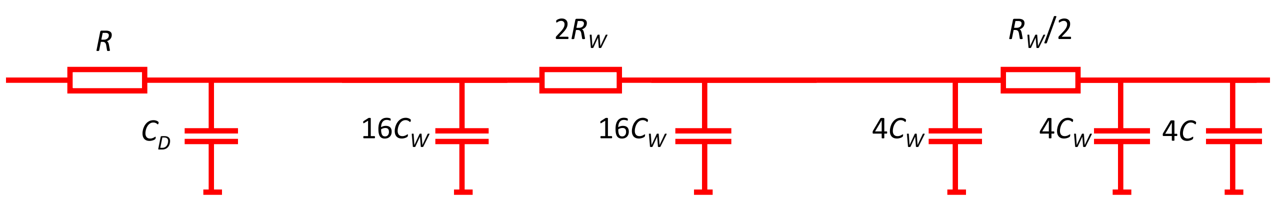
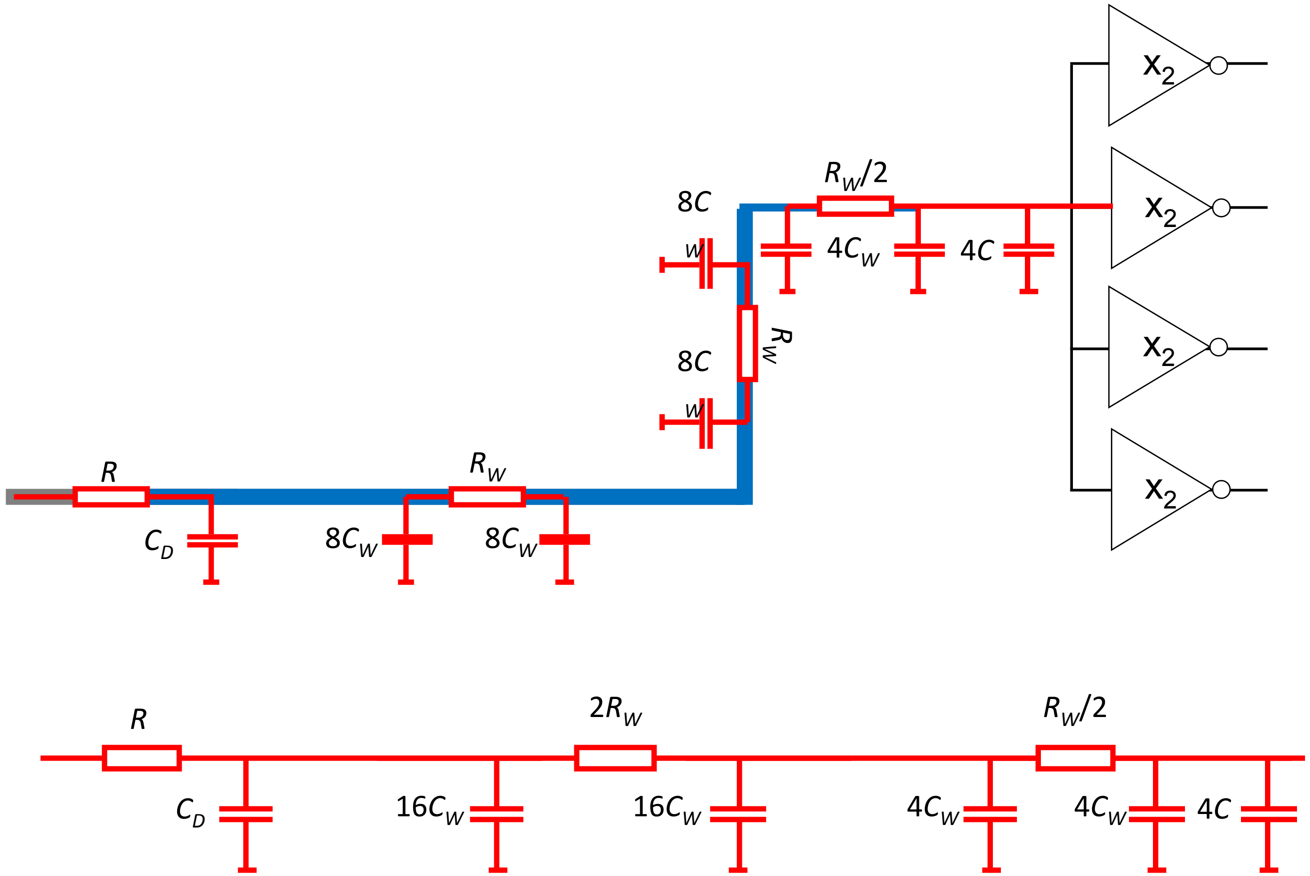


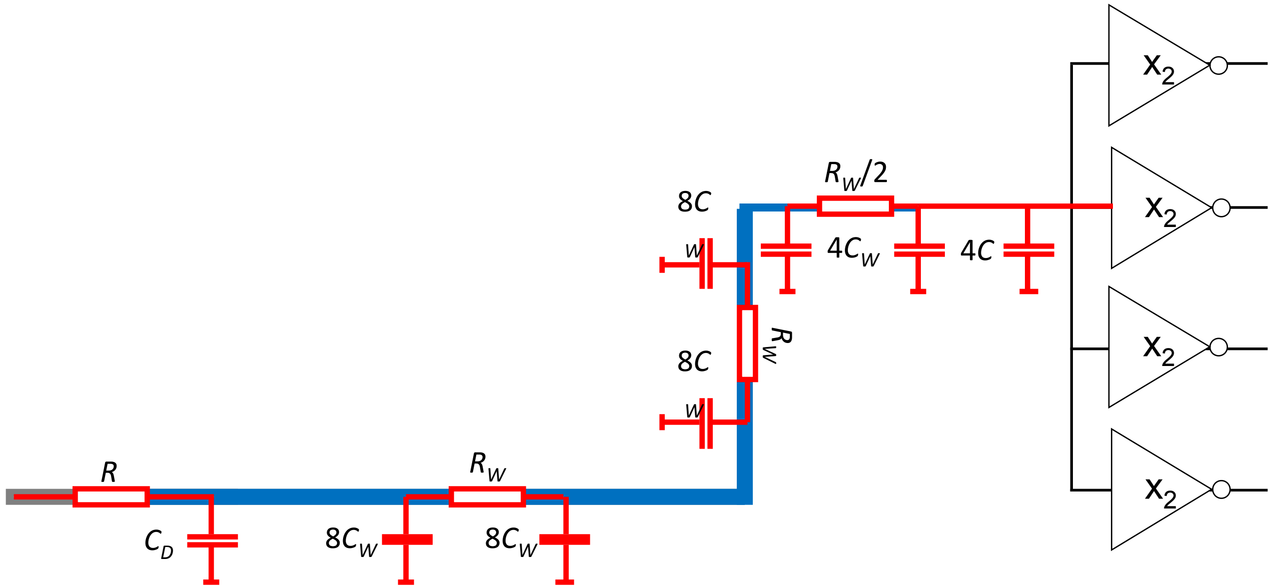


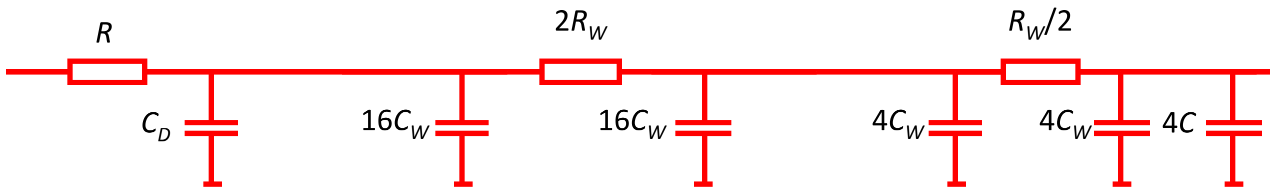
Identify the critical timing path



$$T_E = R\left(C_D + 4C\right) + R \times 40C_W + 2R_W\left(4C + 24C_W\right) + \frac{R_W}{2}\left(4C + 4C_W\right)$$

Identify the critical timing path





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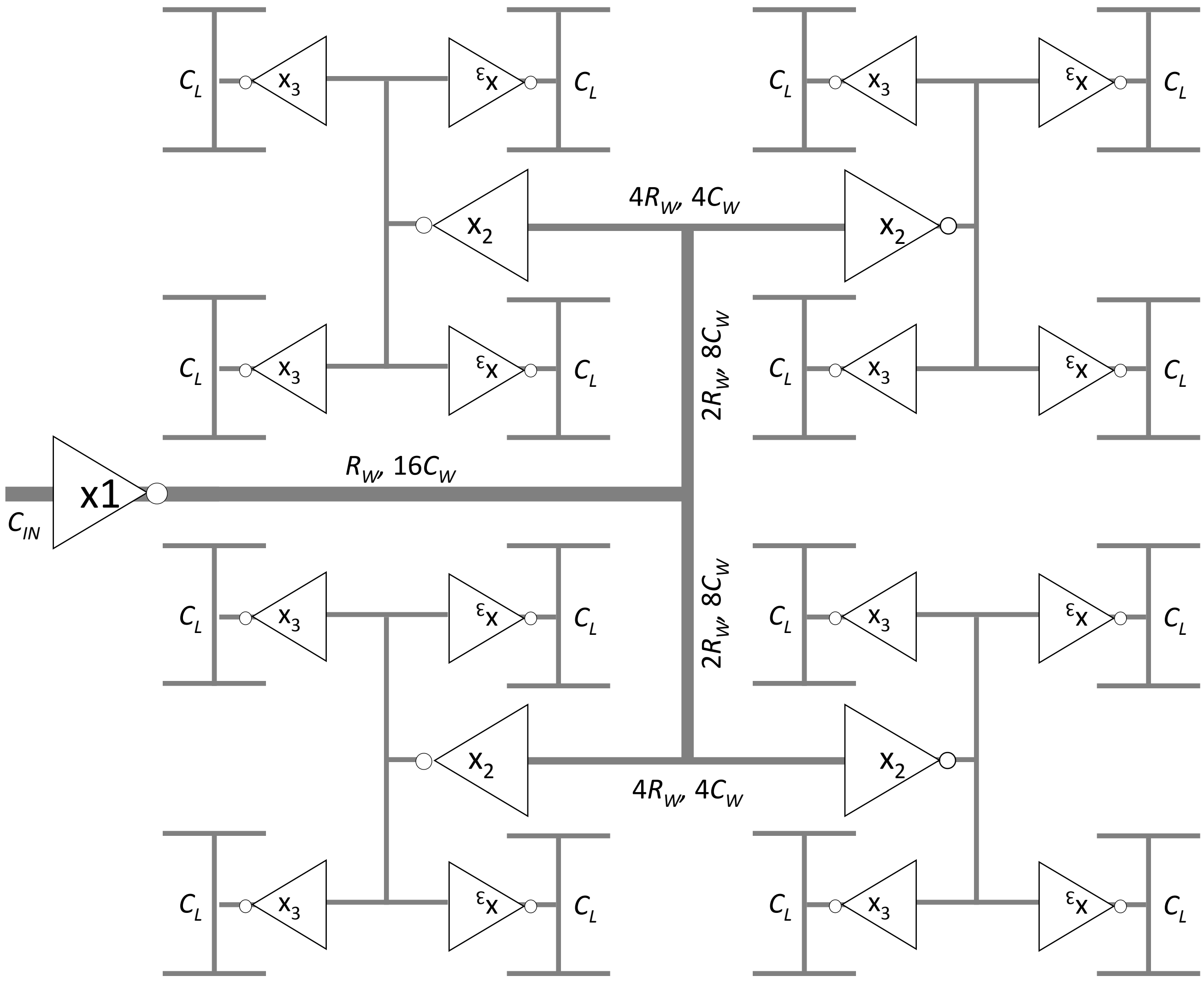
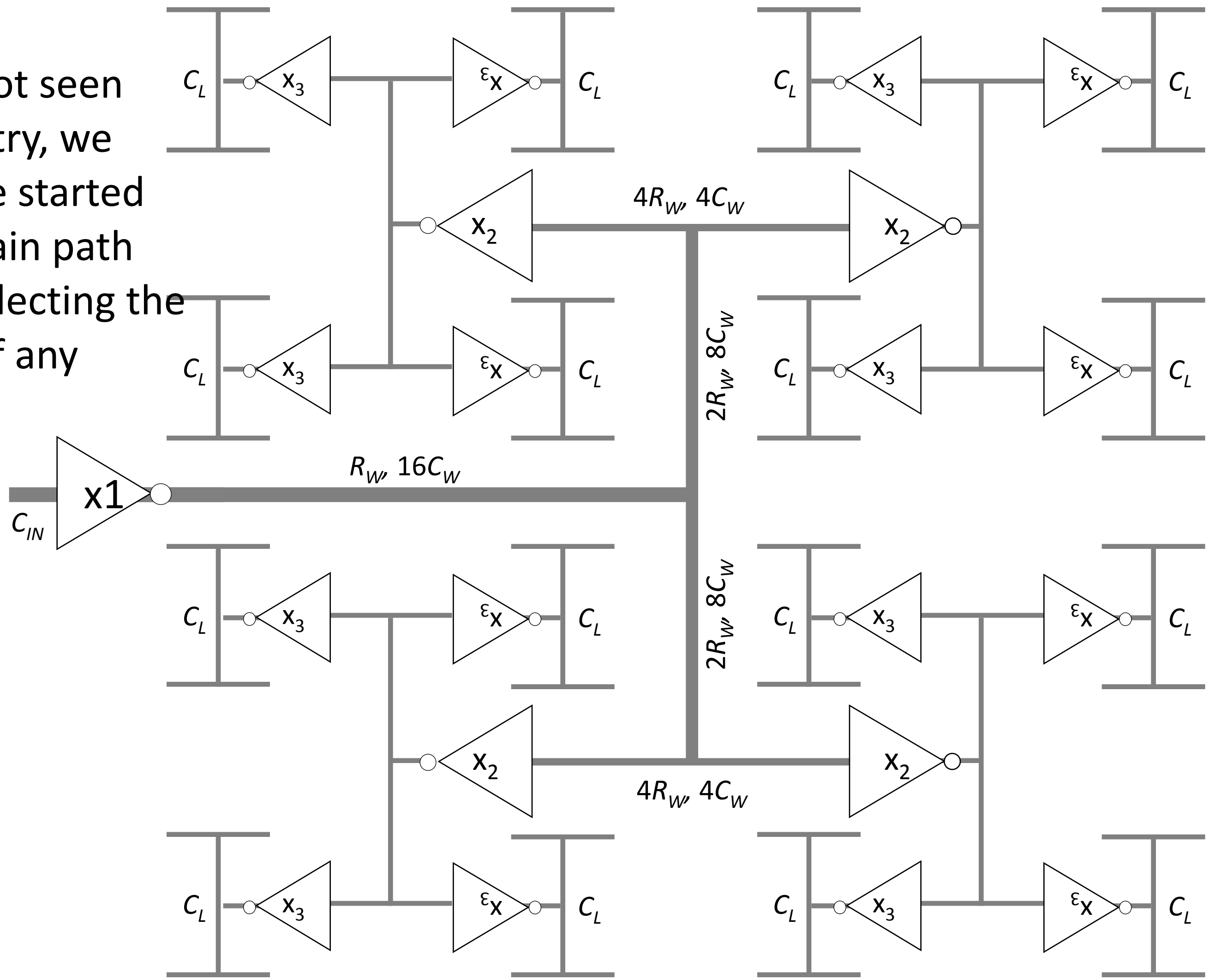
$$T_E = R\left(C_D + 4C\right) + 40RC_W + 10R_W C + 50R_W C_W$$

H-tree clock distribution



H-tree clock distribution

If we had not seen the symmetry, we should have started with the main path delay – neglecting the influence of any branches



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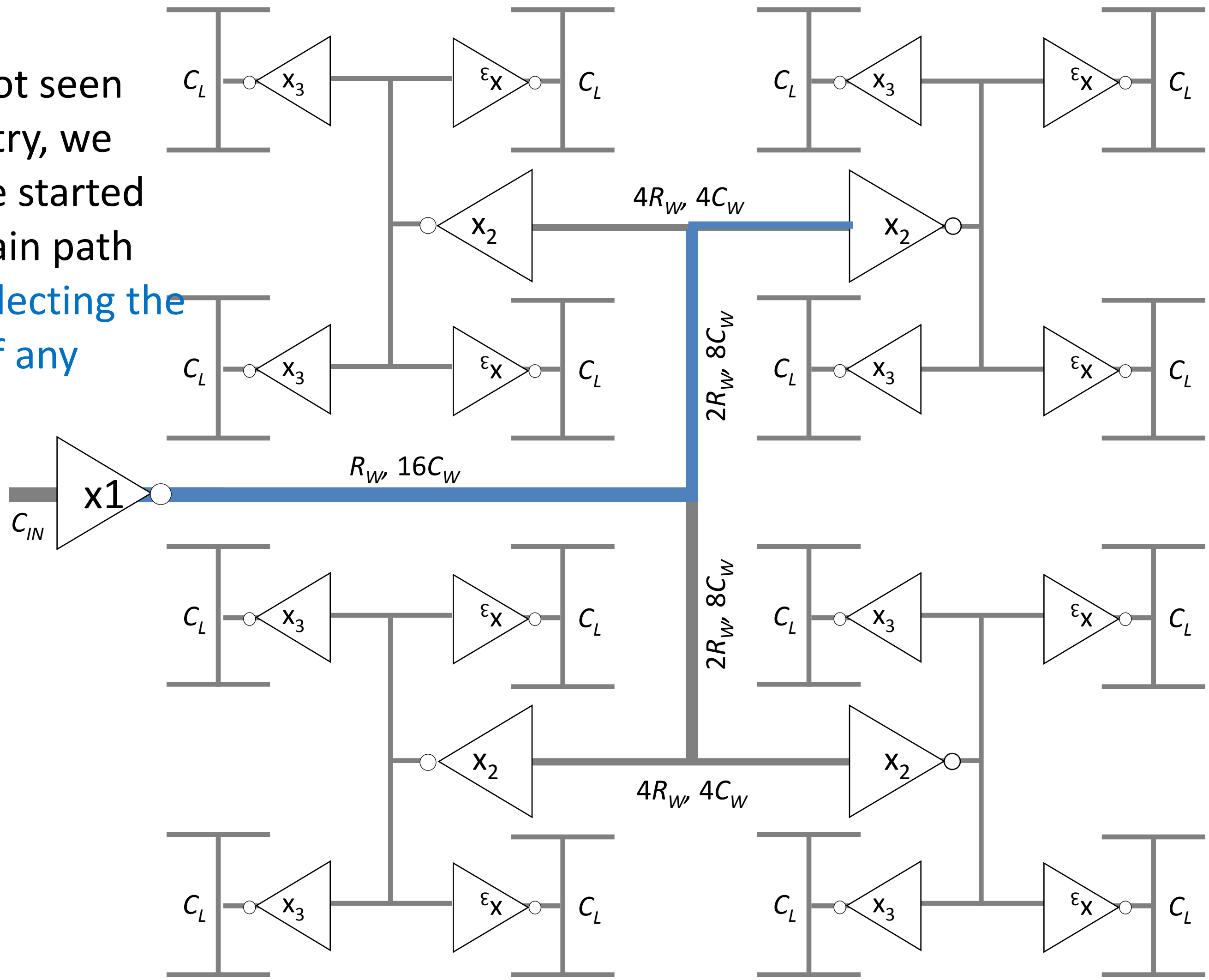

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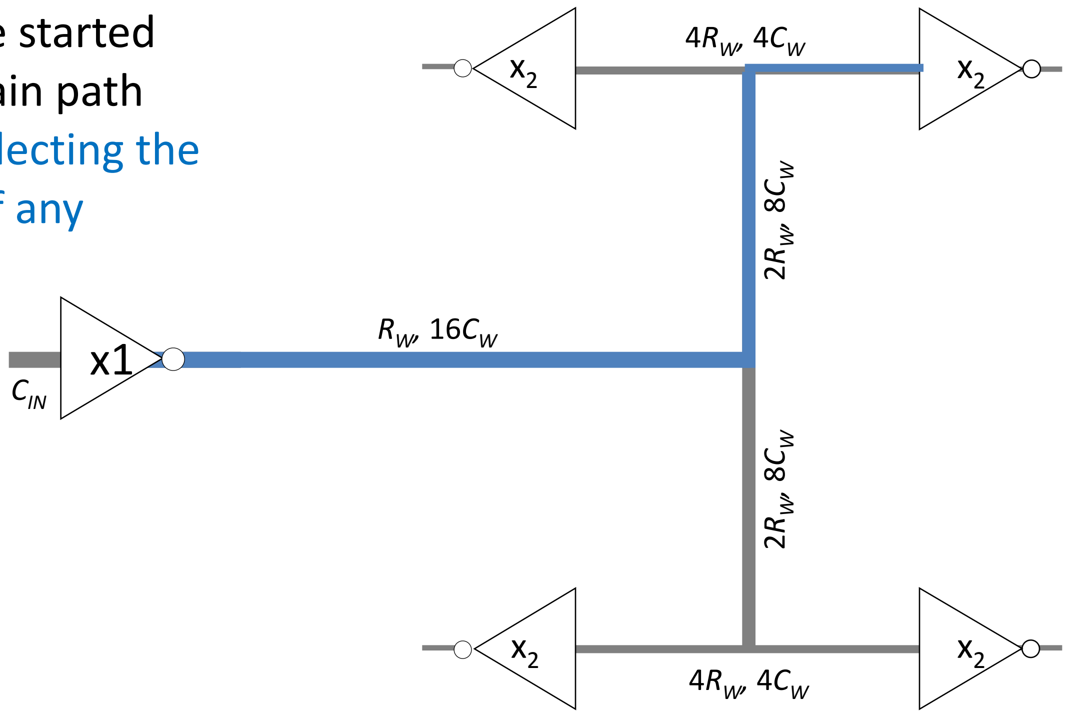

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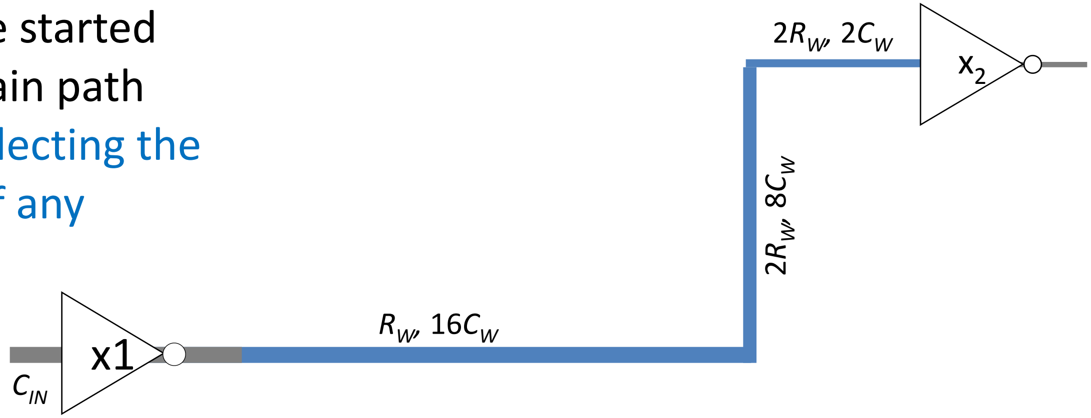

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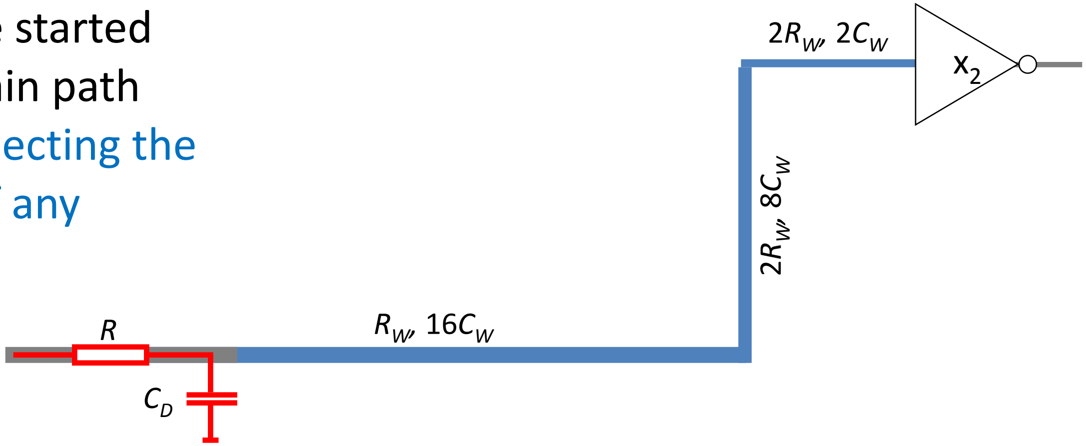

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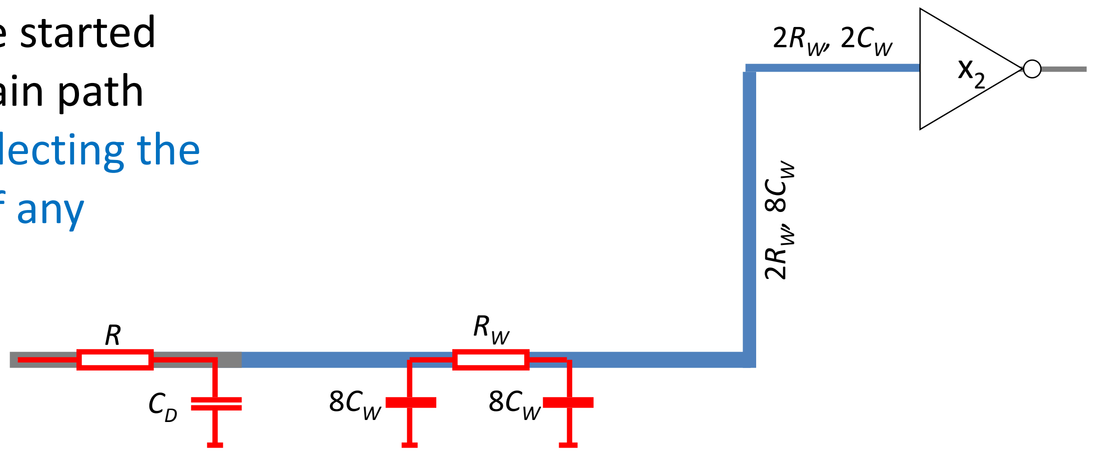

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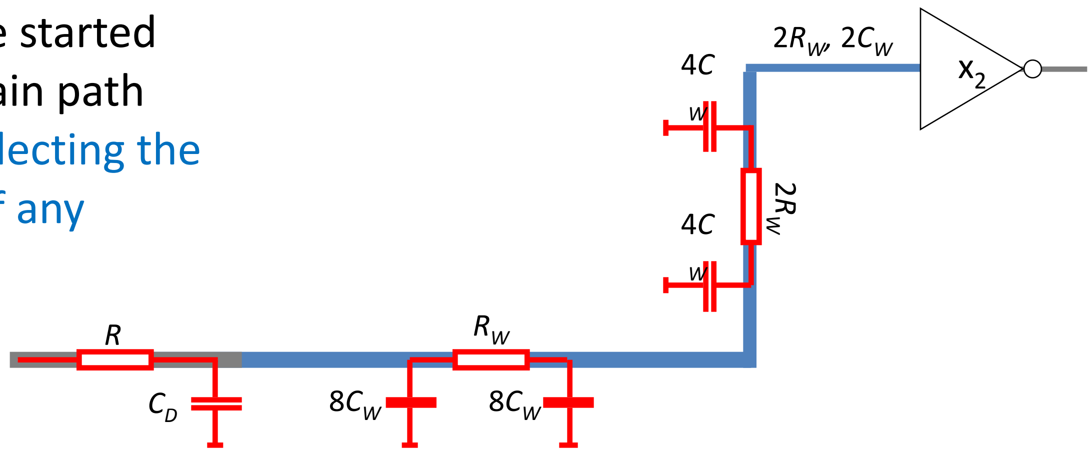

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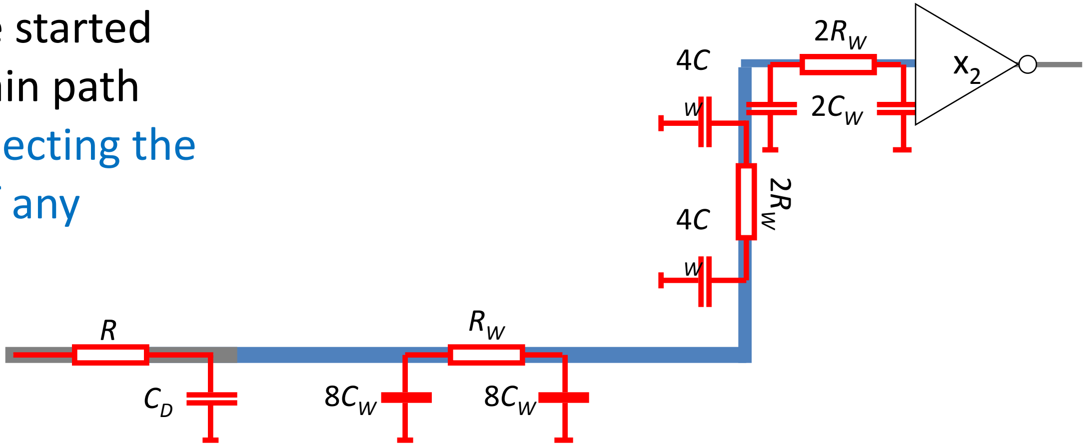

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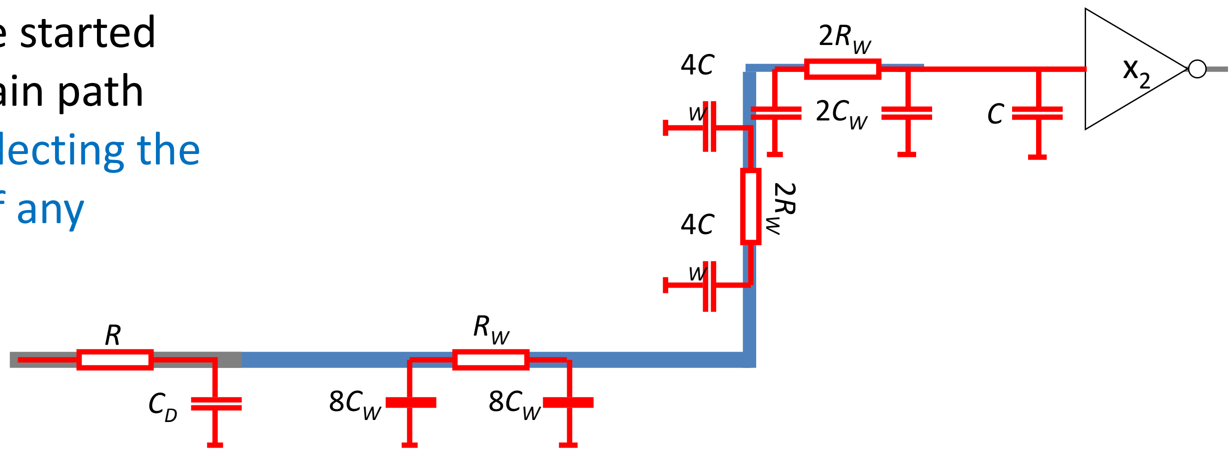

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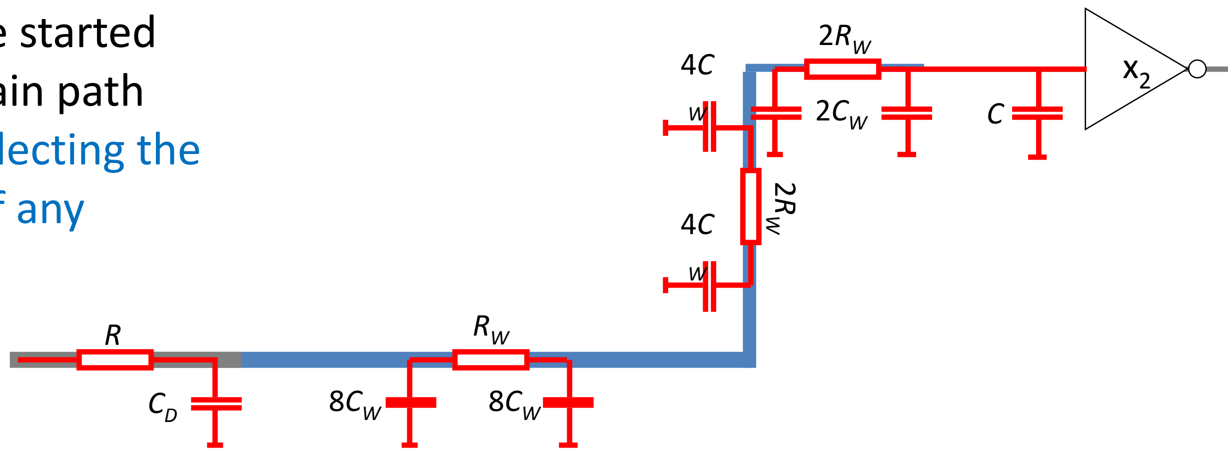
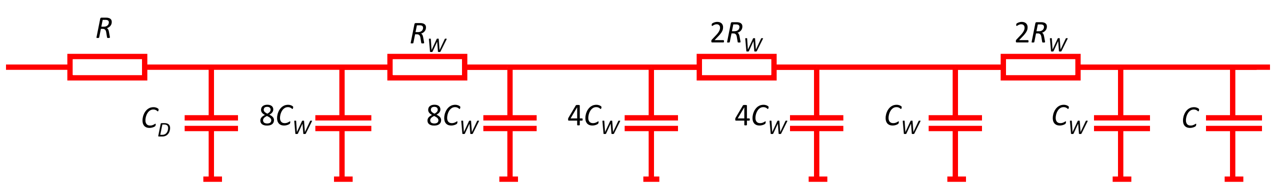


Electrical RC wire model (neglecting branches)



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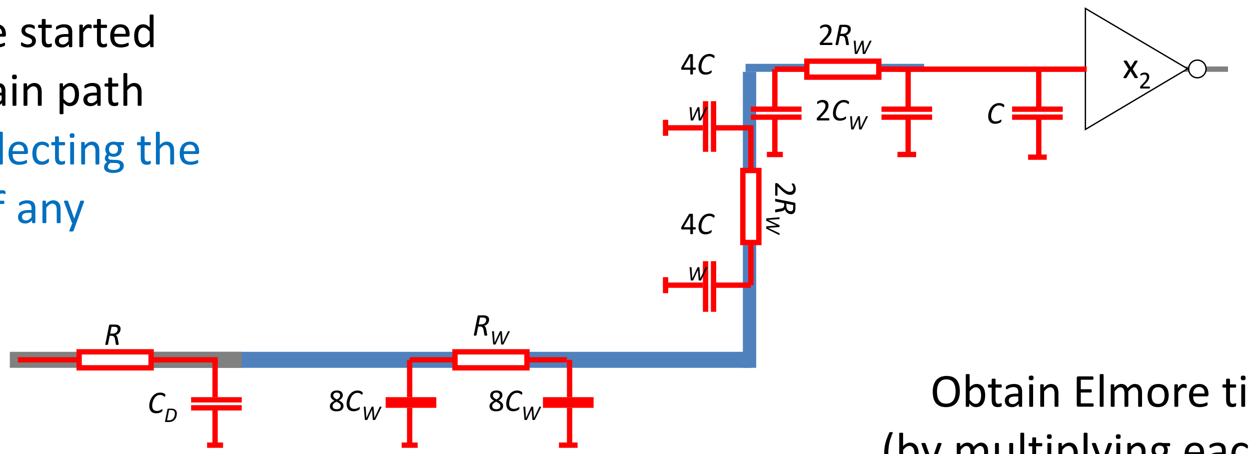
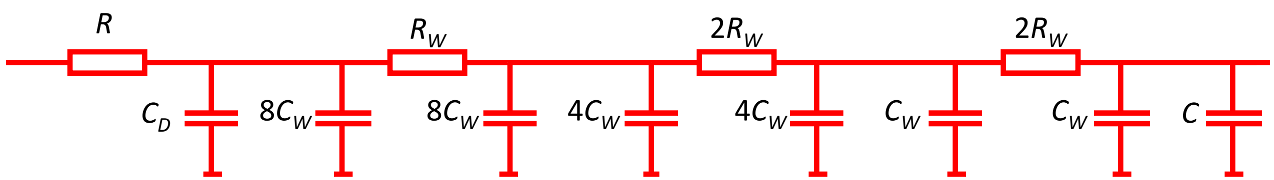


Electrical RC wire model (neglecting branches)

Obtain Elmore time constant (by multiplying each resistance by all downstream capacitances)



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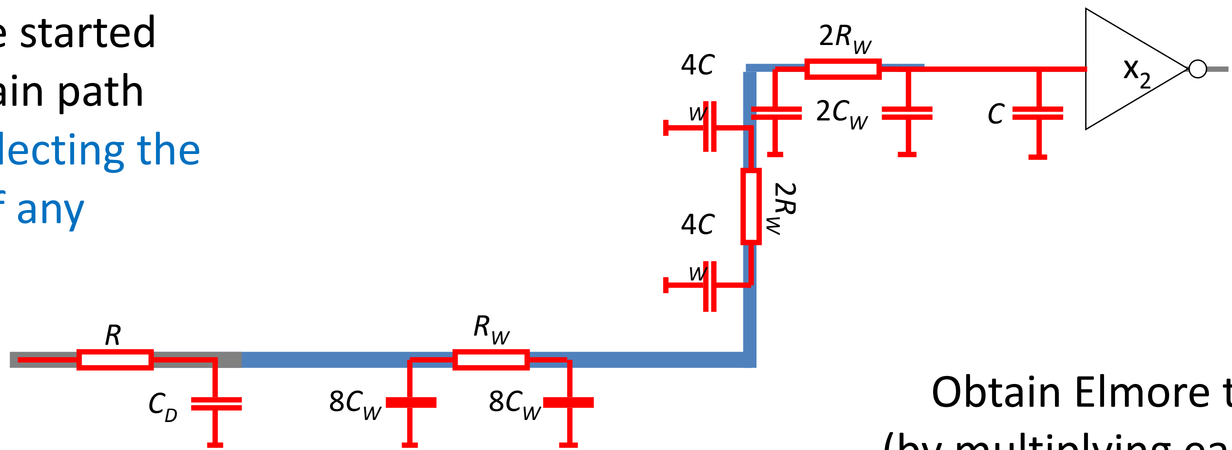

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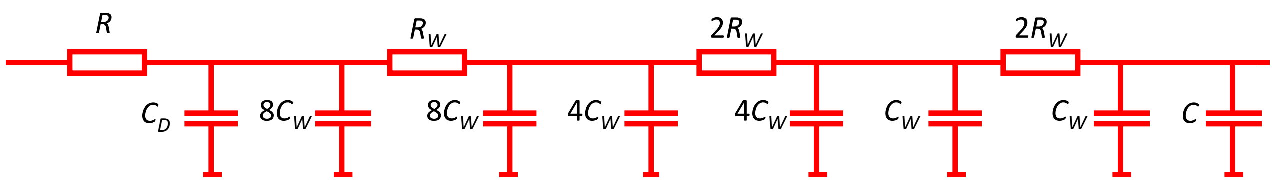

$$T_{E,main} = R\left(C_D + C\right) + R \times 26C_W + R_W\left(C + 18C_W\right) + 2R_W\left(C + 6C_W\right) + 2R_W\left(C + C_W\right)$$

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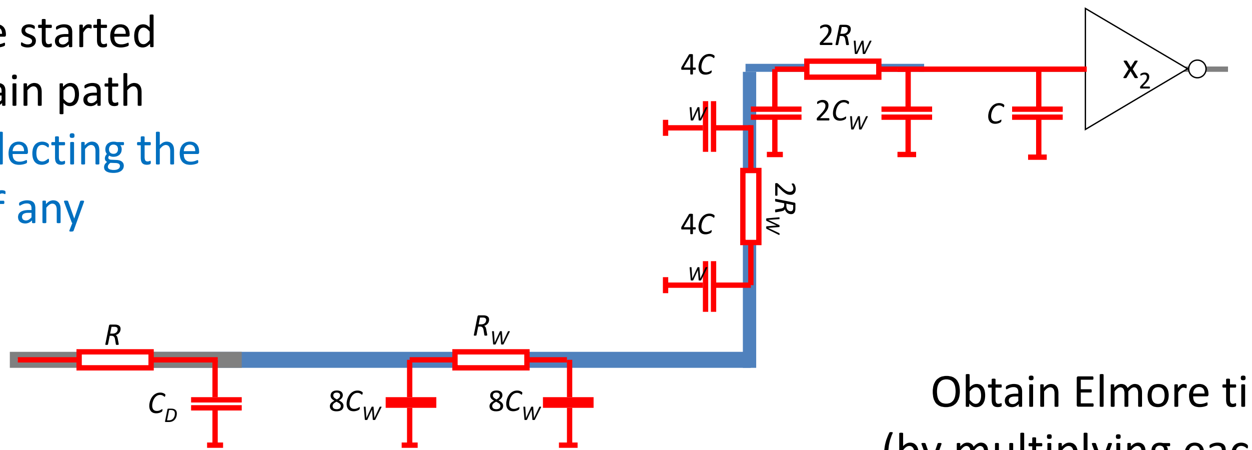

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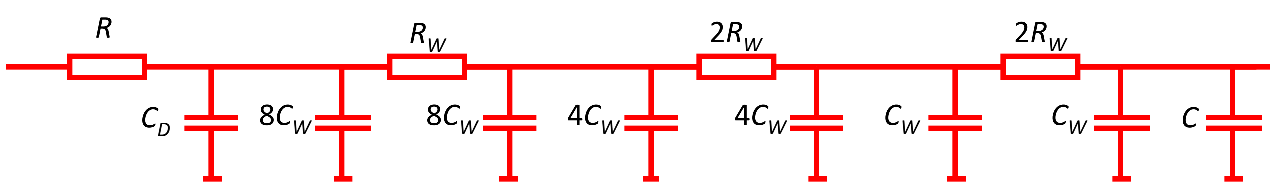

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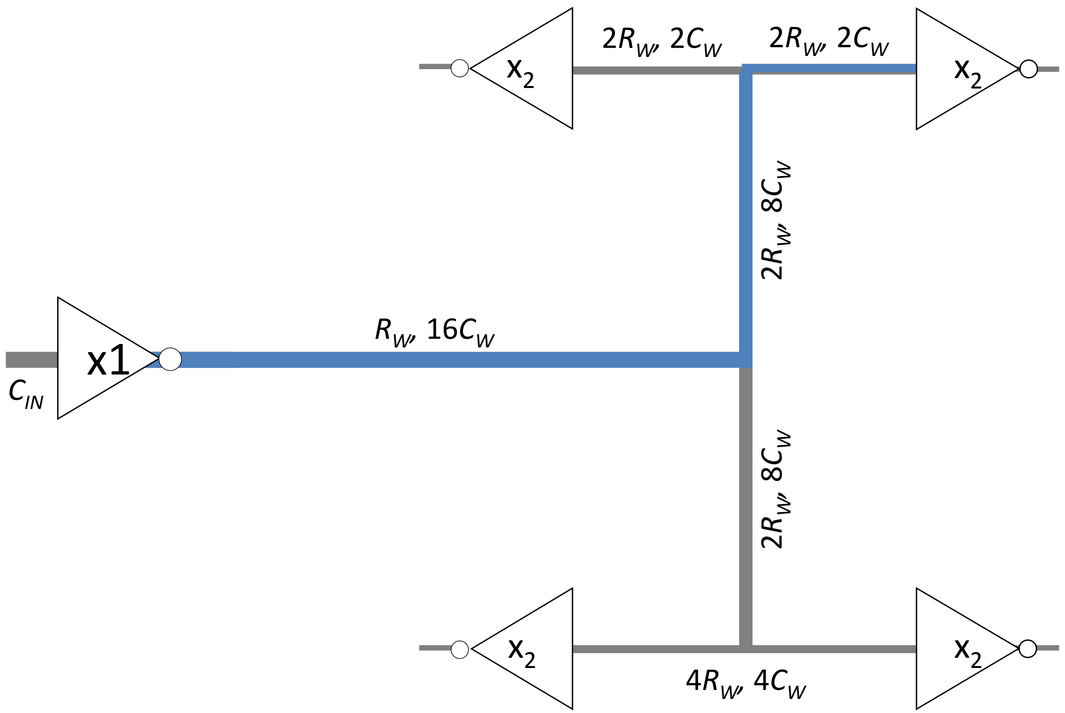
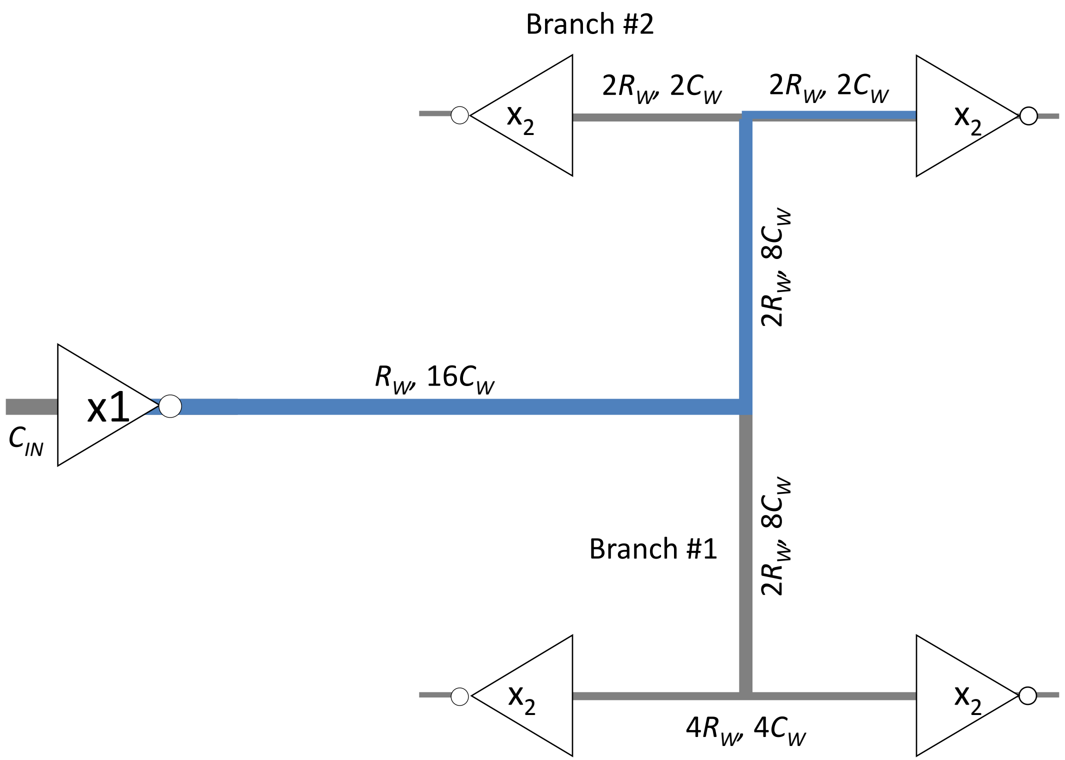
$$T_{E,main} = R\left(C_D + C\right) + 26RC_W + 5R_W C + 32R_W C_W$$

Then consider neglected branches



Then consider neglected branches

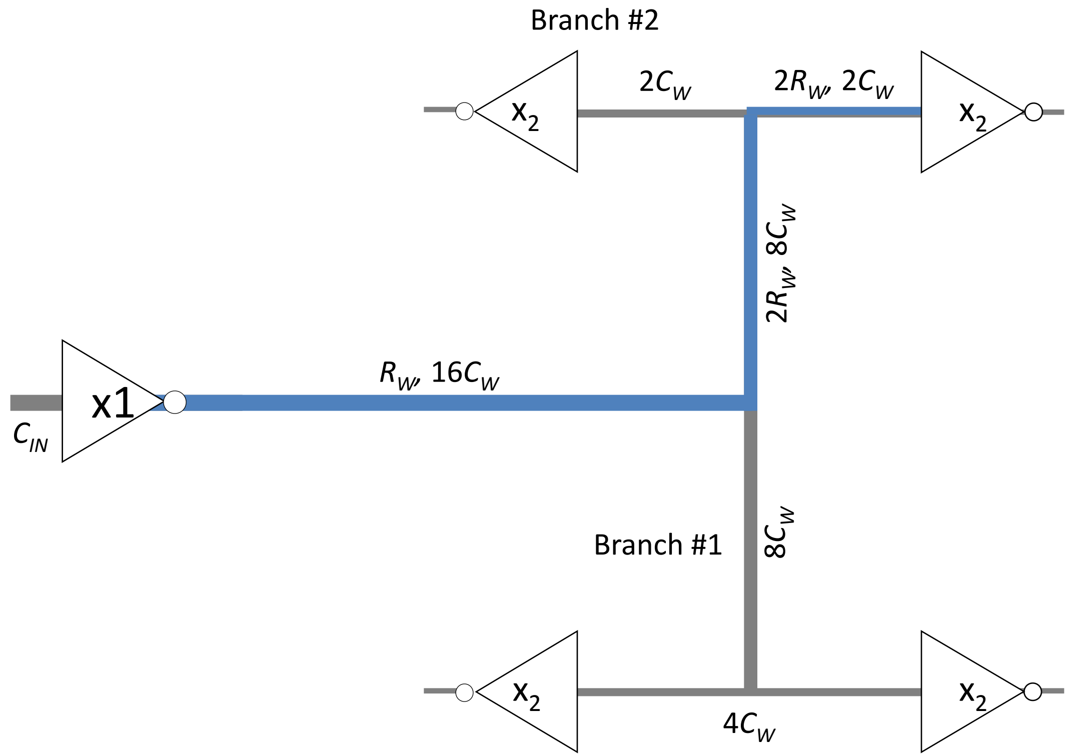
We have two neglected branches: #1 and #2



Then consider neglected branches

We have two neglected branches: #1 and #2

Rule of thumb: Forget about the branch resistances, only consider branch capacitances



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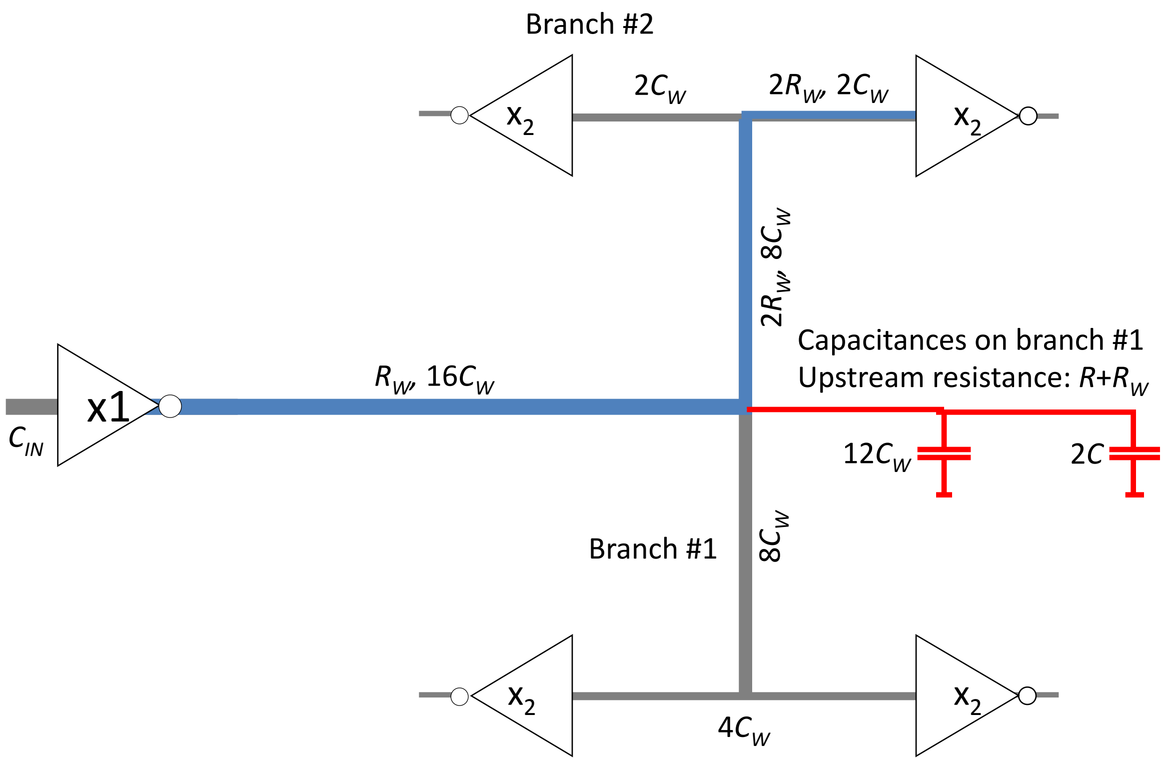

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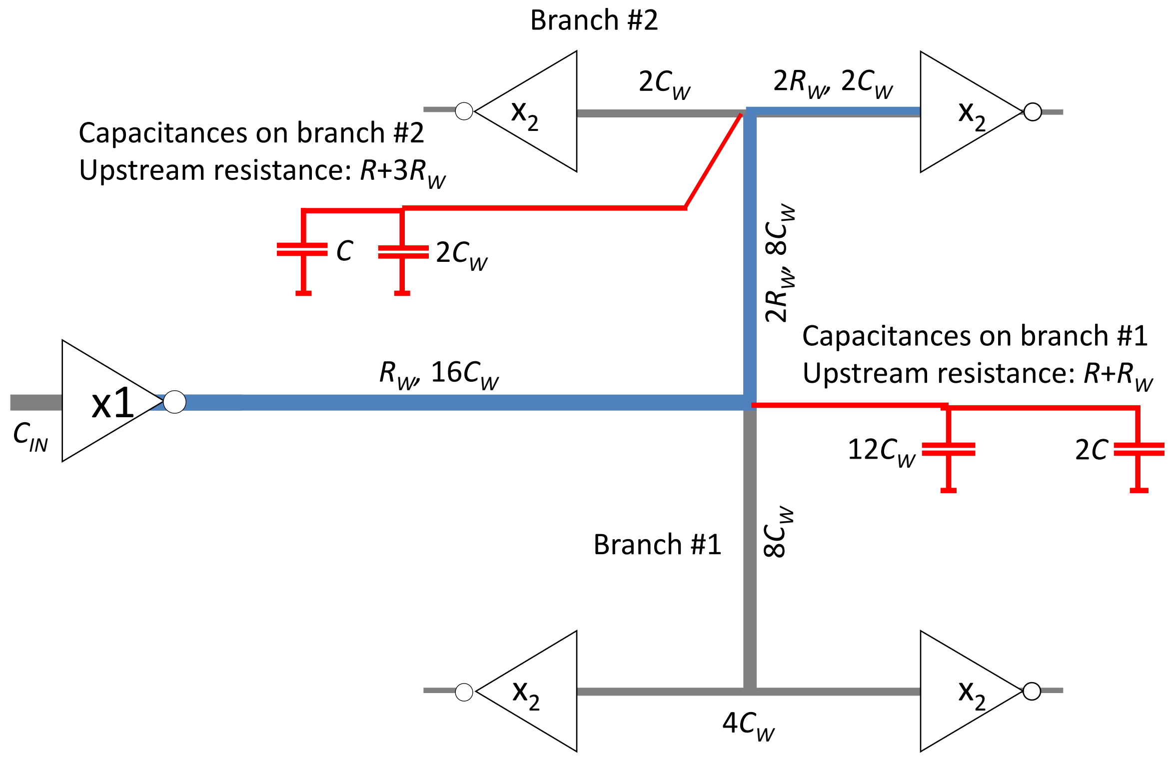

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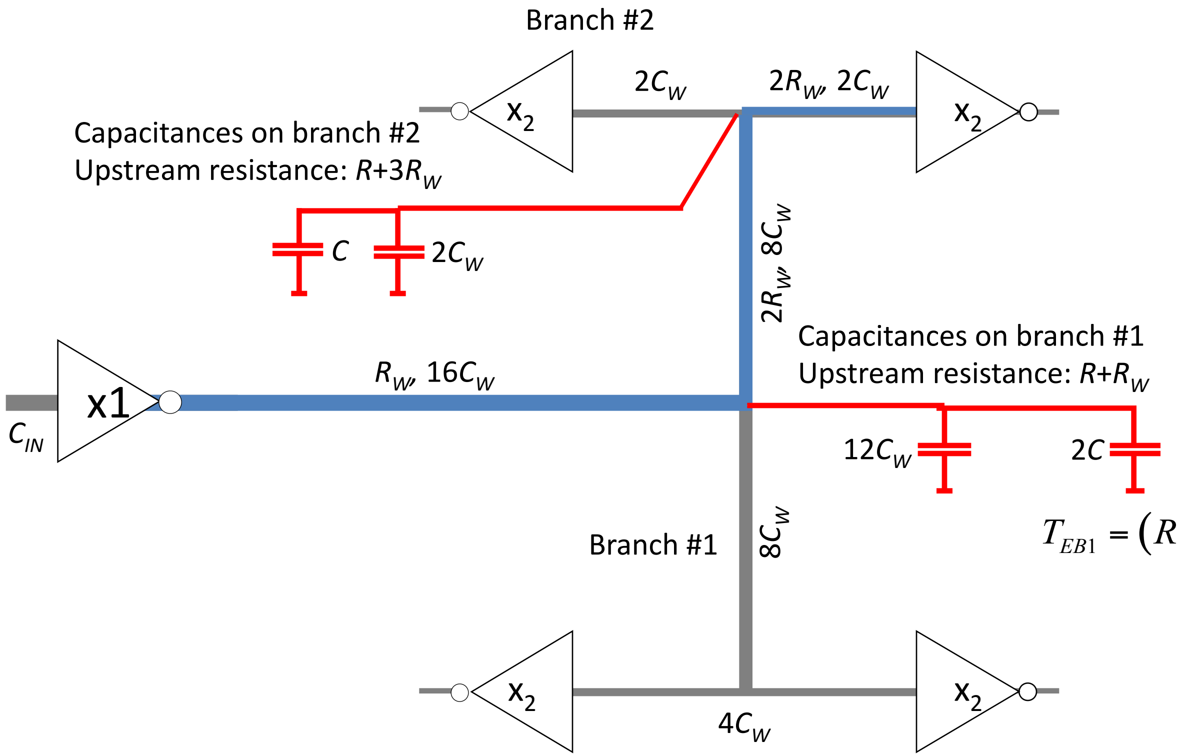

$$T_{EB1} = (R + R_W)(12C_W + 2C)$$

Then consider neglected branches

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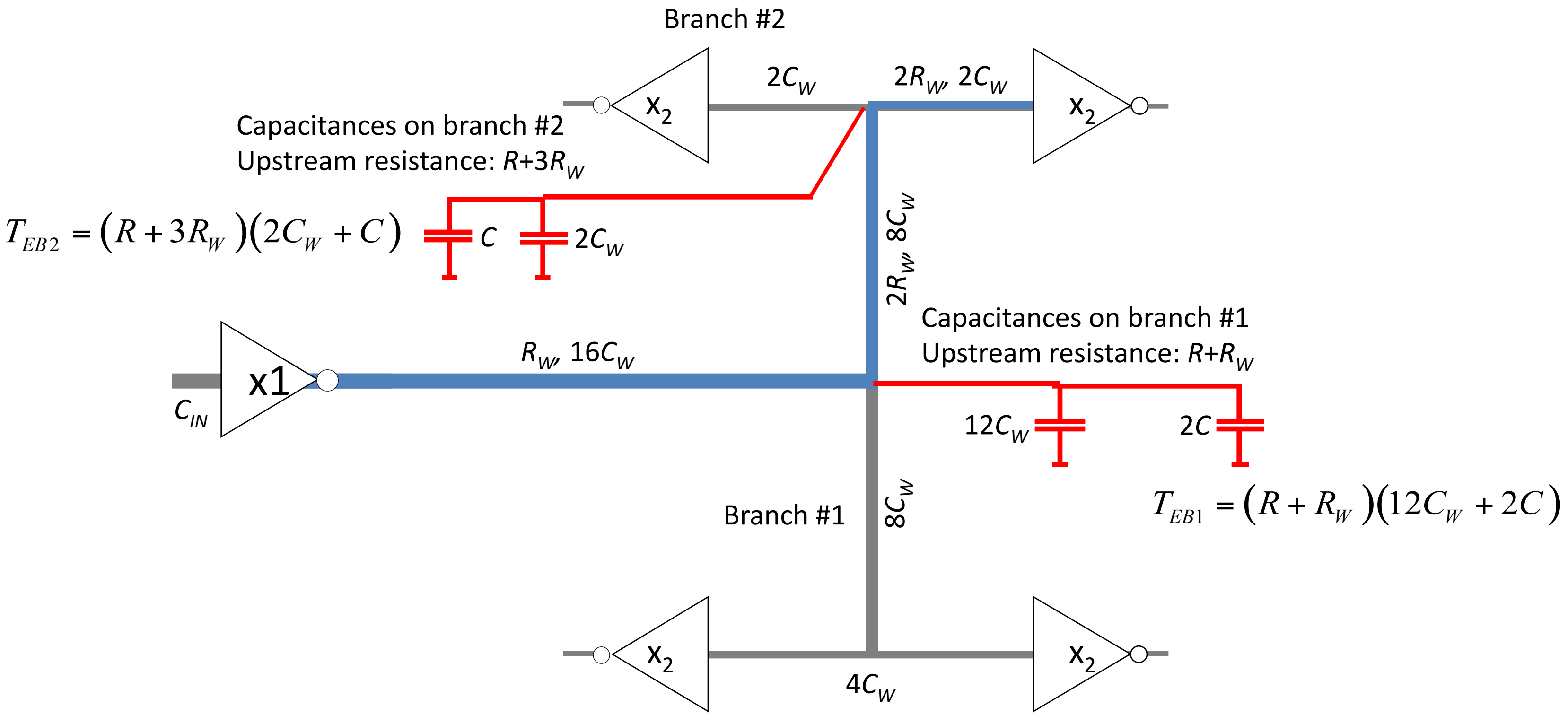

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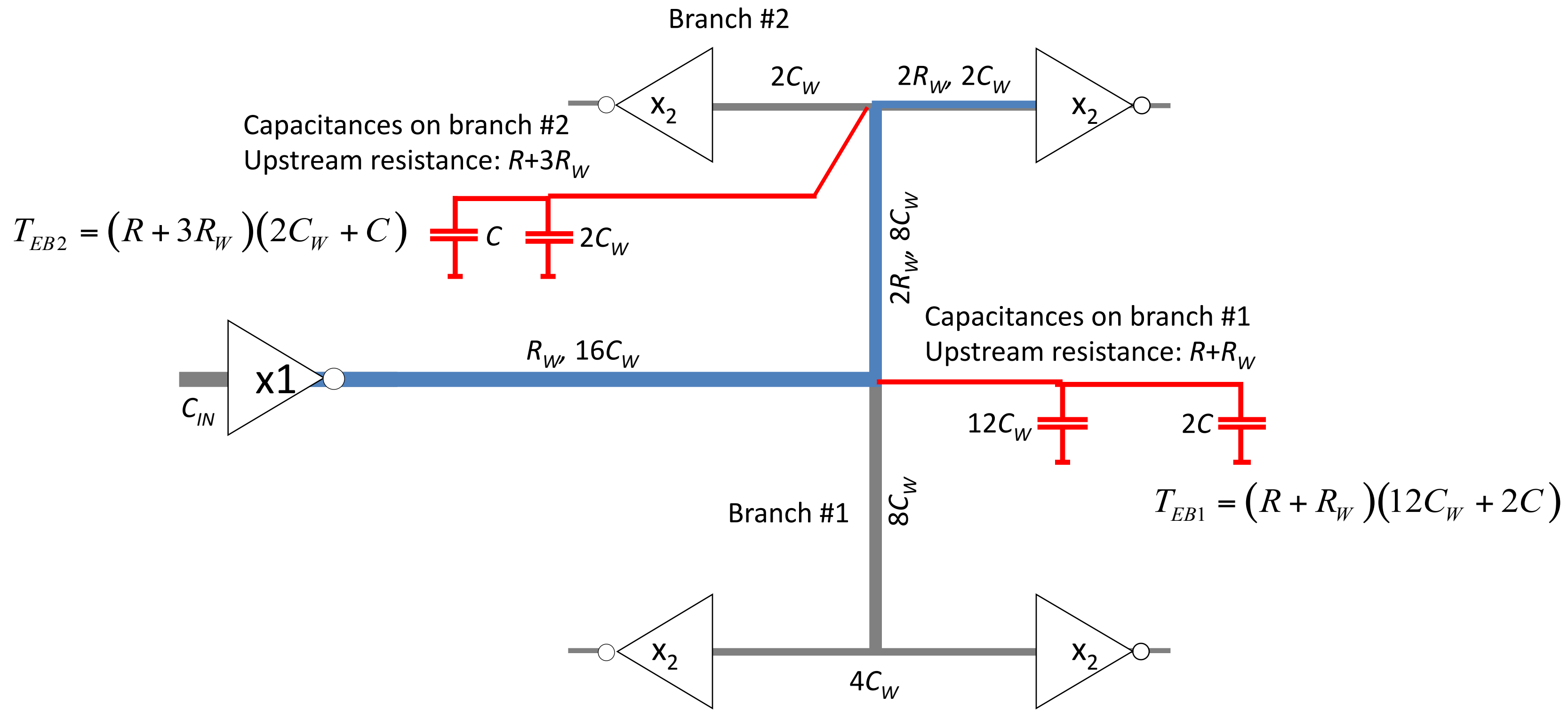

Branch contribution to Elmore time constant: $T_{E,branches} = 3RC + 14RC_W + 5R_WC + 18R_WC_W$

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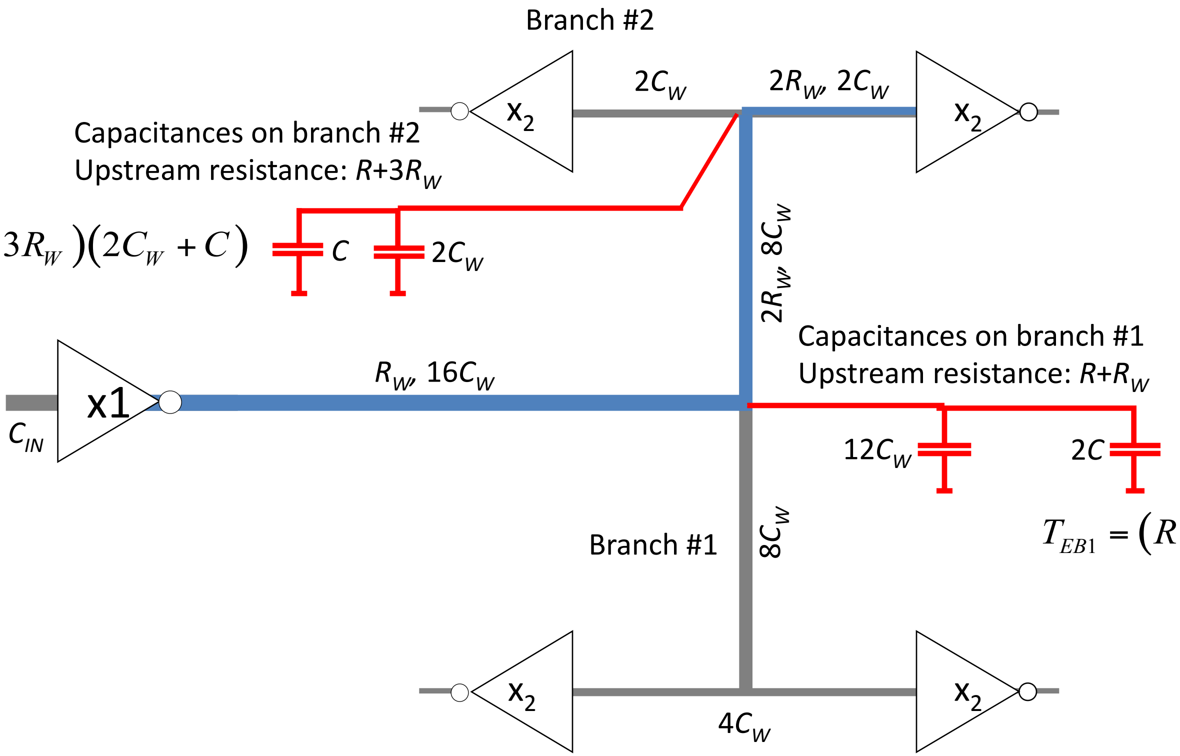


$$T_{EB2} = (R + 3R_W)(2C_W + C)$$

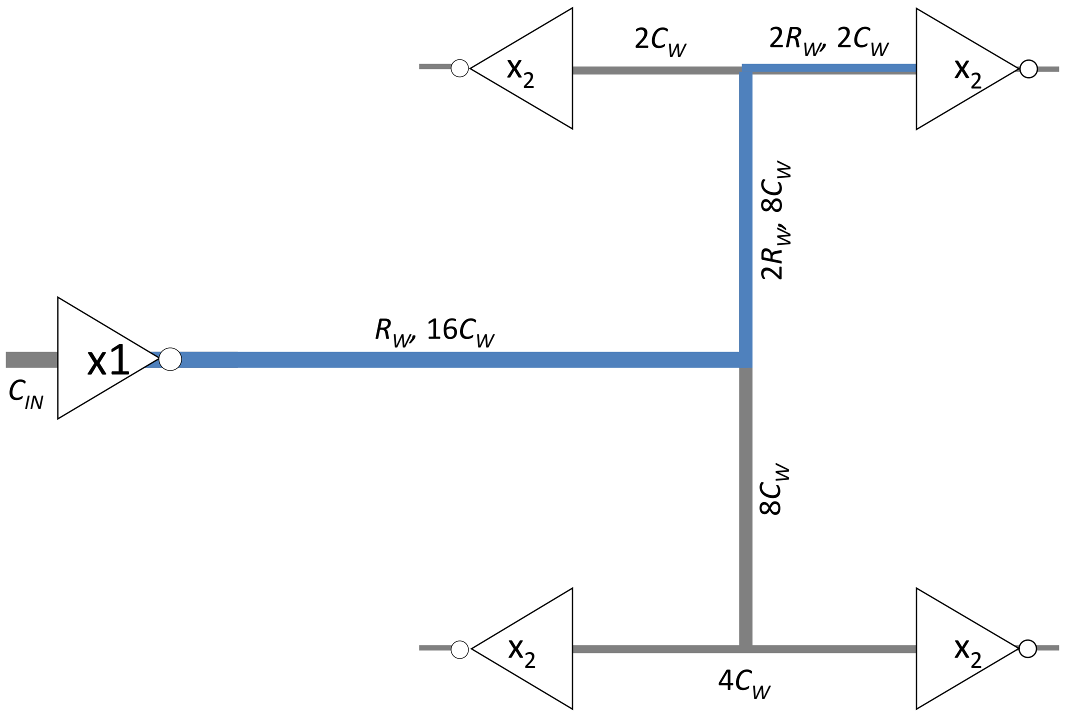
$$T_{EB1} = (R + R_W)(12C_W + 2C)$$

Branch contribution to Elmore time constant: $T_{E,branches} = 3RC + 14RC_W + 5R_W C + 18R_W C_W$

Total Elmore time constant: $T_E = R(C_D + 4C) + 40RC_W + 10R_W C + 50R_W C_W$

Inverter sizing

How to size driver inverter to minimize the time constant, and hence the wire delay?



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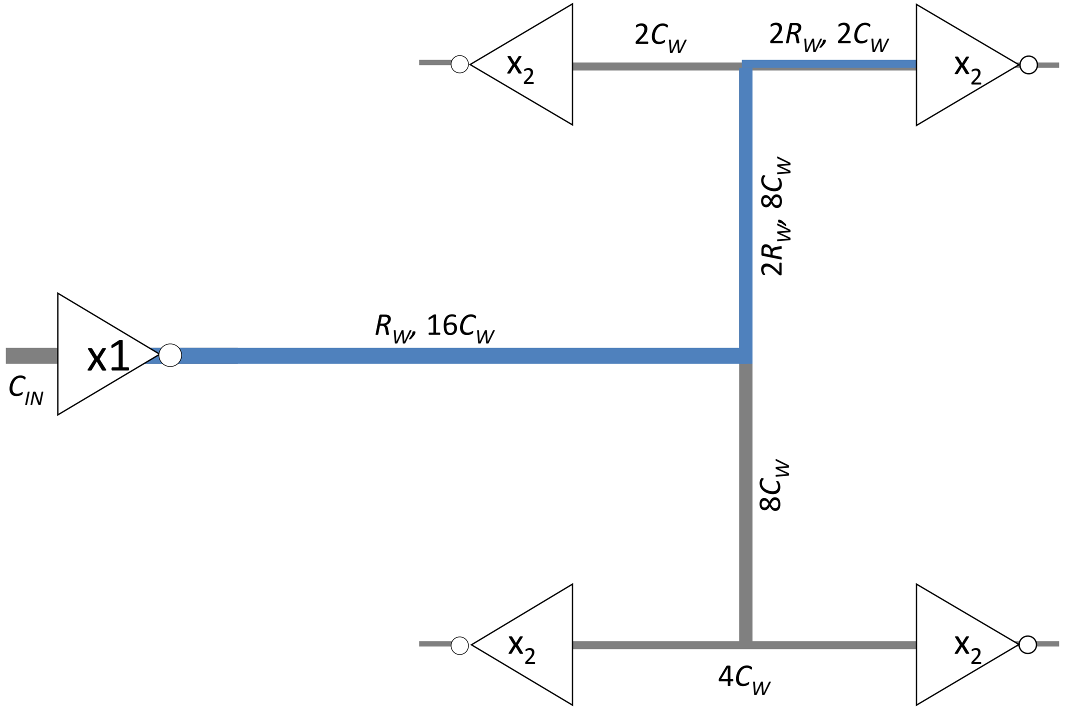
How to size driver inverter to minimize the time constant, and hence the wire delay?

Normalize Elmore time constant: $d = \frac{T_E}{RC} = p_{inv} + 4 + 40\frac{R_W C_W}{RC}\frac{R}{R_W} + 10\frac{R_W}{R} + 50\frac{R_W C_W}{RC}$



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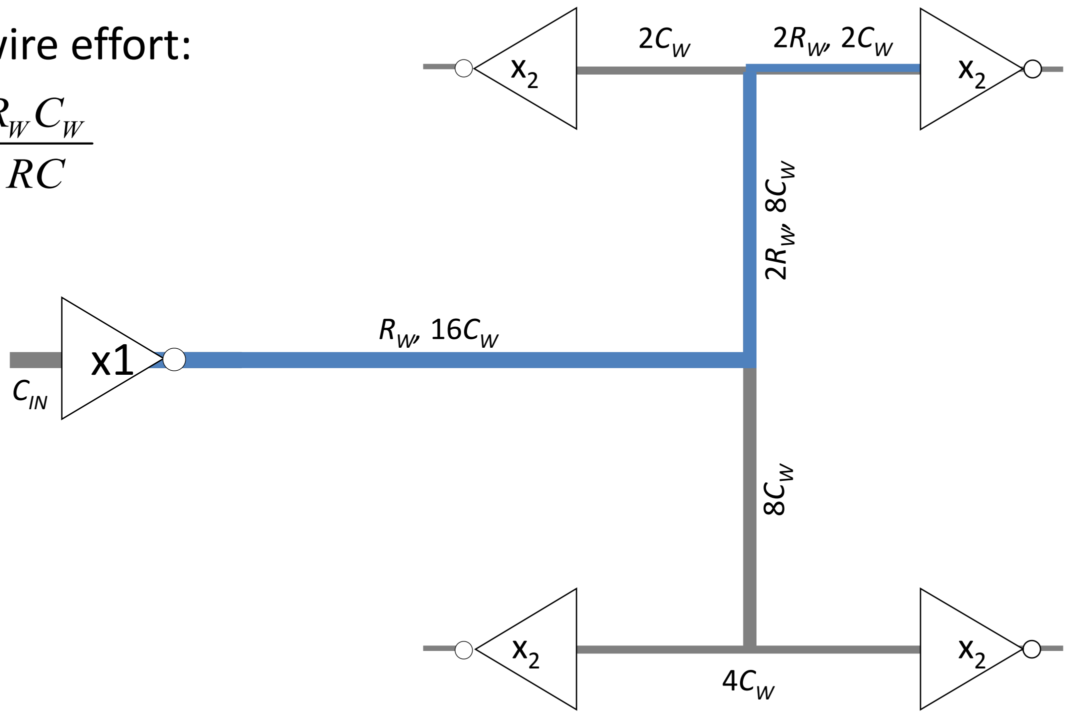
Define wire effort:

$$W_E = \frac{R_W C_W}{RC}$$



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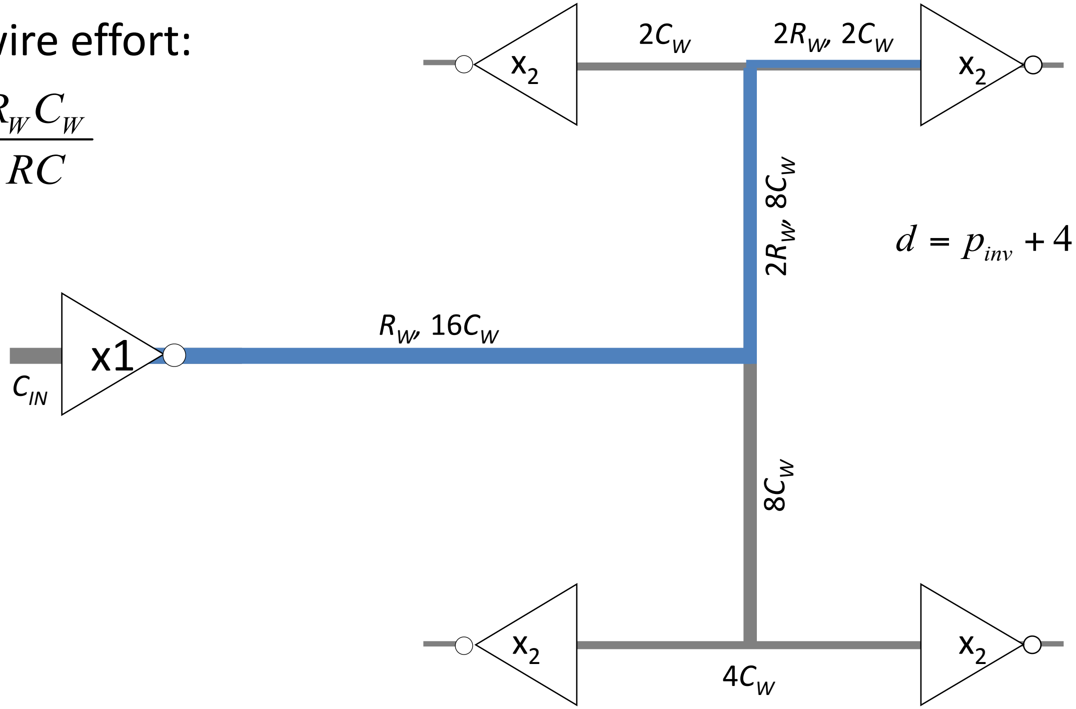

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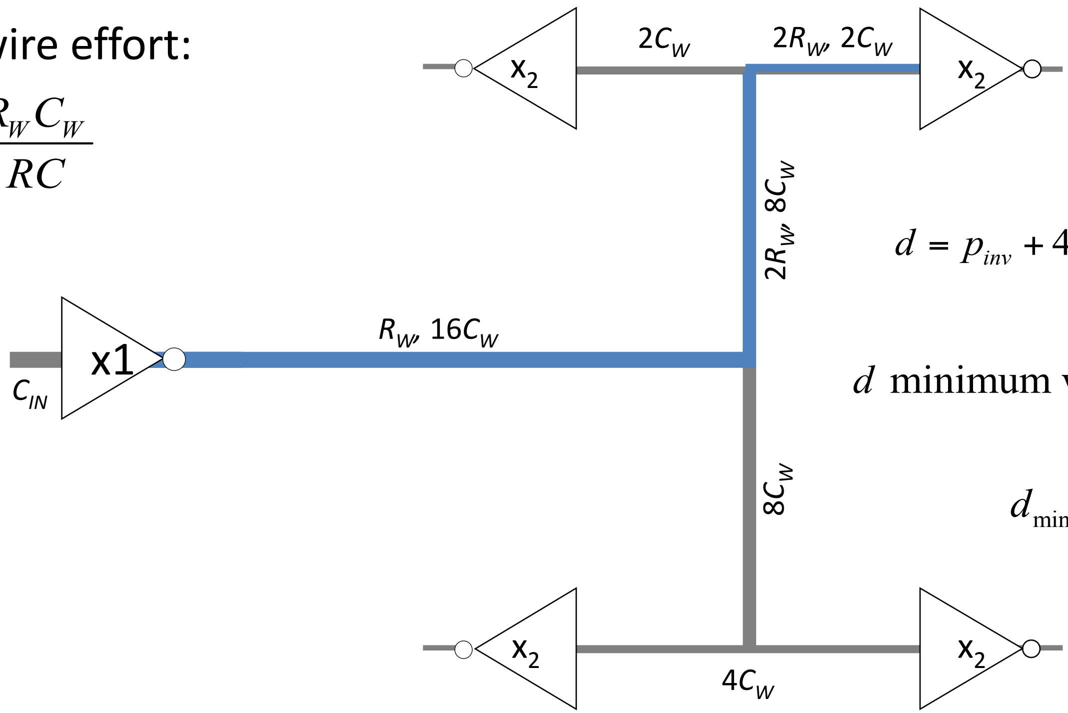

$$d = p_{inv} + 4 + 40W_E\frac{R}{R_W} + 10\frac{R_W}{R} + 50W_E$$

d minimum when $\frac{\partial d}{\partial R} = 4\frac{W_E}{R_W} - 1\frac{R_W}{R^2} = 0$

$d_{\min}$ for $R = \frac{R_W}{2\sqrt{W_E}}$

Total Elmore time constant: $T_E = R\left(C_D + 4C\right) + 40RC_W + 10R_W C + 50R_W C_W$

Conclusion

We have learnt two things:

- The Elmore model – a generalized delay model
  - Relying on the existence of a dominating time constant
- How to handle the influence on delay of branches
  - Main timing path first, neglecting branches
  - Then consider branches neglecting any branch resistances only considering branch capacitances!

Thanks a lot for listening!