Quantization 1

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What?

- Restrict signal values to a limited set
  - Cf. sampling: restrict time to limited set
- Select set member closest to “real” value
  - Use an index/code to refer to value
- # levels: “resolution”
  - Often 2^N levels: “N bits”
- Often equidistant levels: “uniform quantization”

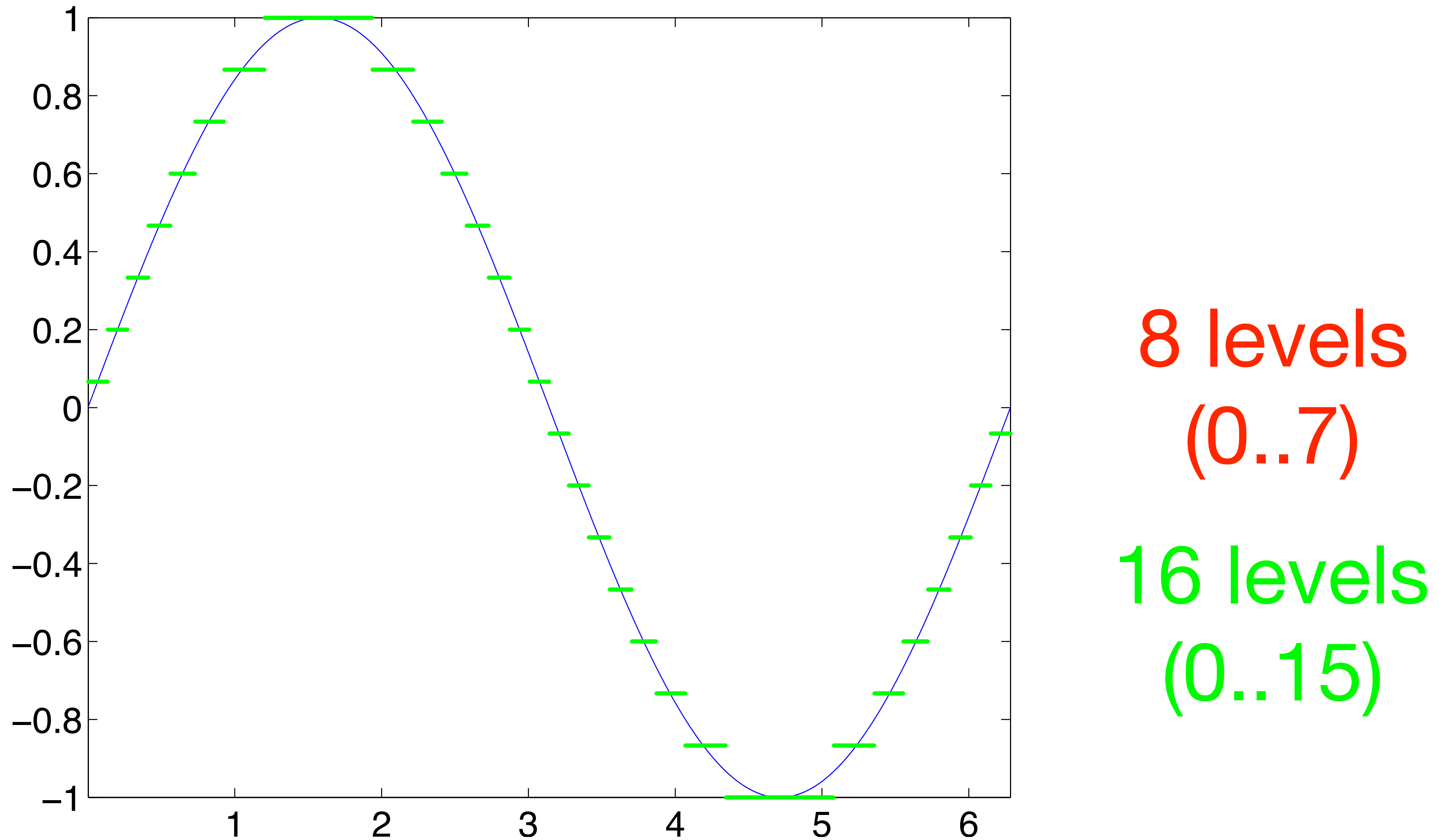
Discretization of time and value

181119 LJS

Where?

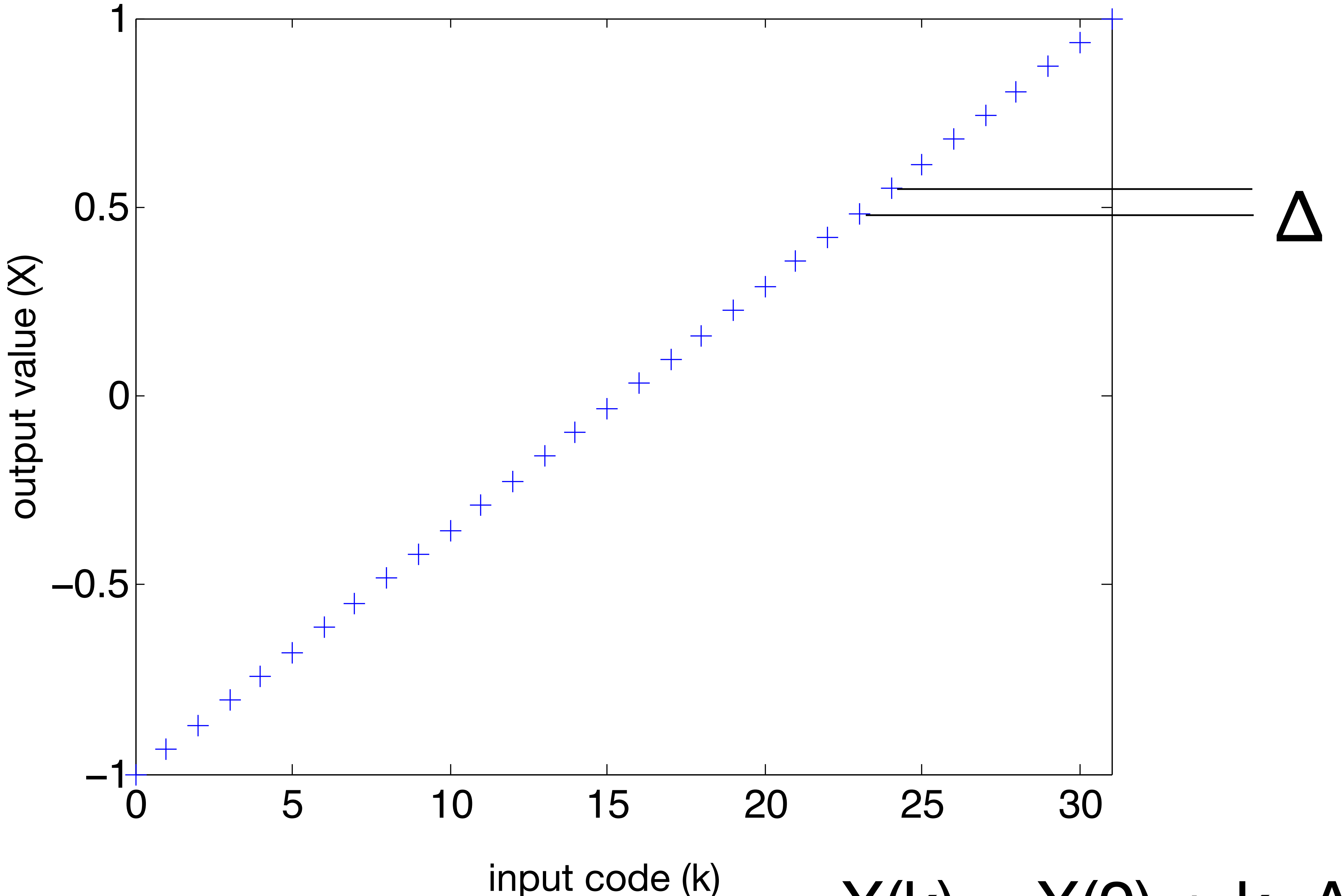
- Conversion from continuous values to digital codes (A/D conversion, ADC)
- Conversion <u>to</u> continuous values <u>from</u> digital codes (D/A conversion, DAC)
- Mirror-image <u>operations</u>
  - <u>Not</u> mirror-image <u>implementations</u>
  - Most ADCs contain a DAC
- More on implementations Thursday

Uniform quantization



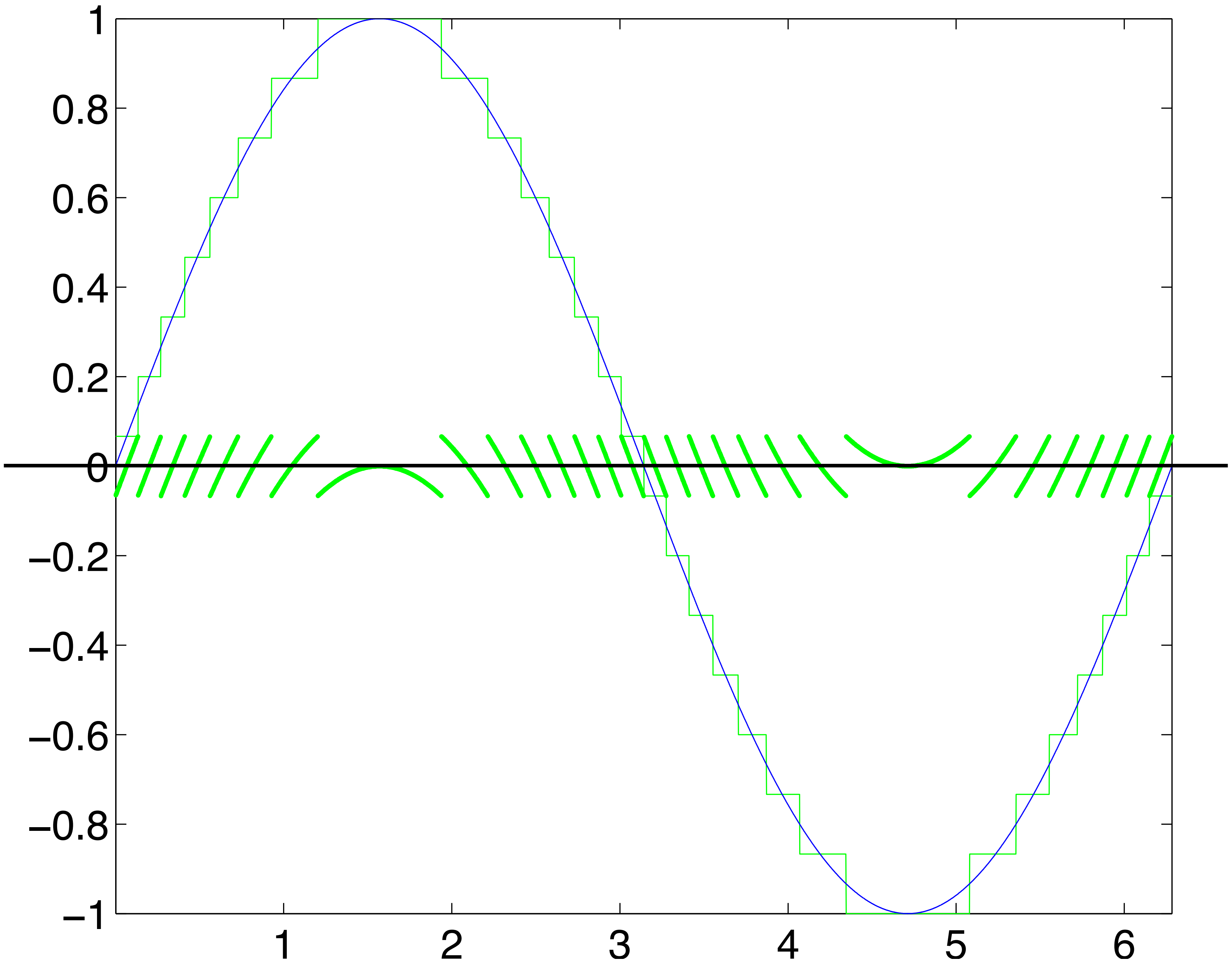
- Convention: converter range is ±1, like a sine wave
- Convention: may use continuous scale also for discrete values
- Difference between levels: Δ

Linear conversion

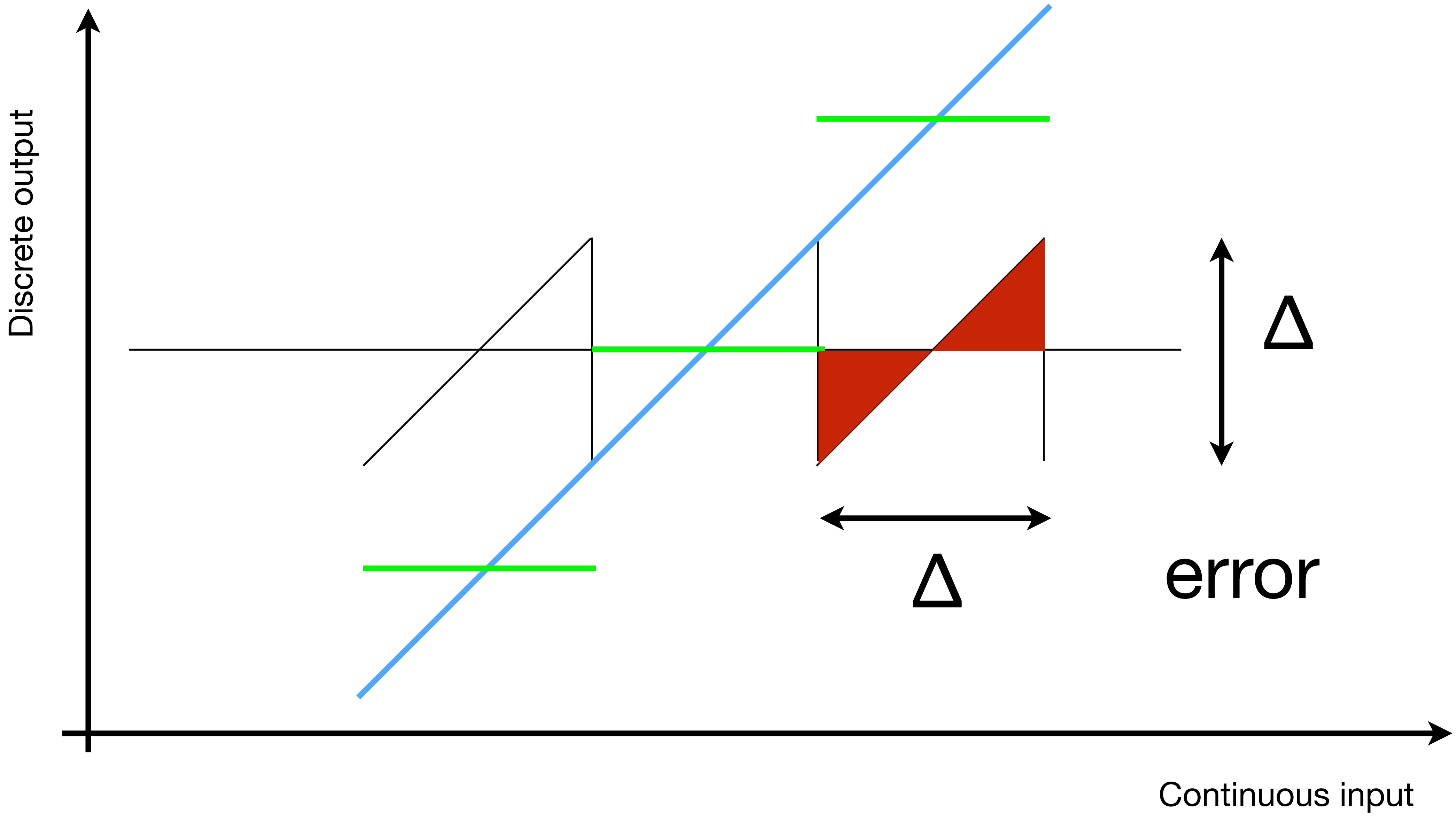


$$X(k) = X(0) + k \cdot \Delta$$

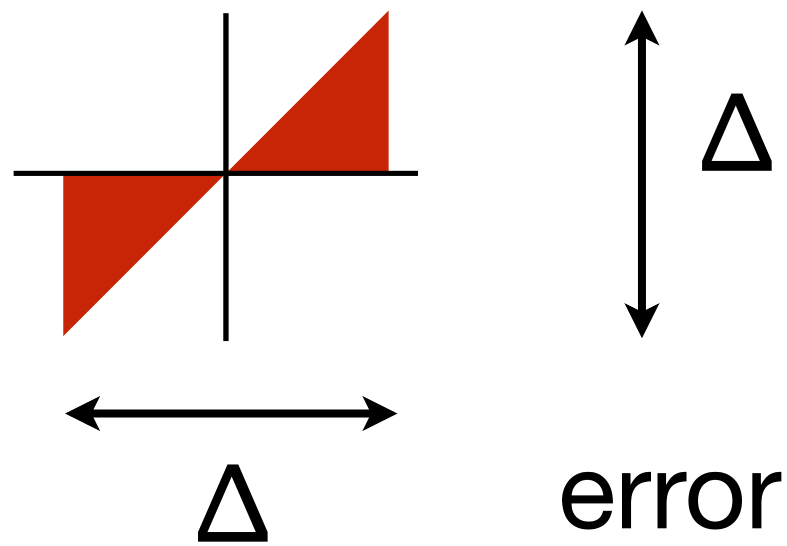
Quantization error



Error characteristics



Maximum error



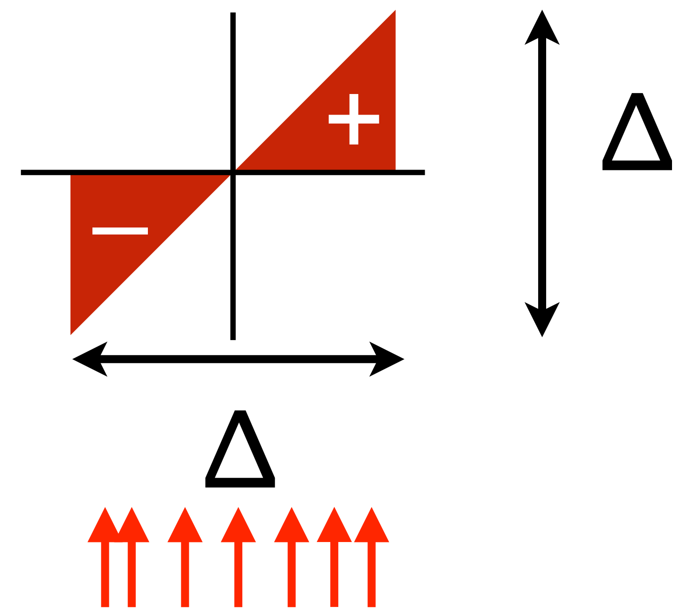
- Consider ±Δ / 2 interval around one level
- Clearly, largest error is ±Δ / 2
- If signal range is ±1, then

$$\Delta = 2 / (2^N - 1) \approx 2 / 2^N$$

- Maximum error: (±)1 / 2^N

Shrinks when N grows

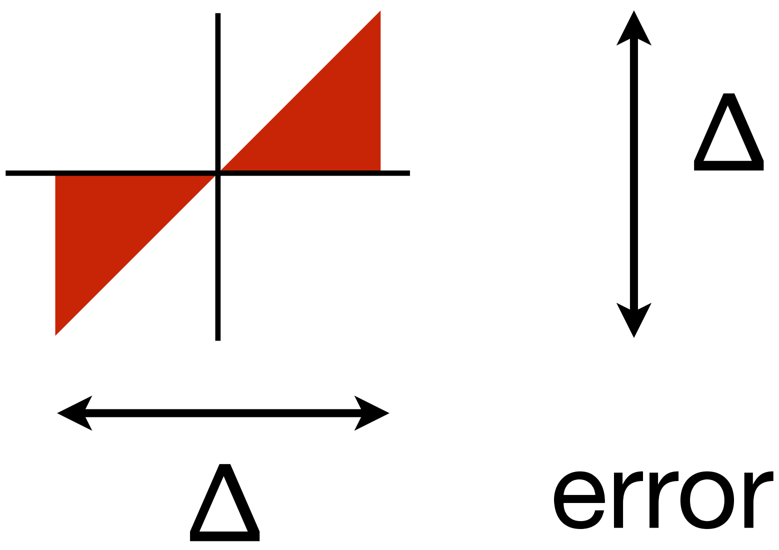
Average error



- Assume <u>any</u> analog value is equally probable
  - All values <u>within interval</u> equally probable
  - Every ±Δ/2 interval equivalent
  - Enough to analyze single interval
- Integrate red area (with sign) from –Δ/2 to Δ/2
  - Divide by Δ (total probability = 1)
- Then, average error = 0

Good!

Error power



- Same thing, only integrate square of error!
- Square of error ≥ 0, for any error
  - Error power > 0, for any Δ

Average error power

- Error: $x, -\frac{\Delta}{2} < x < \frac{\Delta}{2}$
- Maximum error magnitude: $\frac{\Delta}{2} = \frac{1}{2}\frac{2}{2^N}$
- Average power (across one Δ, and thus across range):

$$\frac{1}{\Delta}\int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} x^2 \mathrm{d}x = \frac{1}{\Delta}\left[\frac{x^3}{3}\right]_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} = \frac{1}{\Delta}\frac{1}{3\cdot 8}\left(\Delta^3 - (-\Delta^3)\right) = \frac{\Delta^2}{12}$$

SNR$_Q$

- Signal to Noise Ratio (ratio of powers, expressed in dB)
- If signal is full-scale sine wave:

$$P_{signal} = \frac{1}{2}$$

$$P_{noise} = \frac{\Delta^2}{12} = \frac{1}{3}\frac{1}{2^{2N}}$$

$$SNR = 10 \cdot \log_{10}(\frac{3 \cdot 4^N}{2}) = 6.02 \cdot N + 1.76$$

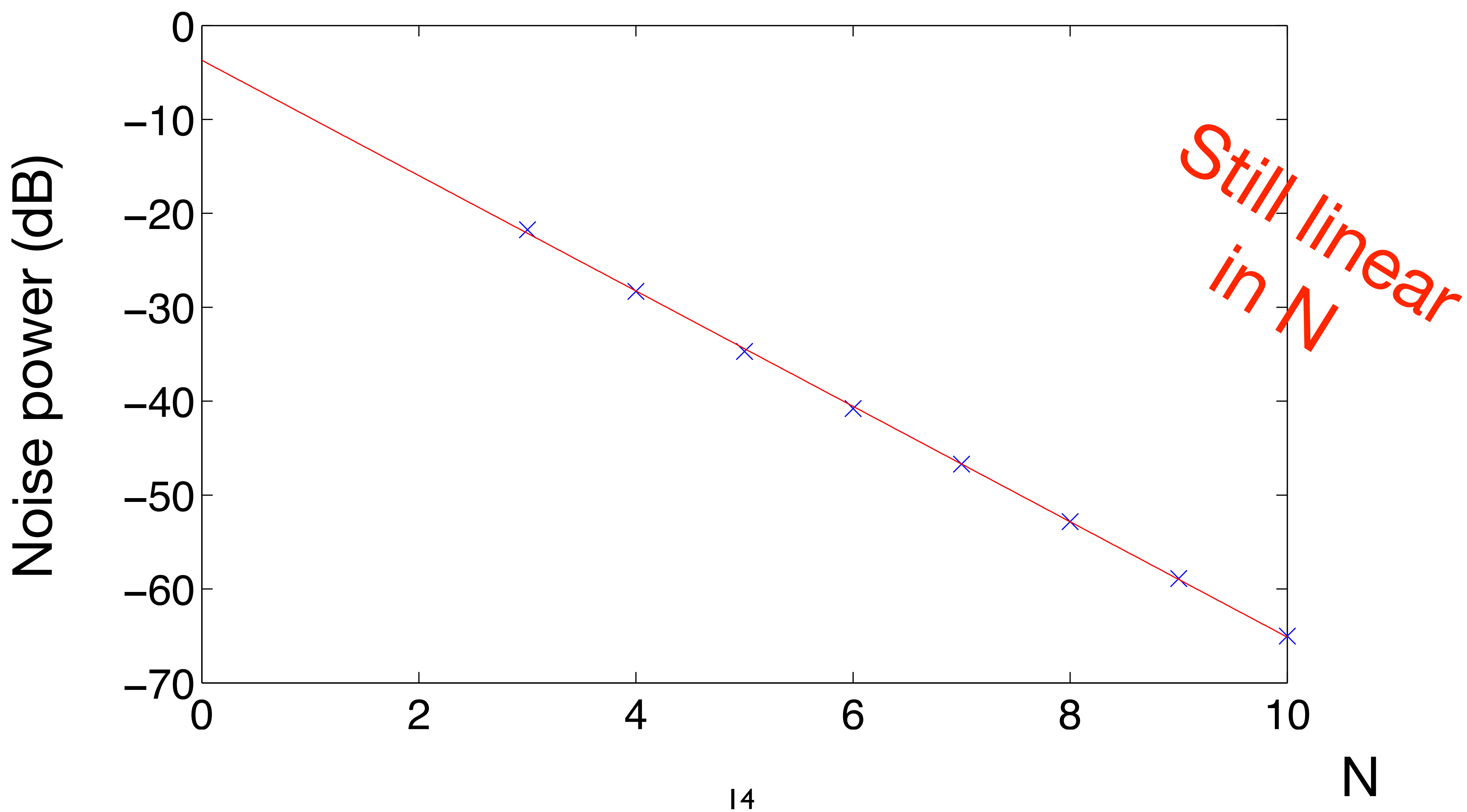
SNR linear in N!

Factor 4

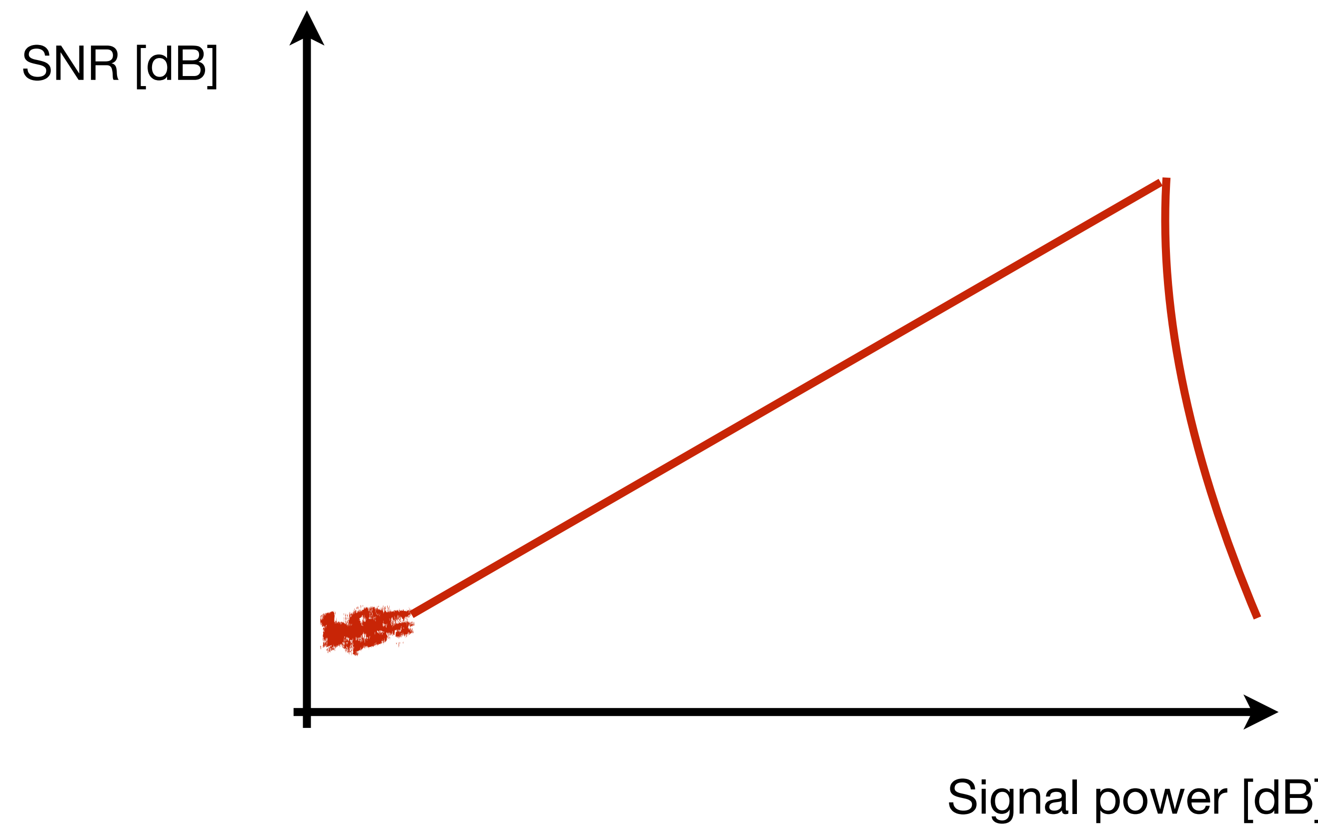
Depends on sinewave

Other signals

- Example: quantization of some wideband signal (not single sine)
- Noise power in dB as function of number of bits



SNR vs signal power?

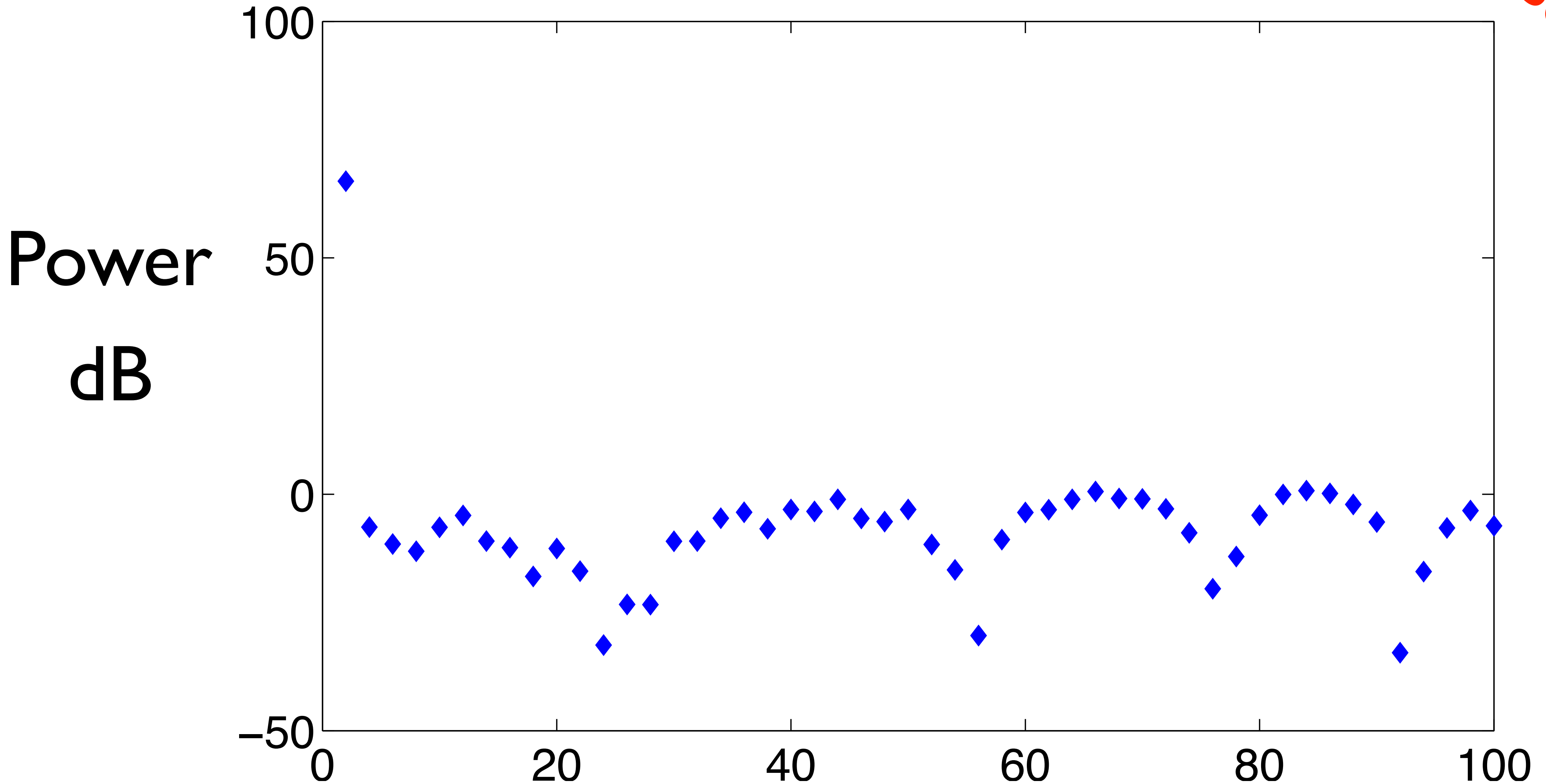


- Noise power constant (depends only on Δ)
- SNR grows linearly with signal power
    - Top, bottom of scale?

Error spectrum

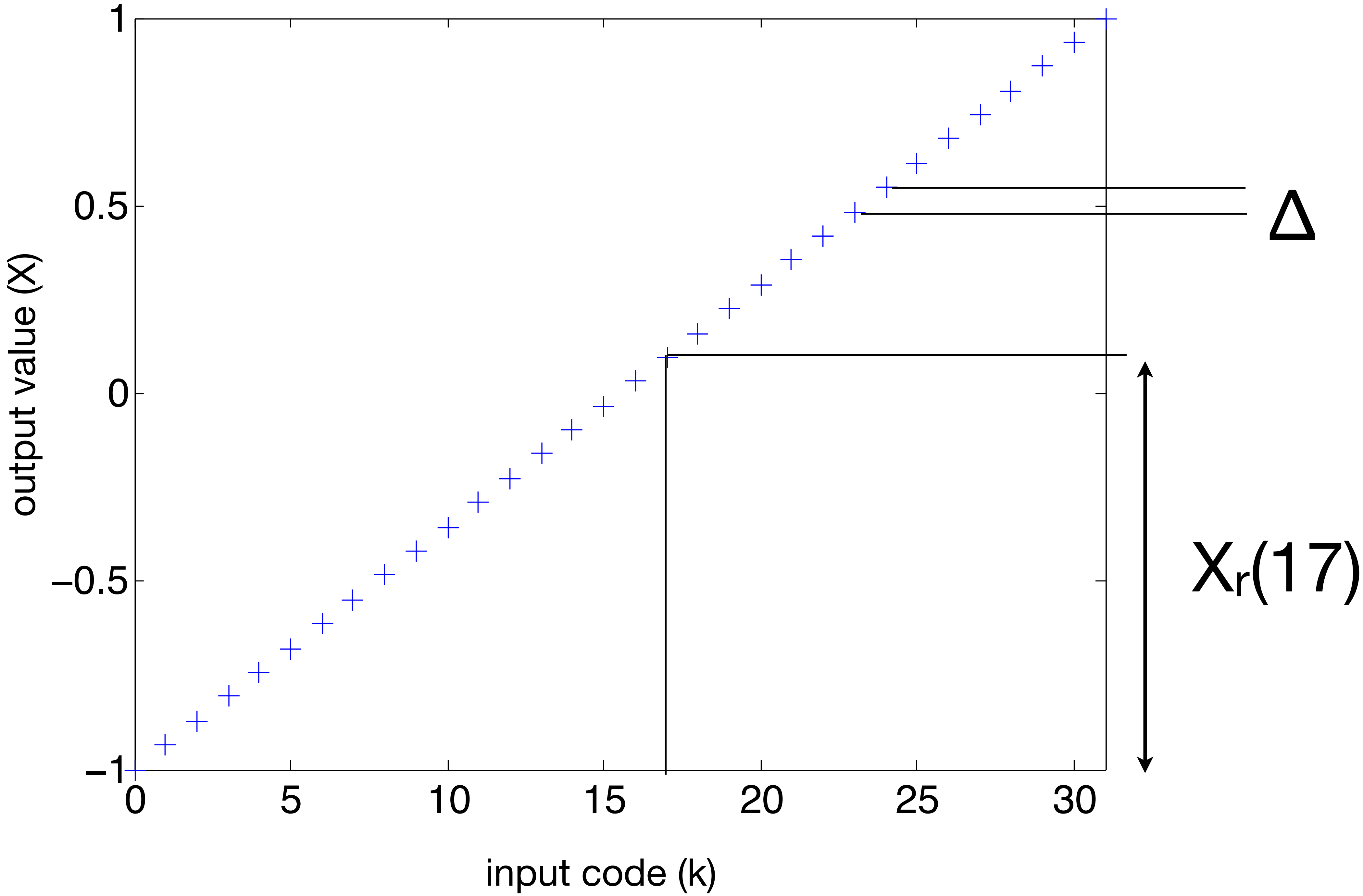
- Error appears as added "white noise"
  - "Quantization noise"

Assuming high sample rate, resolution



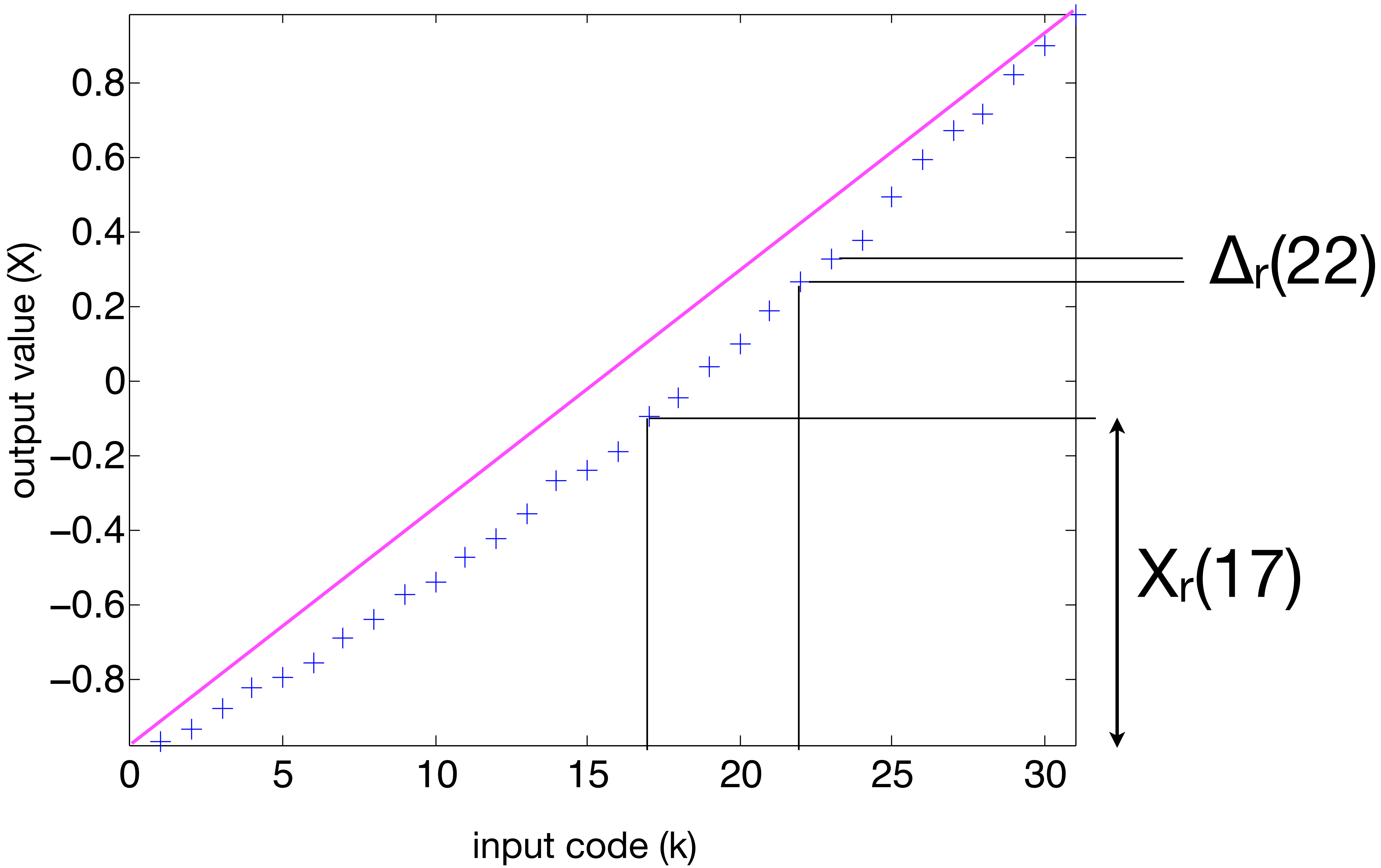
(Non)uniform quantization

Linear D/A conversion

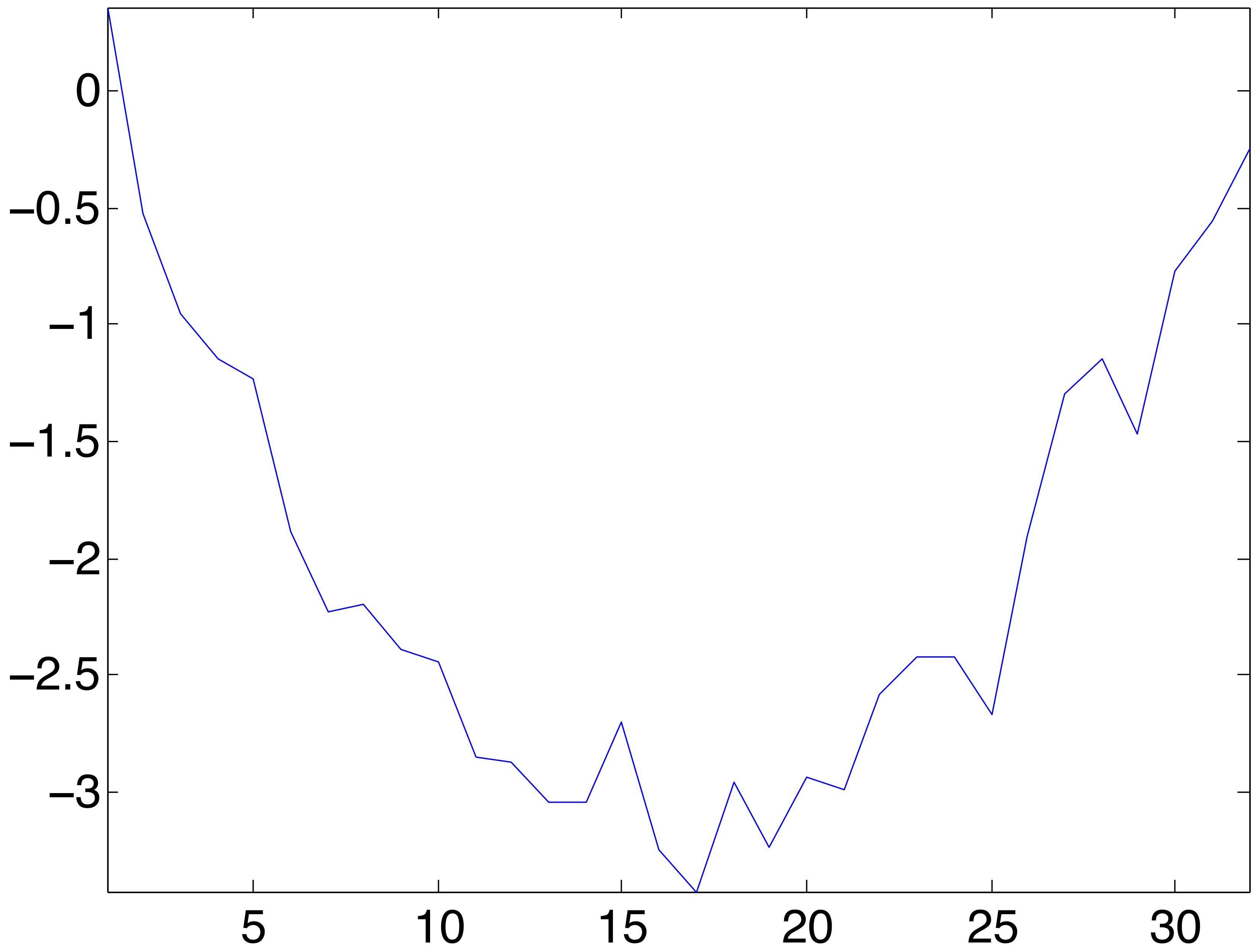


$$X_r(k) = X_r(0) + k \cdot \Delta$$

Non-linearities



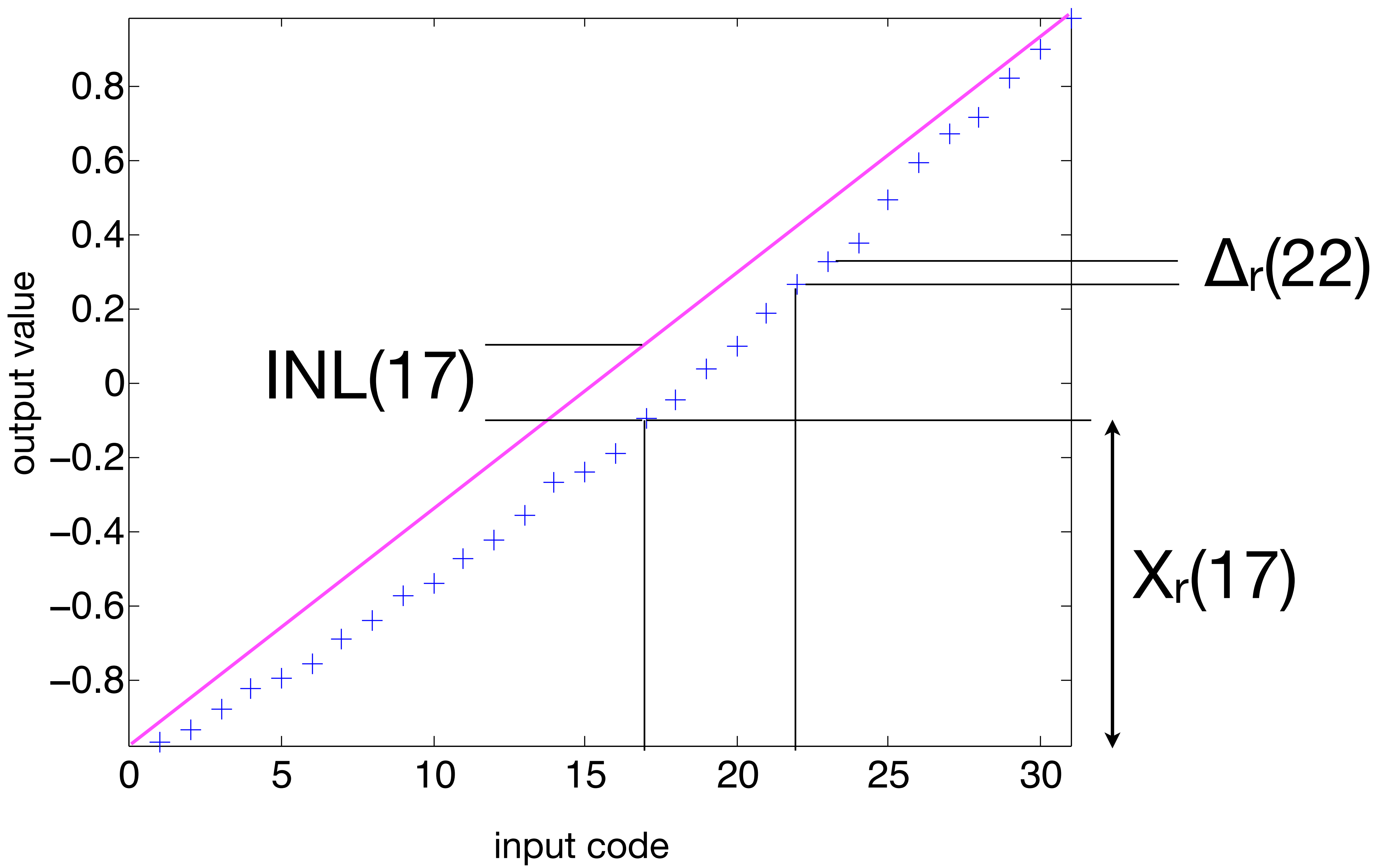
Integral NL



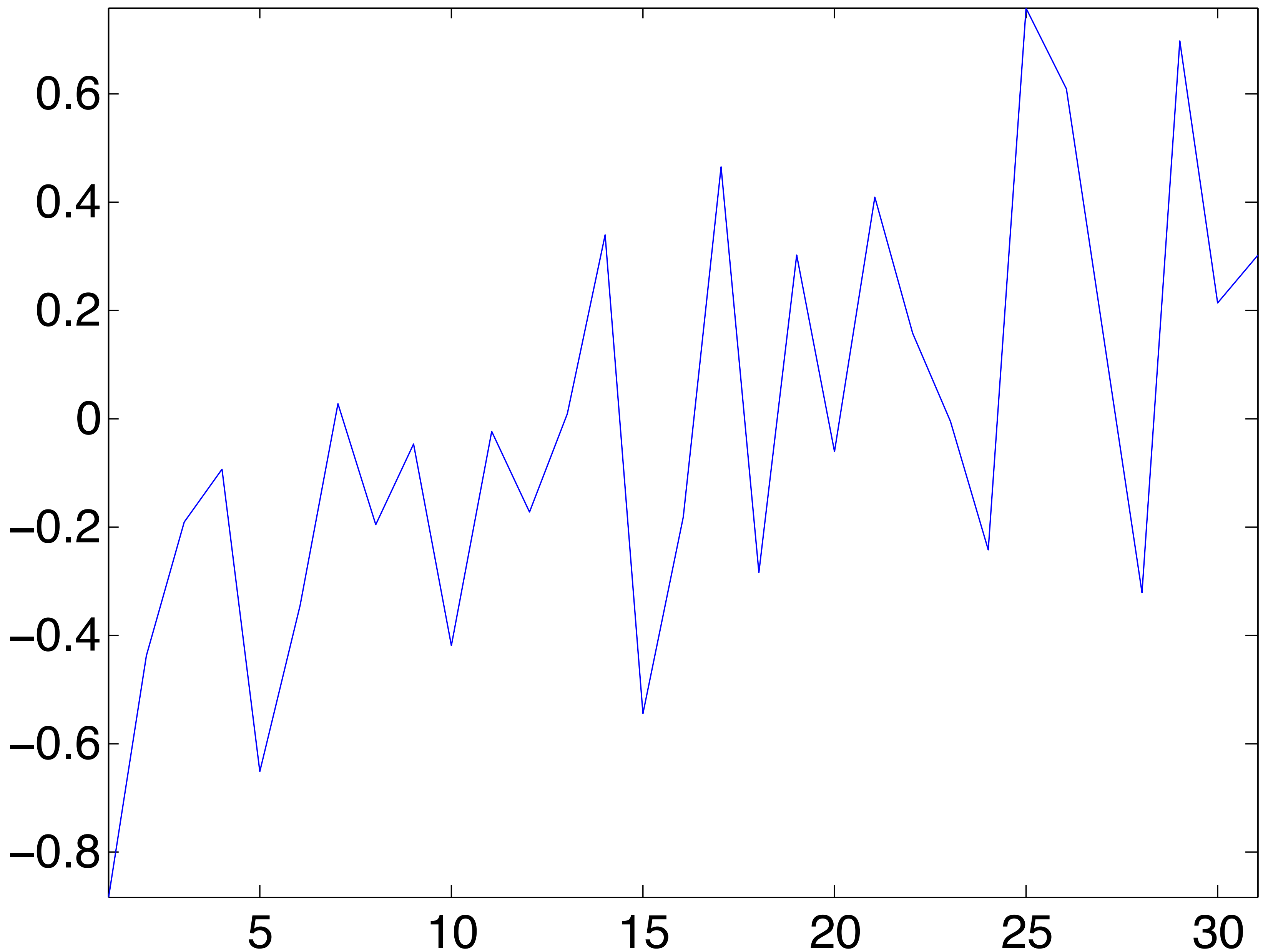
$$INL(k) = \frac{X_r(k) - (X_r(0) + k\cdot\Delta)}{\Delta}$$

Intended value

Non-linearities



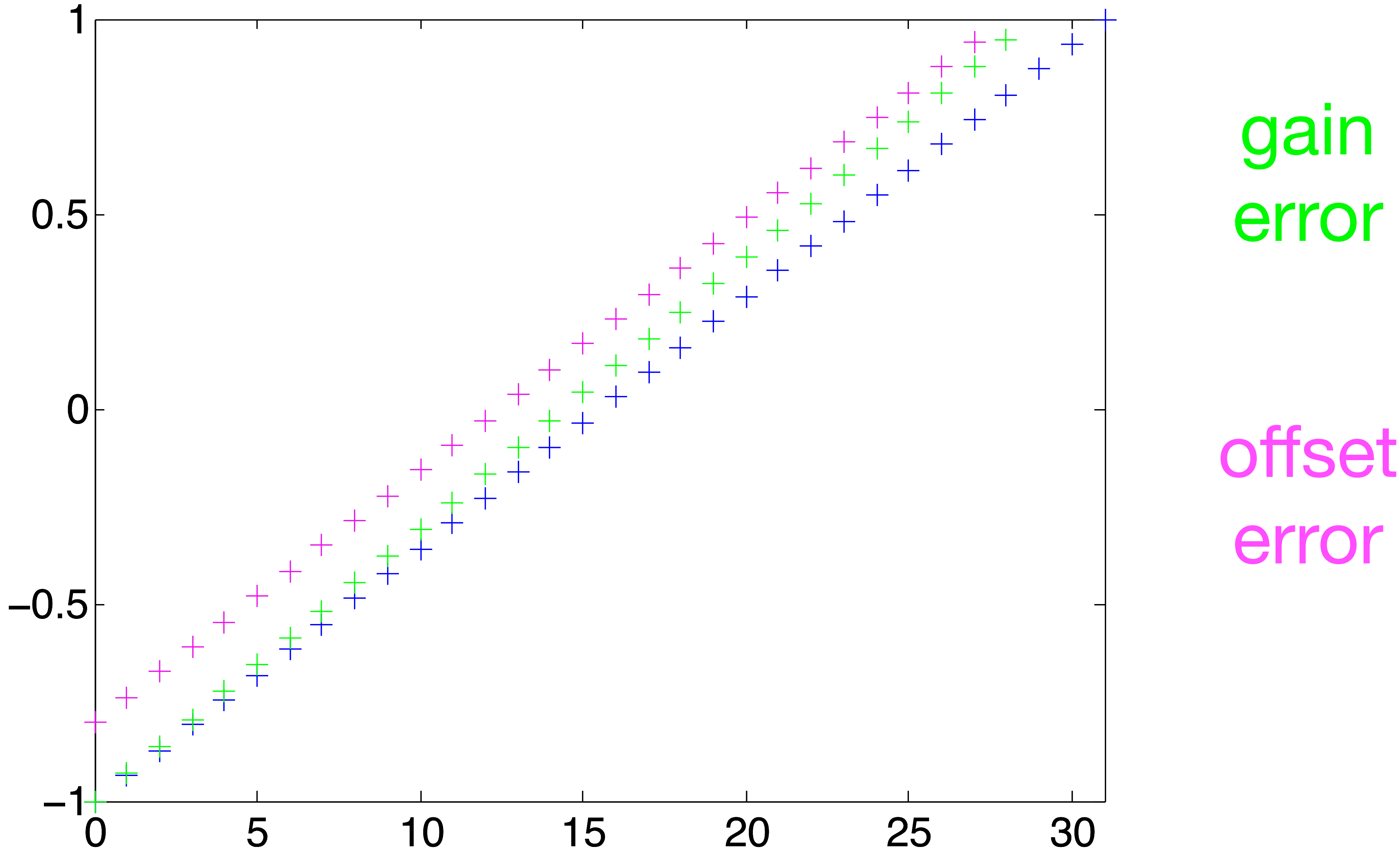
Differential NL



$$DNL(k) = \frac{\Delta_r(k) - \Delta}{\Delta}$$

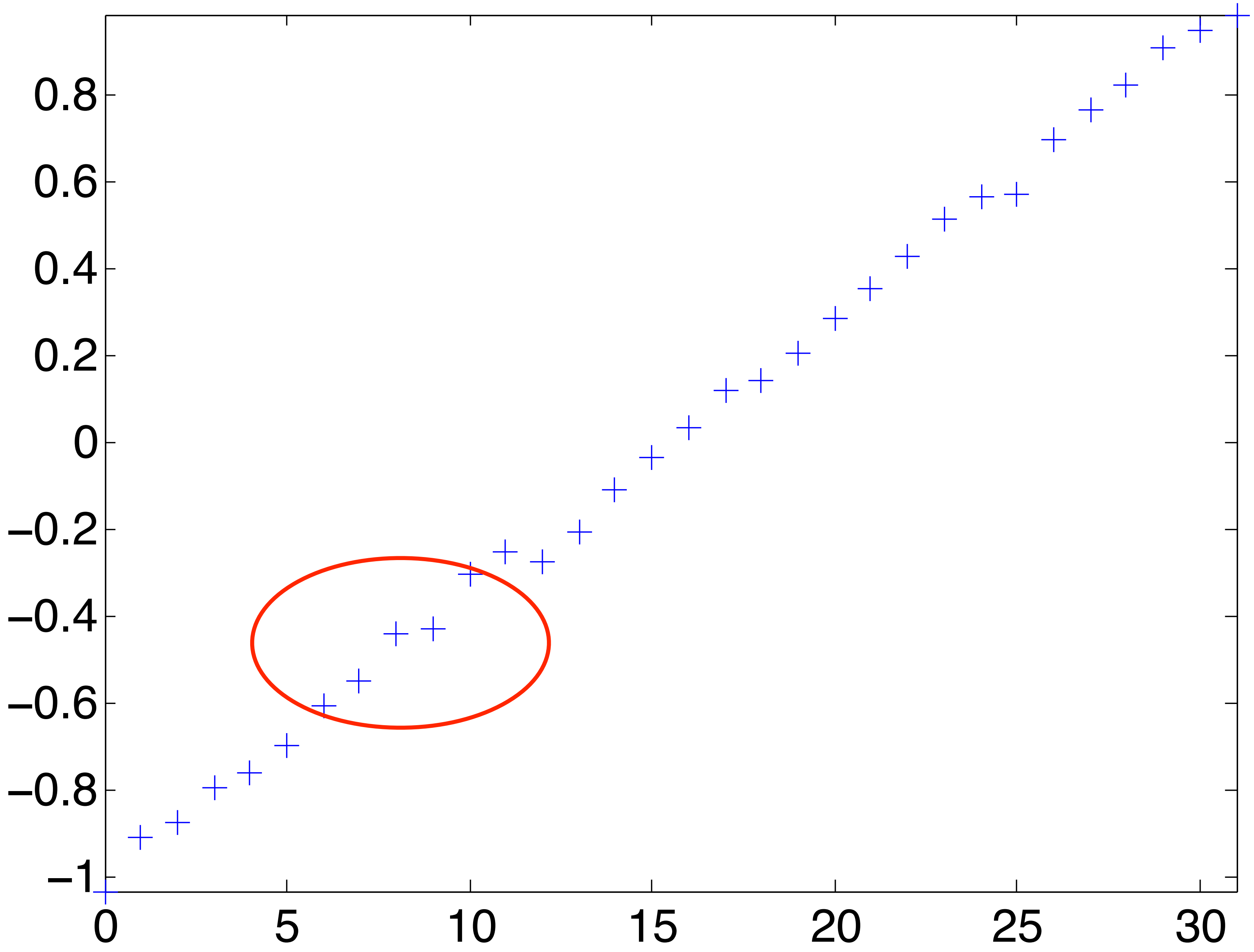
Intended value

Gain, offset errors



Often removed before INL, DNL calculation!

Bad case of DNL

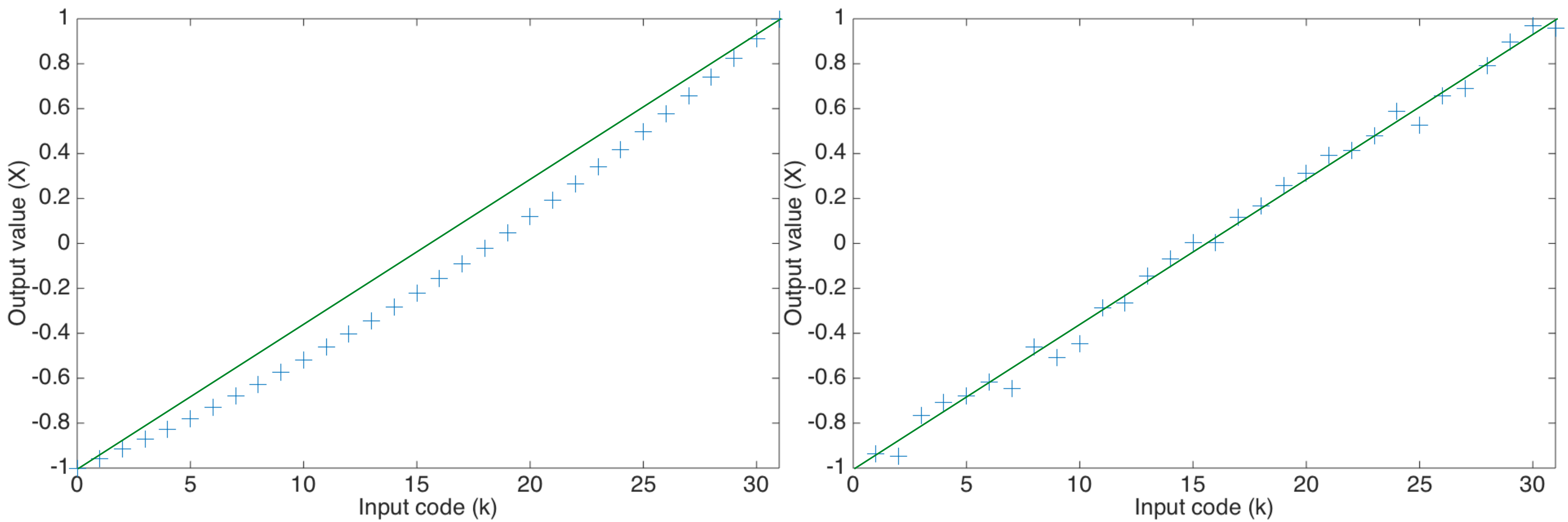


Non-monotonic characteristic

Linearity for ADC?

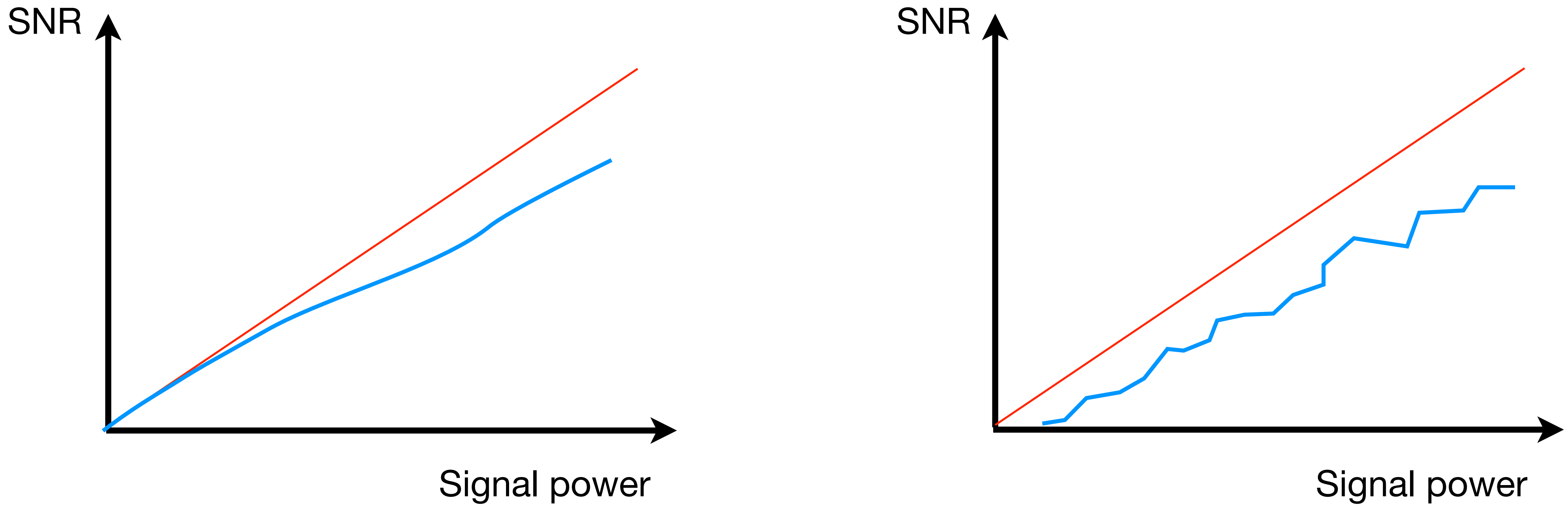
- DNL, INL
  - Similar definitions
- Bad case: missing code
  - Some digital code not emitted for <u>any</u> input value

INL? DNL?



- Clearly related since $X(k) = X(0) + \sum \Delta_{i<k}$
- Either may dominate the total inaccuracy
    - Large DNL is bad also at small signal amplitudes
- Different causes emphasize one or the other (next time)

SNR vs signal level



- High INL but low DNL: “smooth” SNR plot
    - SNR will not to improve as expected with level
- High DNL but low INL: “noisy” straight line
    - Generally high noise level; SNR worse for all input levels

Spectrum influence?

- Large INL:
  - Harmonic distortion, esp. low harmonics
  - OK at low input power levels
- Large DNL:
  - Harmonic distortion also at high harmonics, low amplitudes
  - In limit, spectral properties like noise
    - …if resolution high enough

Figures of merit

- # of bits of resolution
- SNR or SNDR, dB
- ENOB: *effective* number of bits
  - $(SNR_{\text{Full-scale-sine}} - 1.76) / 6.02$
  - Ideally same as resolution

- Conversion frequency

Summary

- Quantization introduces errors
  - "Quantization noise"
  - Not actually random, but may be treated as such
- Errors in quantization exacerbate noise
  - High resolution "useless" if too much INL/DNL
  - ENOB summarizes "usable" performance