Jitter and error estimation

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Why study jitter?

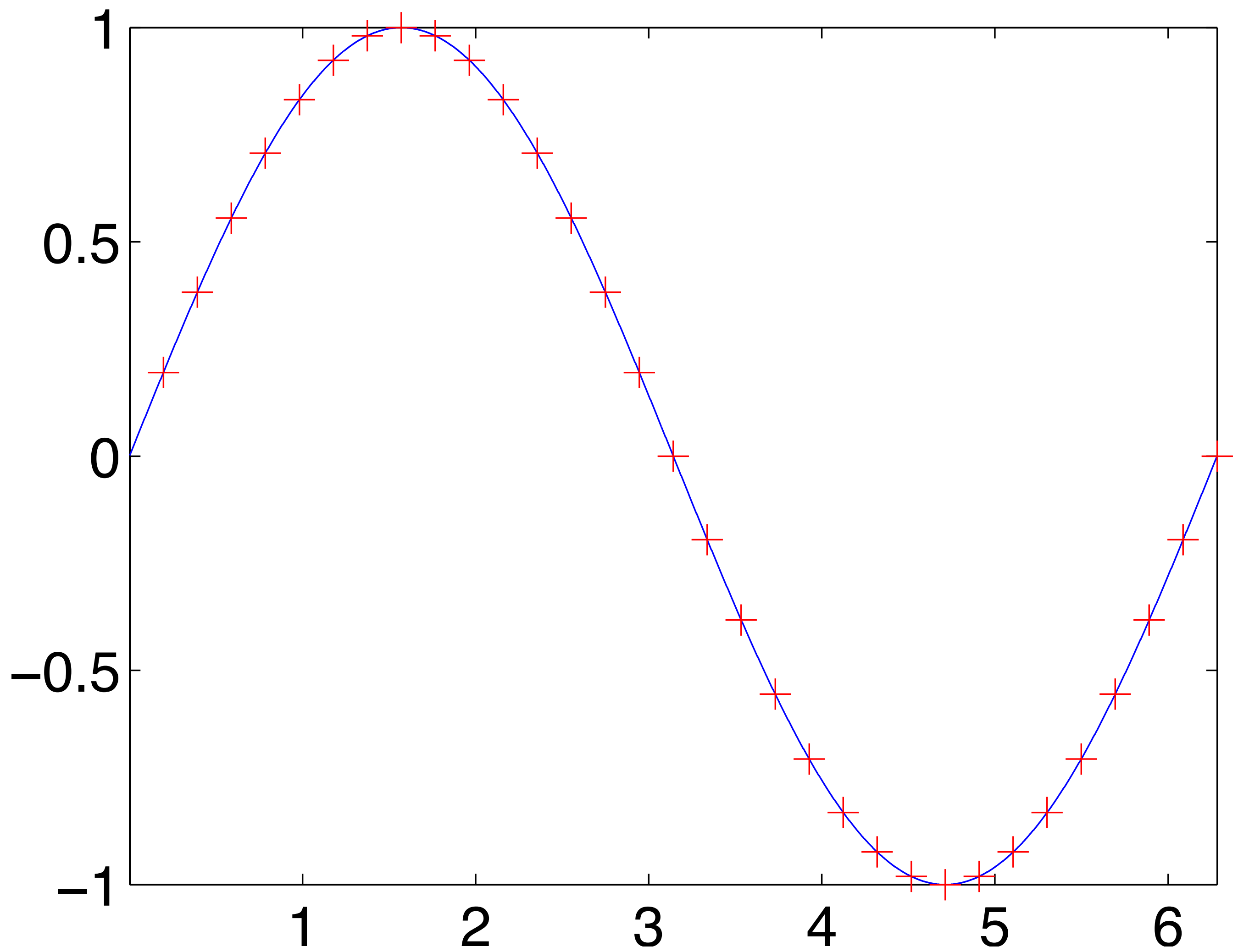
- Sampled representation typically assumes uniform sample intervals
  - Assumption often unspoken
- In practice, sampling never perfectly uniform!
  - Inaccuracy limits performance
- Accuracy improvement may be expensive
  - Designer awareness important

Why error estimation?

- Balanced designs require distribution of requirements across subdesigns
  - Need to be able to compare influence of different non-idealities
  - Avoid costly overdesign
- Jitter used as first example today
  - Principle will reoccur during course

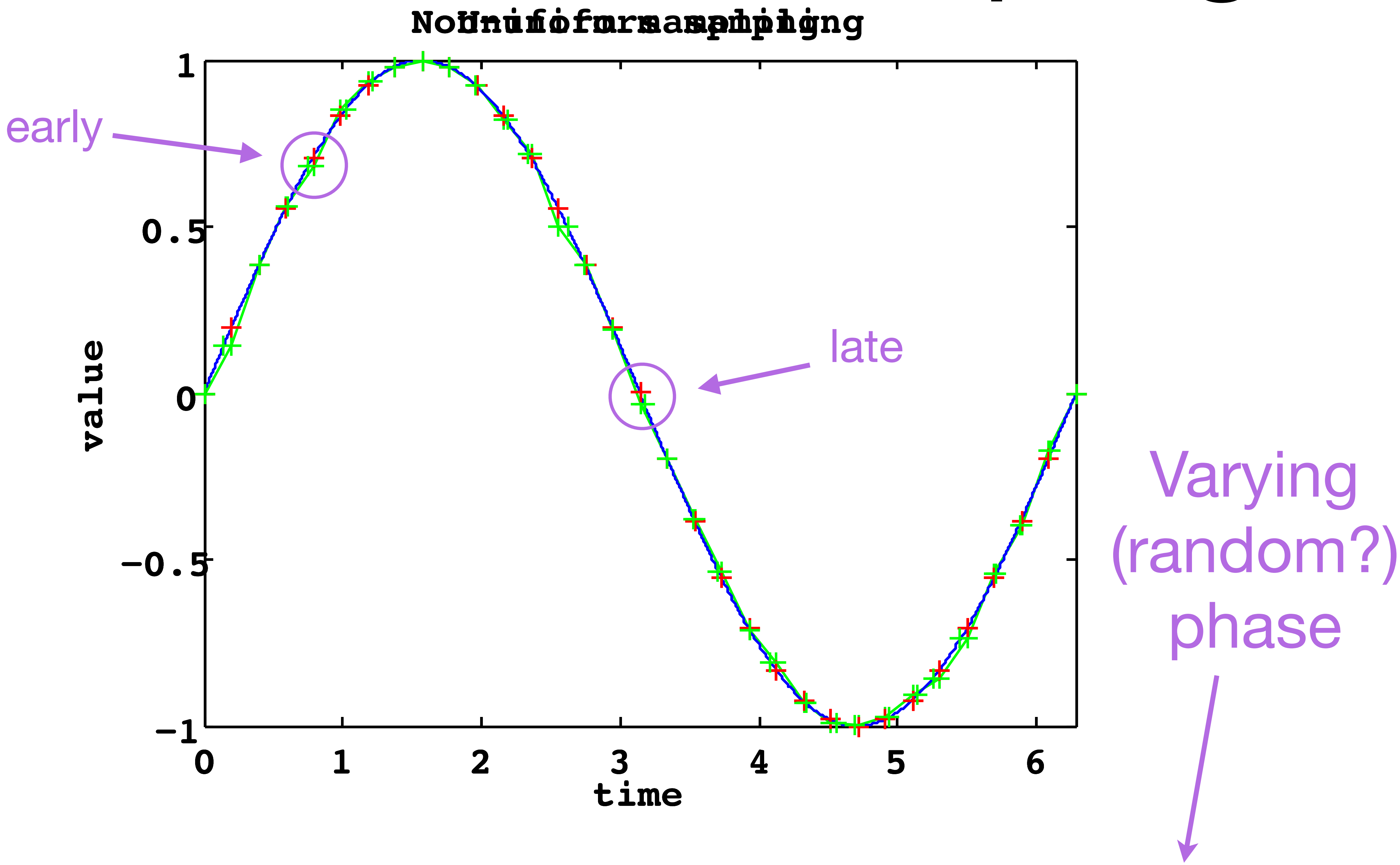
Sampling inaccuracies

- Ideally (uniform sampling):



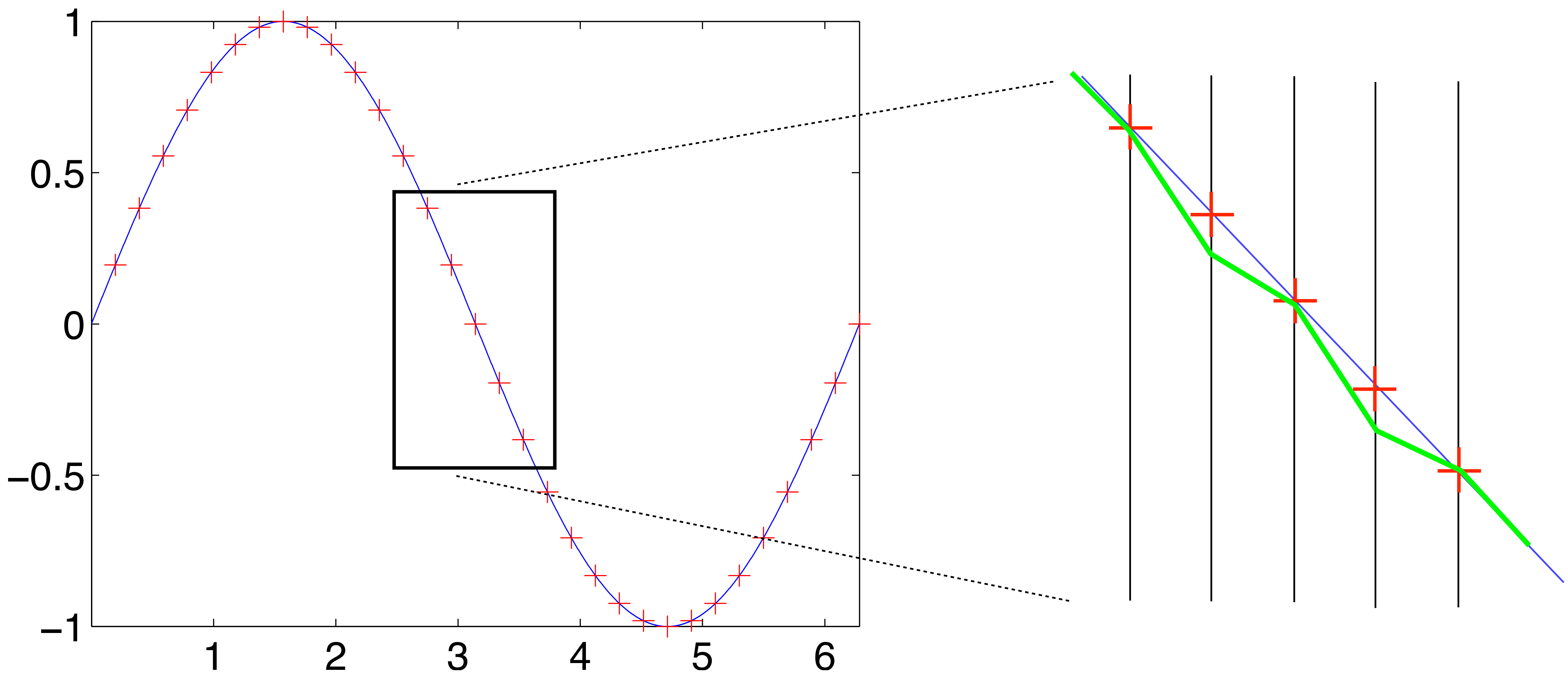
$$\boxed{y_k = sin(x_k), x_k = (2\pi/f_s) \cdot k}$$

Non-uniform sampling



$$y_k = sin(x_k), x_k = (2\pi/f_s) \cdot k + \phi_k$$

Closeup

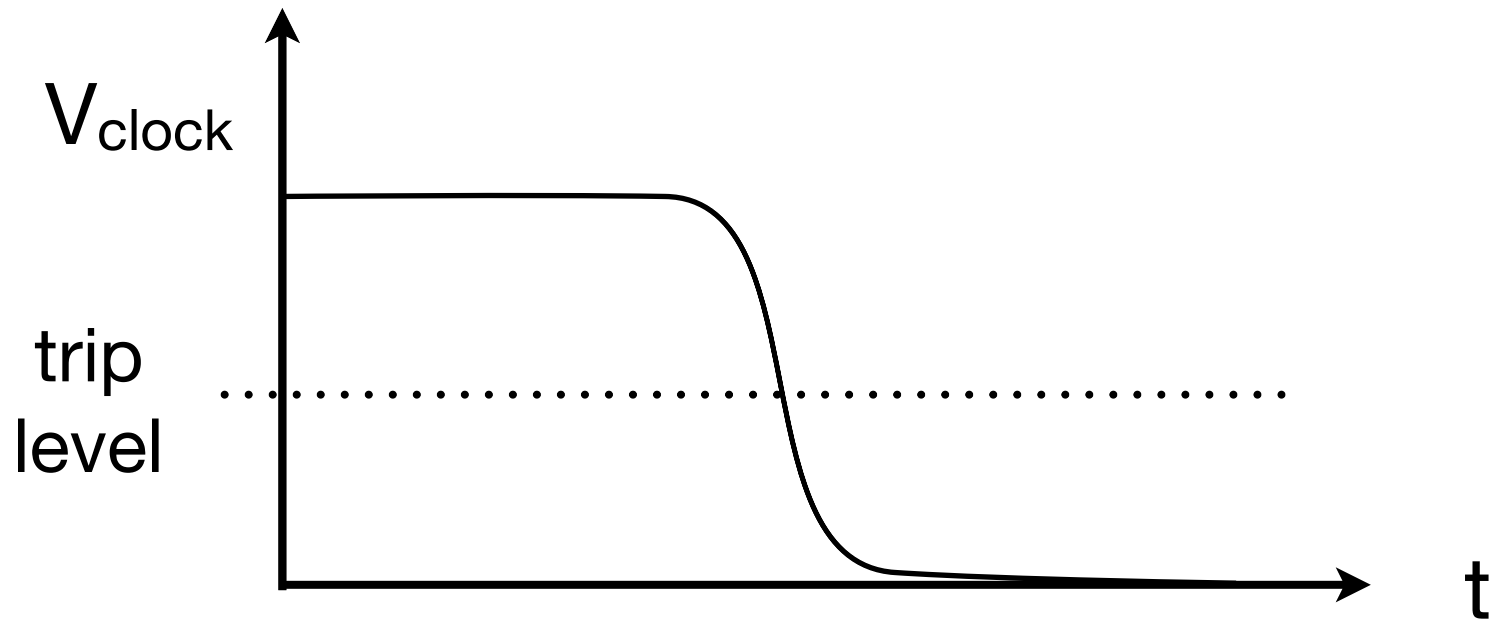


- Uniform-sampling assumption makes non-uniform sampling appear to cause <u>additive noise</u>

Jitter causes?

- Here are some:
  - Supply voltage noise
  - Sampler nonlinearity
  - Other signal coupled into sample clock
  - Digitally generated sample clock
  - Random phase noise (in all oscillators)
  - …
- Avoid over-engineering to remove one cause!

Example: sample clock level



- Capacitive coupling from other signals or coupling from supply may affect level of clock signal
- Will affect sample instance
- Fast sample edge helps, but $, W

Deterministic jitter

- Ex: sample clock generated by flip-flop to get 50% duty cycle
- FF asymmetry delays every other edge

Purely random jitter

- Ex: Oscillator phase noise
- Unavoidable; needs $, W to reduce

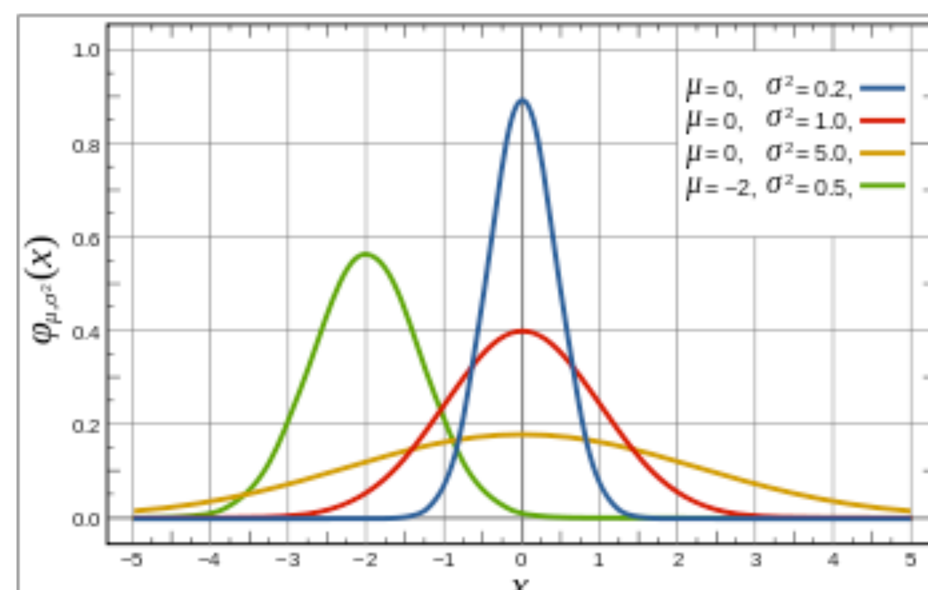
Error estimation

- SNR: signal to noise ratio (also SNDR, etc)
  - Ratio of signal power to error power : $SNR = P_{sig} / P_{err}$
- Express ratio in dB
  - Log scale: $SNR_{dB} = 10 \cdot \log_{10}(SNR)$
    - Ex: 1mW is 30 dB below 1W

Why error <u>power</u>?

- …rather than, say, maximum or average error?
  - Power of a sum is sum of powers (if signals uncorrelated)
    - Handle composed signals by parts (easier!)
    - Cf. linearity: $P(ax + b) = a^2\, P(x) + P(b)$
  - Covers practically important phenomena such as <u>additive Gaussian noise</u>
    - No maximum error, but

      power still well-defined



Why a power <u>ratio</u>?

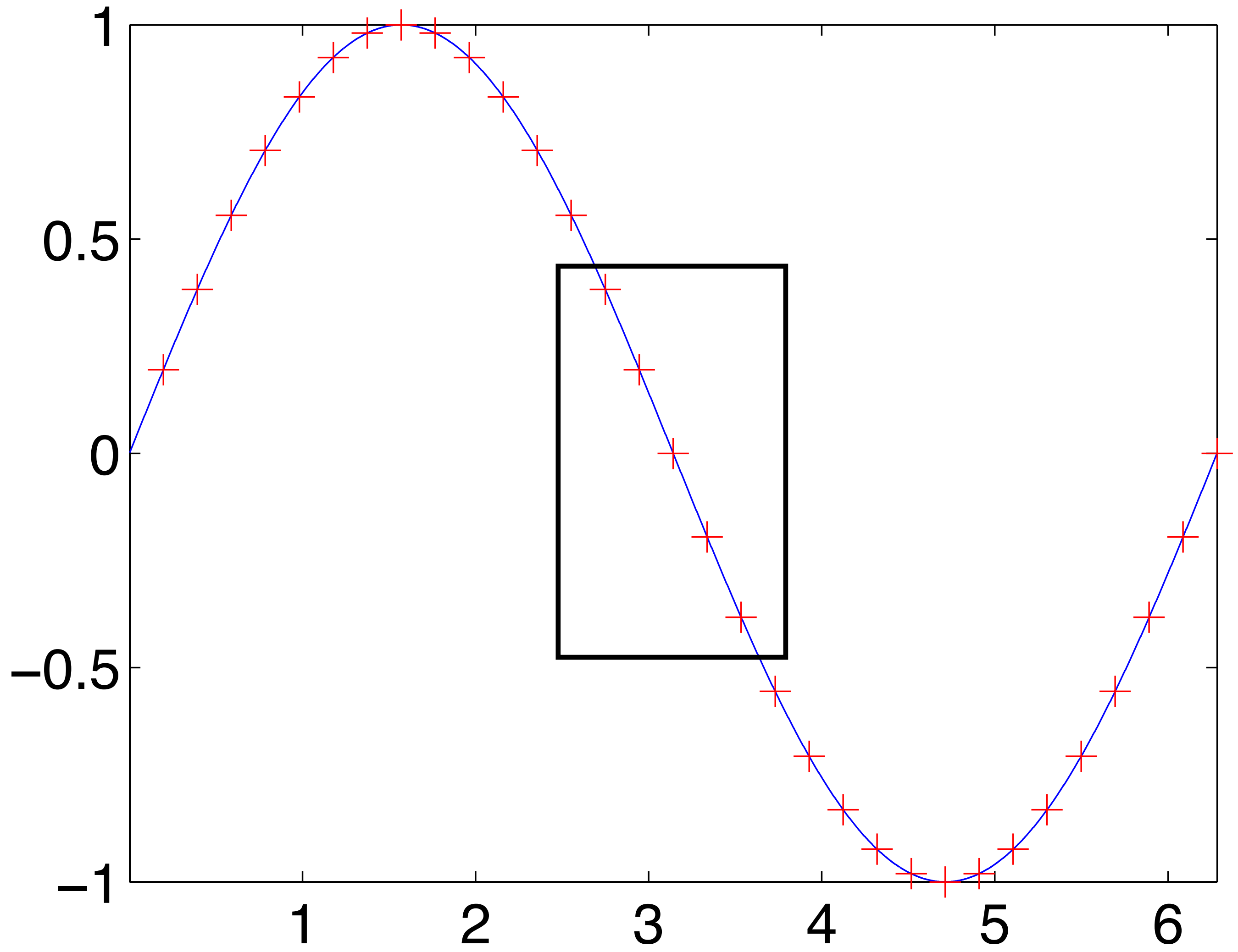
- SNR unchanged by scaling
  - Same SNR before and after (ideal) amplifier
- Allows us to represent power with square of voltage, disregarding R (cf. lab 0!)
  - $P_{sig} = V_{sig}^2 / R$; $P_n = V_n^2 / R$; $P_{sig} / P_n = V_{sig}^2 / V_n^2$
- Often good (not perfect) predictor for "performance"
  - Sensory (sight, hearing) masking
  - Telecommunication bit-error rates
  - …

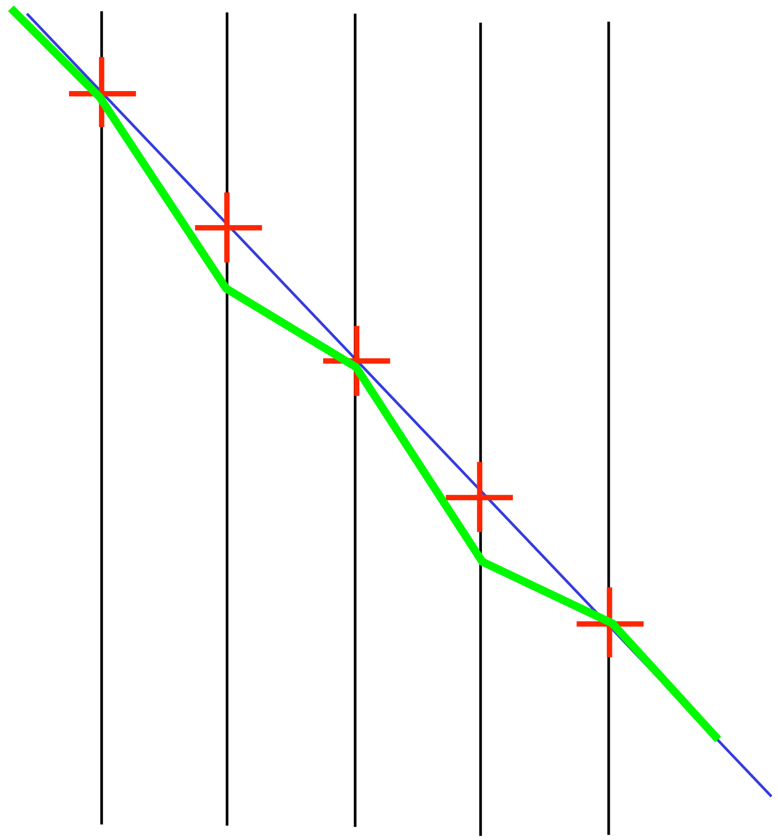
Why dB scale?

- Ubiquitous in signal processing
- Actual ratios (SNRs, gains) often numerically big
  - Gain A = 1000000 is easy to misread
  - Cf. A = 120 dB
- Mental add/sub is easier than mul/div (for most people)
  - Example: two-stage amp; need gain $A_{tot} = A_1 \cdot A_2$; if A_2 given, then $A_1 = A_{tot} / A_2$

Jitter-limited SNR

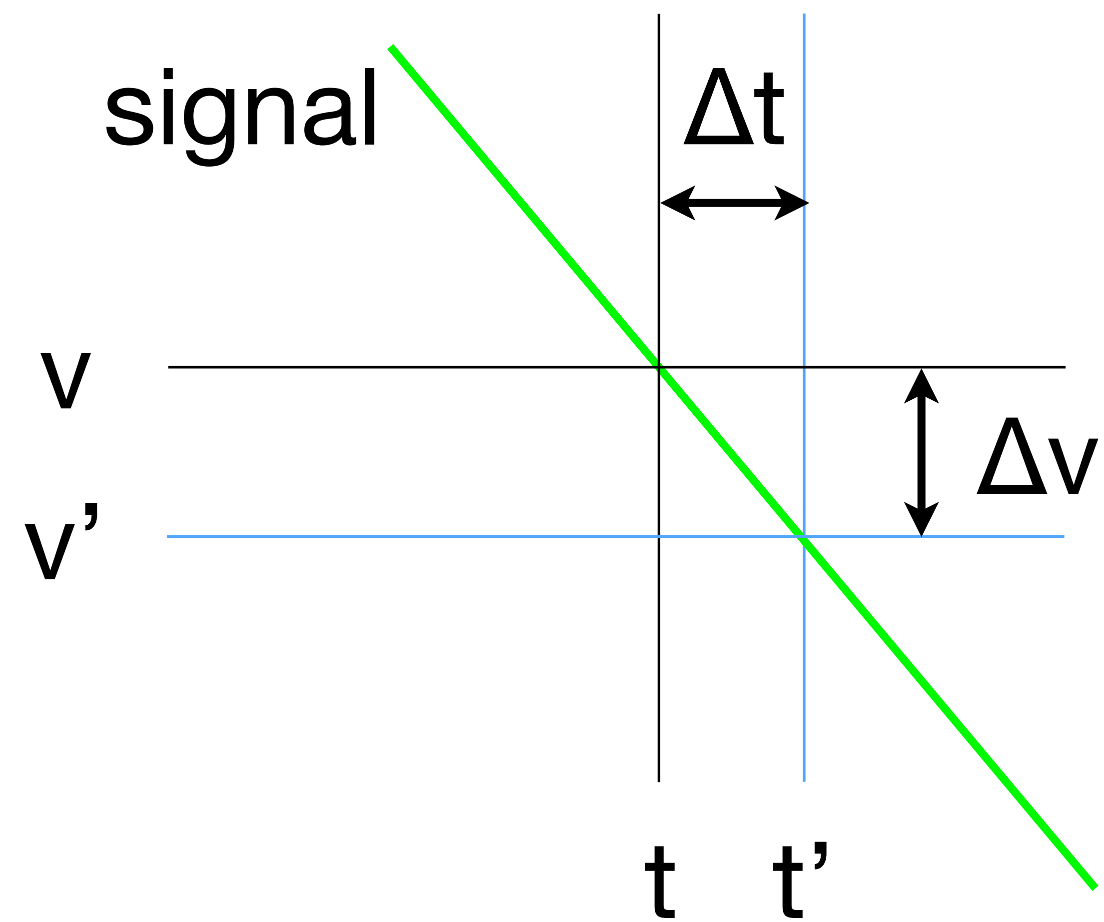
Recall:





- Non-uniform sampling with uniform assumption causes additive noise!

Quantitatively:



- Nominally: sample at t, get v
- Actually: sample at t’, get v’
- At small $\Delta t = t' - t$, then $\Delta v = v' - v \approx \Delta t \cdot \partial v/\partial t$

$\partial v/\partial t$ estimate

- Time derivative, i.e. rate-of-change, of signal v
  - Signal dependent…
- Assume simple sine wave “test” signal

$v = A \sin(2\pi f \cdot t)$

$\partial v / \partial t = A \cdot 2\pi f \cos(2\pi f \cdot t)$

Magnitude grows with A, f

Error power estimate

- $\Delta v \approx \Delta t \cdot \partial v/\partial t = \Delta t \cdot A \cdot 2\pi f \cos(2\pi f \cdot t)$
- $(\Delta v)^2 = (\Delta t)^2 \cdot (\partial v/\partial t)^2$

  $= (\Delta t)^2 \cdot A^2 \cdot (2\pi f)^2 \cos^2(2\pi f \cdot t)$
- $P_{err} = \text{avg}((\Delta v)^2)$

  $\approx \text{avg}((\Delta t)^2) \cdot \text{avg}(A^2 \cdot (2\pi f)^2 \cos^2(2\pi f \cdot t))$

  $= \text{avg}((\Delta t)^2) \cdot A^2 \cdot (2\pi f)^2\ \boxed{\text{avg}(\cos^2(2\pi f \cdot t))}$

  $= \text{avg}((\Delta t)^2) \cdot A^2 \cdot (2\pi f)^2\ \boxed{/ 2}$
- Assumption: Δt uncorrelated with v

Signal power, SNR

- $v = A \sin(2\pi f \cdot t)$
- $v^2 = A^2 \sin^2(2\pi f \cdot t)$
- $P_{sig} = \mathrm{avg}(v^2) = A^2\, \mathrm{avg}(\sin^2(2\pi f \cdot t)) = A^2 / 2$

- SNR: $(A^2 / 2) / (\mathrm{avg}((\Delta t)^2) \cdot A^2 \cdot (2\pi f)^2 / 2)$
- In dB: $-10 \log((2\pi f)^2 \cdot \mathrm{avg}((\Delta t)^2))$
- Maloberti: $-20 \log(2\pi f \cdot \mathrm{sqrt}(\mathrm{avg}((\Delta t)^2))) =$
  $-20 \log(2\pi f \cdot \Delta t_{RMS})$

Observations

- SNR in dB: $-10 \log((2\pi f)^2 \cdot \text{avg}((\Delta t)^2))$
- Jitter <u>error</u> *proportional* to $\partial v/\partial t$
  - Grows with signal amplitude and frequency
- Jitter-induced <u>SNR</u> *independent* of signal amplitude
  - Error power grows with signal power
- What practical signal yields <u>worst-case</u> $\partial v/\partial t$?
  - Full-range sine of highest in-band frequency!
  - Typical “test case” for jitter testing

Two sinewaves

- $v = A_1 \sin(2\pi f_1 \cdot t) + A_2 \sin(2\pi f_2 \cdot t)$
- $\partial v / \partial t = A_1 2\pi f_1 \cos(2\pi f_1 \cdot t) +$

  $+ A_2 2\pi f_2 \cos(2\pi f_2 \cdot t)$
- $(\partial v / \partial t)^2 = (A_1 2\pi f_1 \cos(2\pi f_1 \cdot t))^2 +$

  $+ 2 A_1 2\pi f_1 \cos(2\pi f_1 \cdot t) \cdot$

  $\cdot A_2 2\pi f_2 \cos(2\pi f_2 \cdot t) +$

  $+ (A_2 2\pi f_2 \cos(2\pi f_2 \cdot t))^2$
- $\mathrm{avg}((\partial v / \partial t)^2) = \underline{A_1^2 (2\pi f_1)^2 / 2} + \underline{A_2^2 (2\pi f_2)^2 / 2}$

$f_1 \neq f_2$

N sinewaves

- Two sines:

  $avg((\partial v / \partial t)^2) = A_1^2 (2\pi f_1)^2 / 2 + A_2^2 (2\pi f_2)^2 / 2$

- Generalized:

  $avg((\partial v / \partial t)^2) = 2\pi^2 \sum (A_i f_i)^2$

  - Sum of powers scaled by square frequencies

- “Any” signal can be represented as sum of sinewaves…

  - …so jitter error power for general signal spectrum follows a high-pass-filtered version of signal power

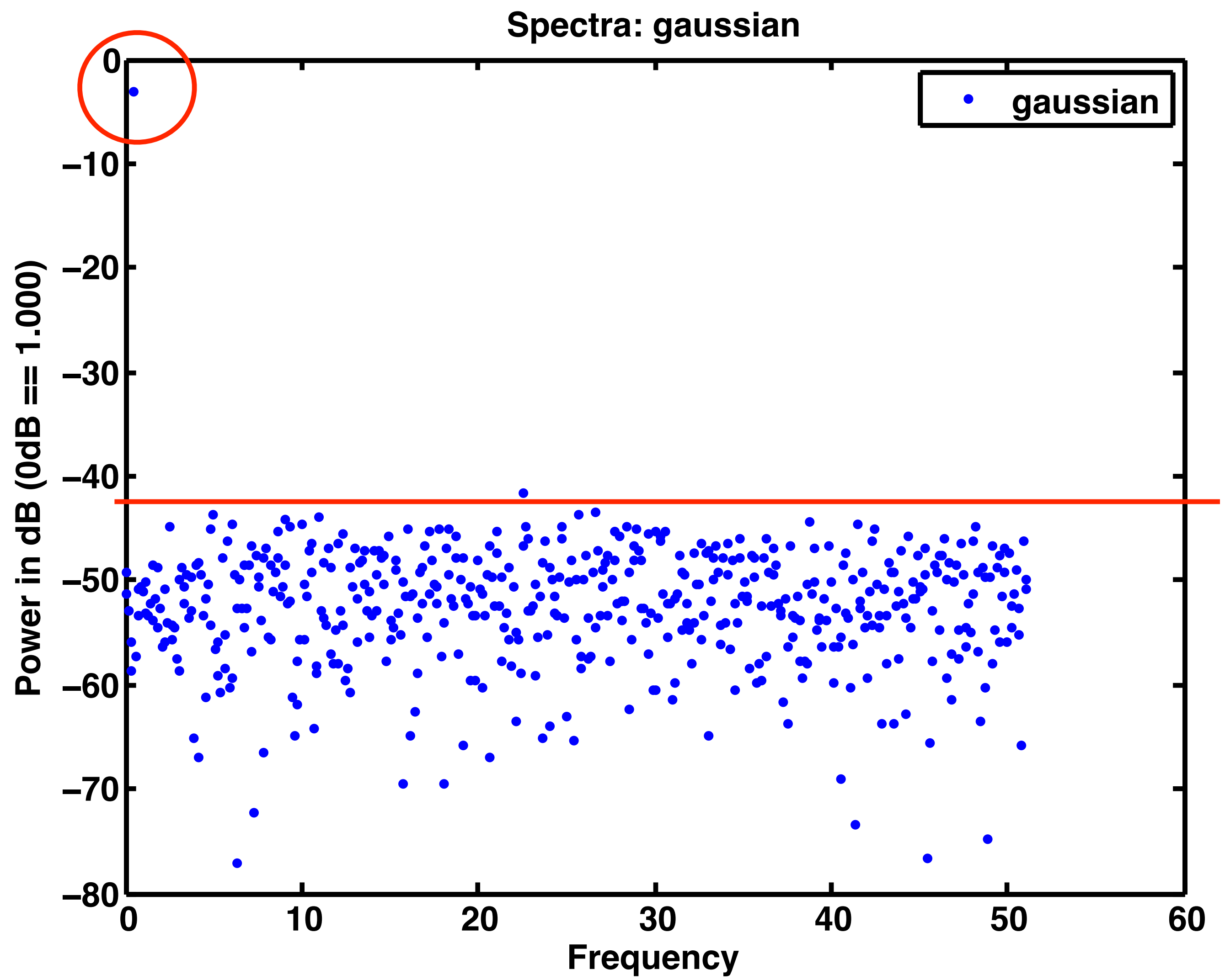
Jitter spectrum

- Jitter-induced error power clearly depends on jitter and on signal
  - Likewise with spectrum of error signal
- Jitter error spectrum useful!
  - Identify cause of insufficient SNR
  - Possibly remove noise by linear filtering
- Will illustrate principles with simple examples

1. “Random” jitter

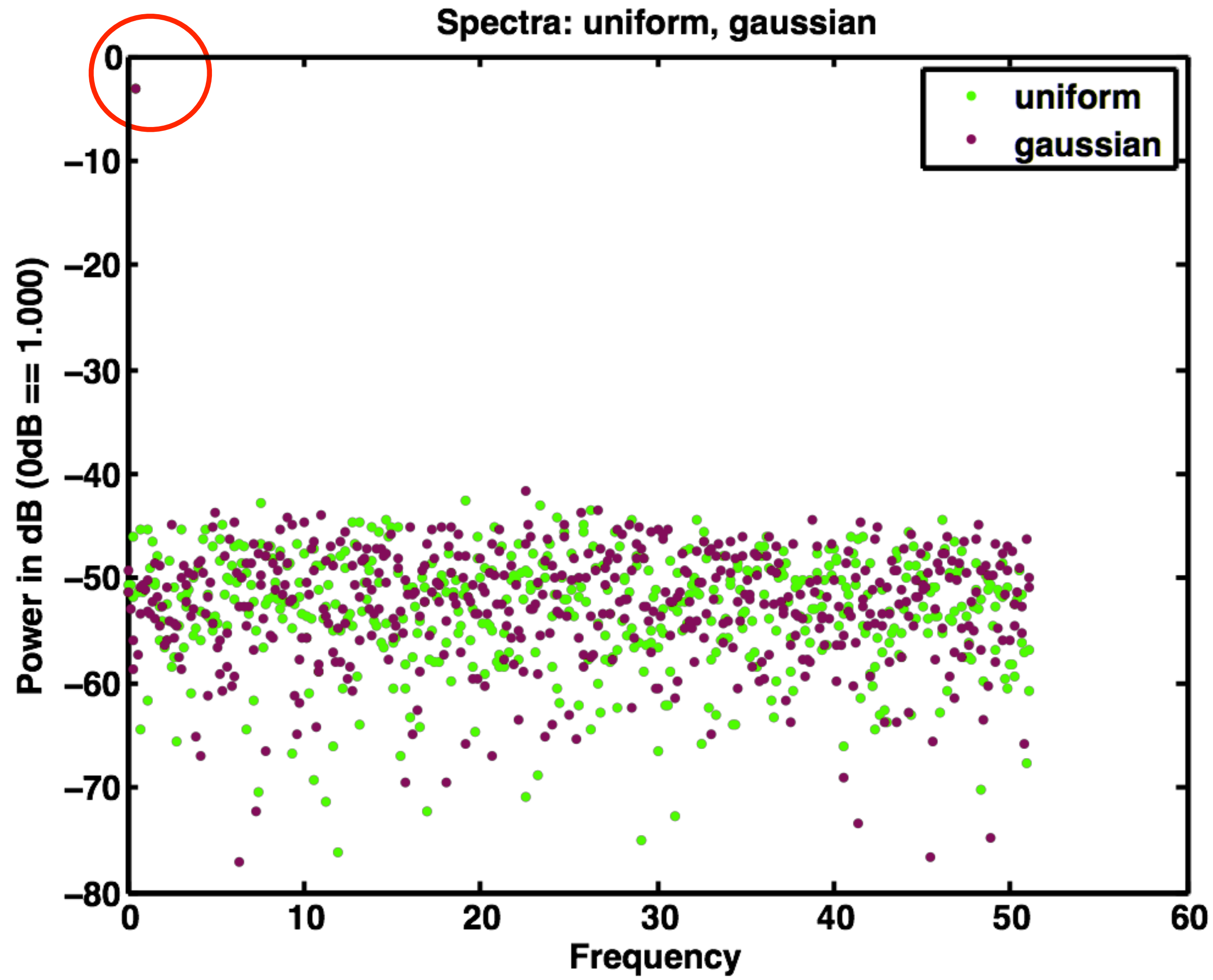
- “White” jitter / phase-noise
  - Constant power in all frequency bands
- No correlation from Δt_i to Δt_{i+1}
  - Expect no correlation from Δv_i to Δv_{i+1}
- Spectrally, should expect “white” noise added to signal

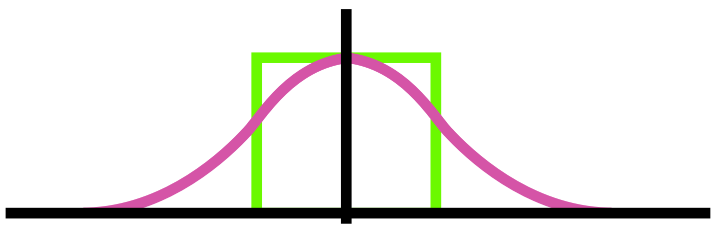
White Gaussian jitter



- Δt has Gaussian distribution
- White "noise floor" under single sine wave

Amplitude distribution?





- Green: Δt <u>uniformly</u> distributed (same avg((Δt)²)
- Large-scale spectral behavior unaffected!

2. Non-random jitter

- Assume the sample time error

  $\Delta t \sim \sin(2\pi\, f_j\, t)\ ,\ f_j \neq f_{sig}$

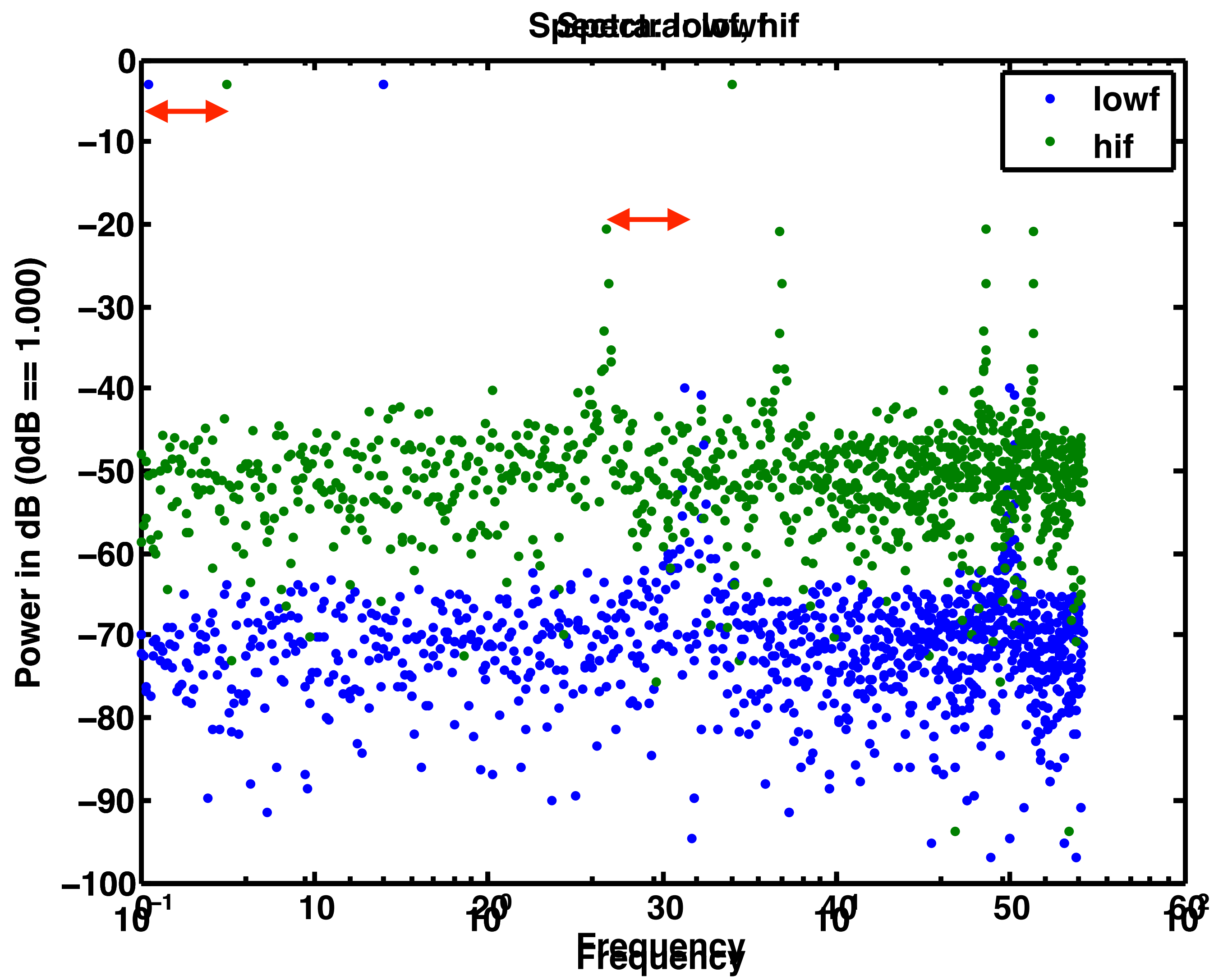
- Then,

  $\Delta v \sim \Delta t \cdot \partial v / \partial t \sim \sin(2\pi\, f_j\, t) \cdot \cos(2\pi\, f_{sig}\, t) \sim$

  $\sin(2\pi\, (f_j + f_{sig})\, t) + \sin(2\pi\, (f_j - f_{sig})\, t)$

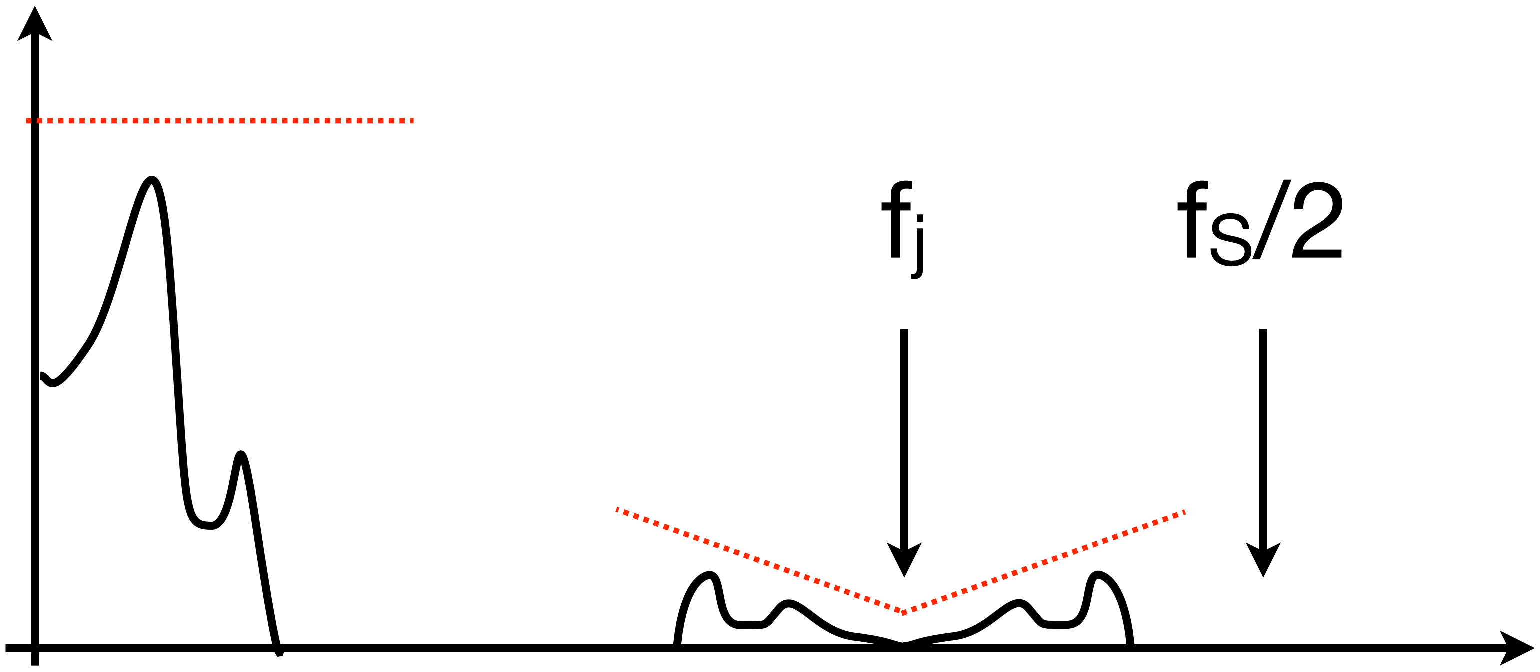
- Frequency sum and difference!

f_{sig} sampled with white + sine jitter



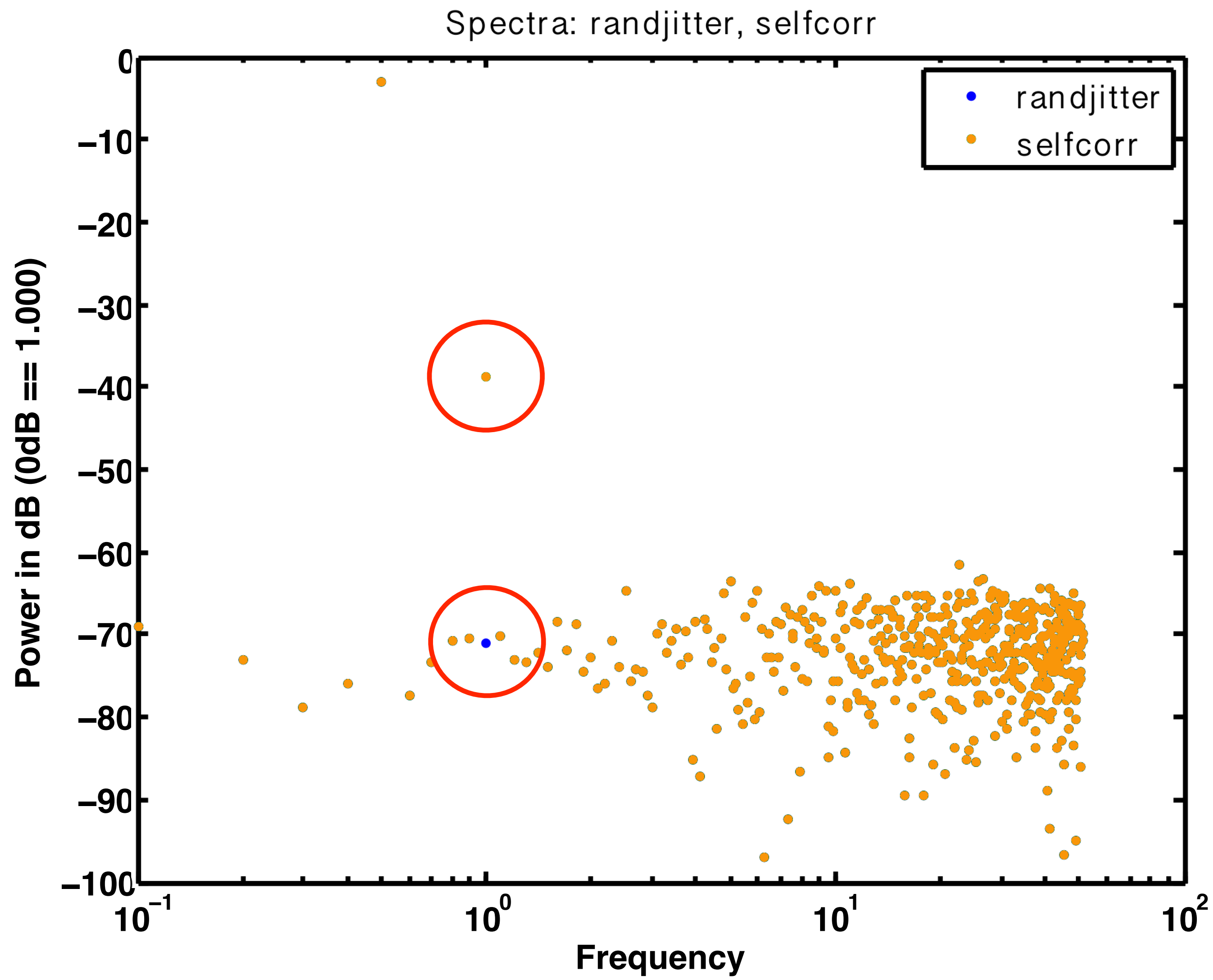
- Sine jitter yields “noise tones”
- Tone separation, noise level follows f_{sig}

“Modulation”



- Sine component at f_j in jitter causes signal band to be mirrored around f_j
  - Higher signal frequencies emphasized
- Distinguishable from f_S aliasing!

3. Signal-correlated jitter



- $\Delta t \sim$ [random] + v
- $\Delta v \sim v$; shows as harmonic distortion (2nd)

Summary

- SNR applicable for many kinds of errors
  - Useful tool for trading off requirements
- Variations such as SNDR also used
  - Signal to Noise and Distortion Ratio
  - Explicitly include harmonics in error

Summary, cont.

- Sample jitter significant source of errors
  - Error grows with signal frequency
  - Especially bad for high-performance systems
- Jitter characteristics reflected in error spectrum
  - Enables diagnostic analysis!