Time-continuous filters: an overview (II)

DAT116, Nov 29, 2018

lenap@chalmers.se

Monday recap

- Characteristics of filters
- Lowpass filters
- Pole/zero placement (classical methods)
  - Butterworth, Chebysjev, Elliptic filters etc...
  - Holds for **all** types of implementations!
- Cascades of first/second order links (often called biquads)
  - Active-RC implementations

Muddy 2018

- How do we know what the purple boxes are?
- Any examples of different filter types that suits different implementations? For example: when is it critical to have a flat pass-band etc.?
- Relationship between Q and poles.
- If Q means “quality” is higher Q always better?
- Q connected to poles.
- The order of the second-order sections in the 10-pole filter.
- What does “sensitive” mean for higher order?
- Biquad diagram - what is it about?
- Transconductor?

An example - anti-aliasing filter

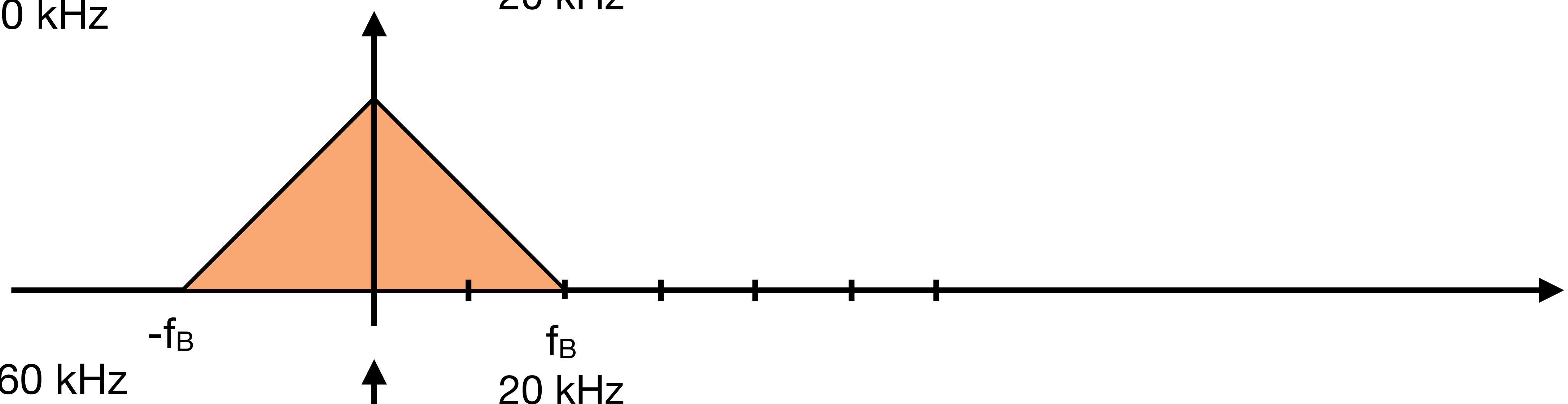
- Assume we are to AD convert an audio signal.
- It has signal of interest up to 20 kHz.
- How do we select fs?
  - fs >=40 kHz according to Nyquist.
- Three choices
  - fs = 40 kHz, fs = 50 kHz, fs = 60 kHz

Complete diagrams

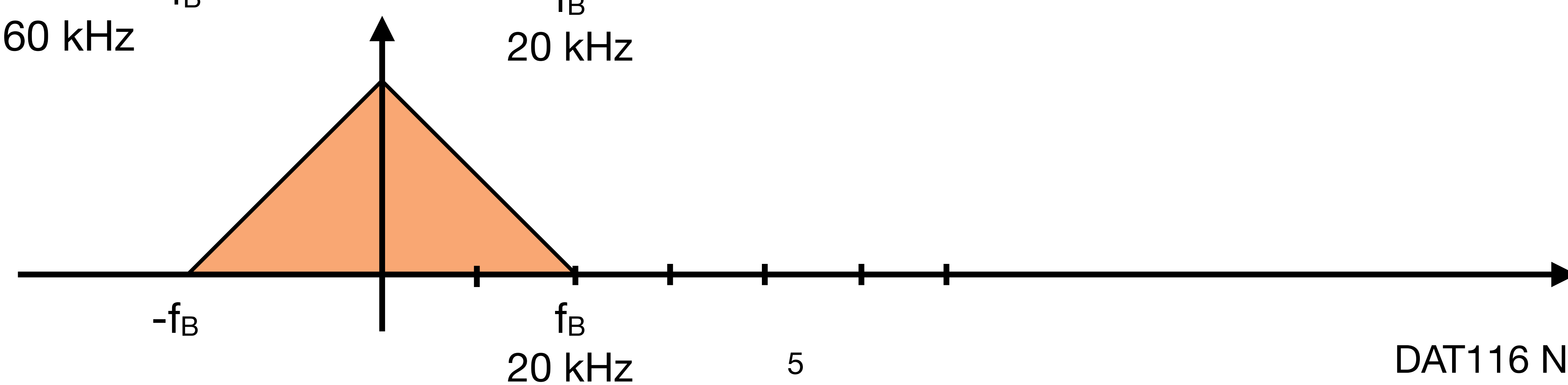
Band on interest: 0 - f_B Add fs, fs/2, draw alias of band of interest, filter spec boxes

fs = 40 kHz

fs = 50 kHz

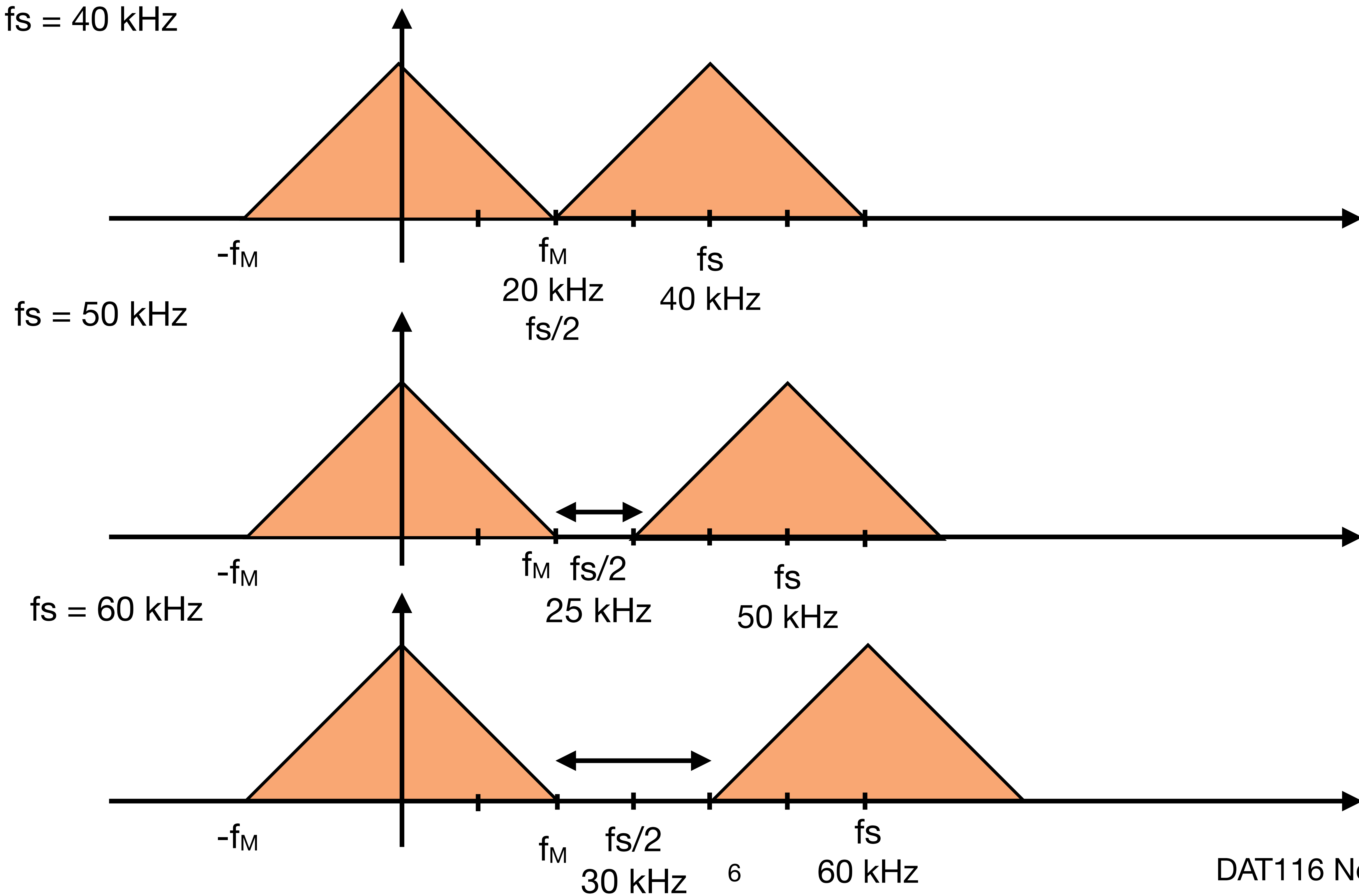


fs = 60 kHz



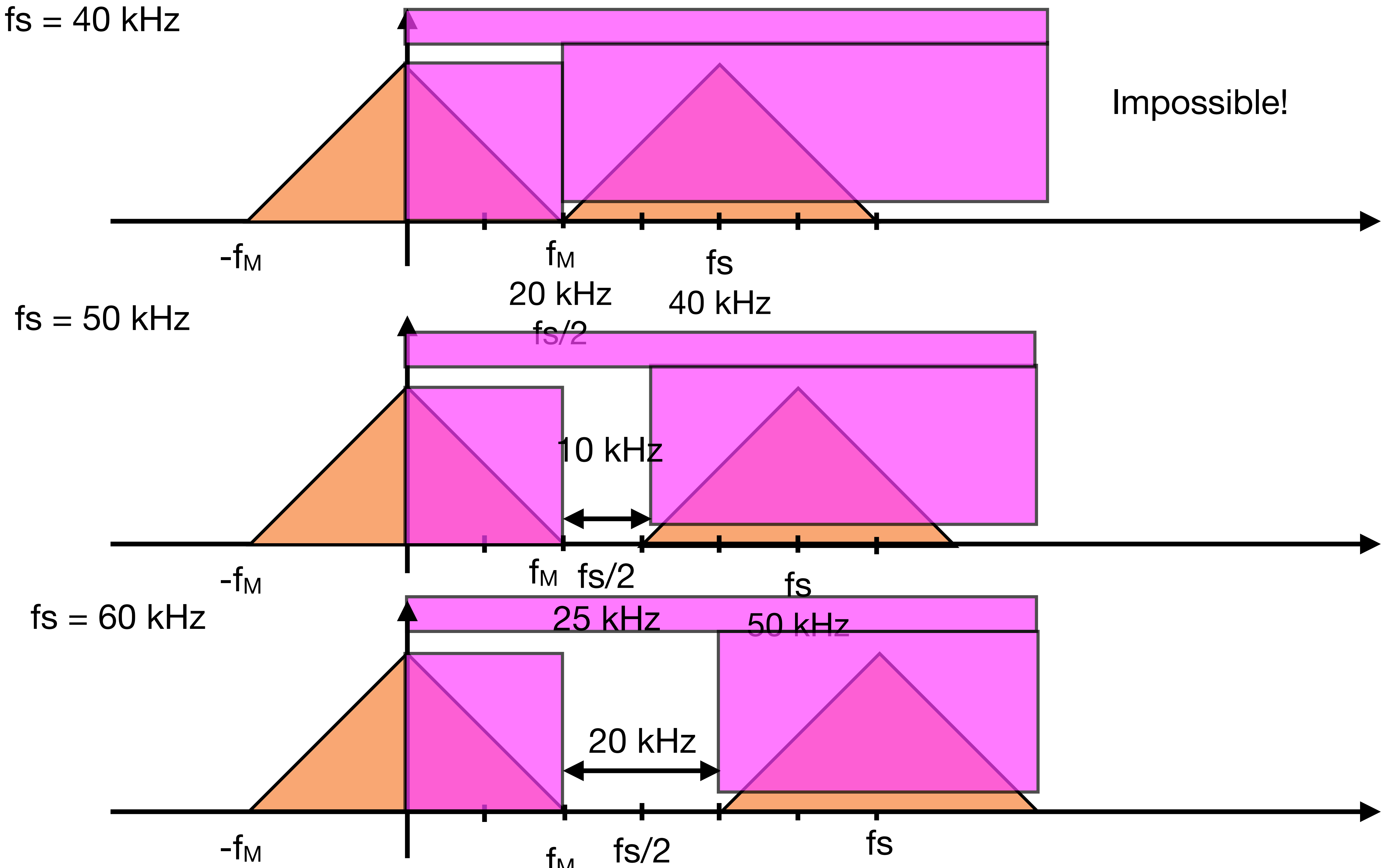
Complete diagrams

Band on interest: 0 - f_M Add fs, fs/2, draw alias of band of interest, filter spec boxes



With filter specs

Band on interest: 0 - f_M Add fs, fs/2, draw alias of band of interest, filter spec boxes



Pole zero demonstrator

- https://www.youtube.com/watch?v=PybGMXKTp7c

Q for the poles

- Q for filters/poles : A measure of the angle that describes the pole pair
- $Q = 1/(2\cos \psi)$ where ψ is angle

Design example (more Monday)

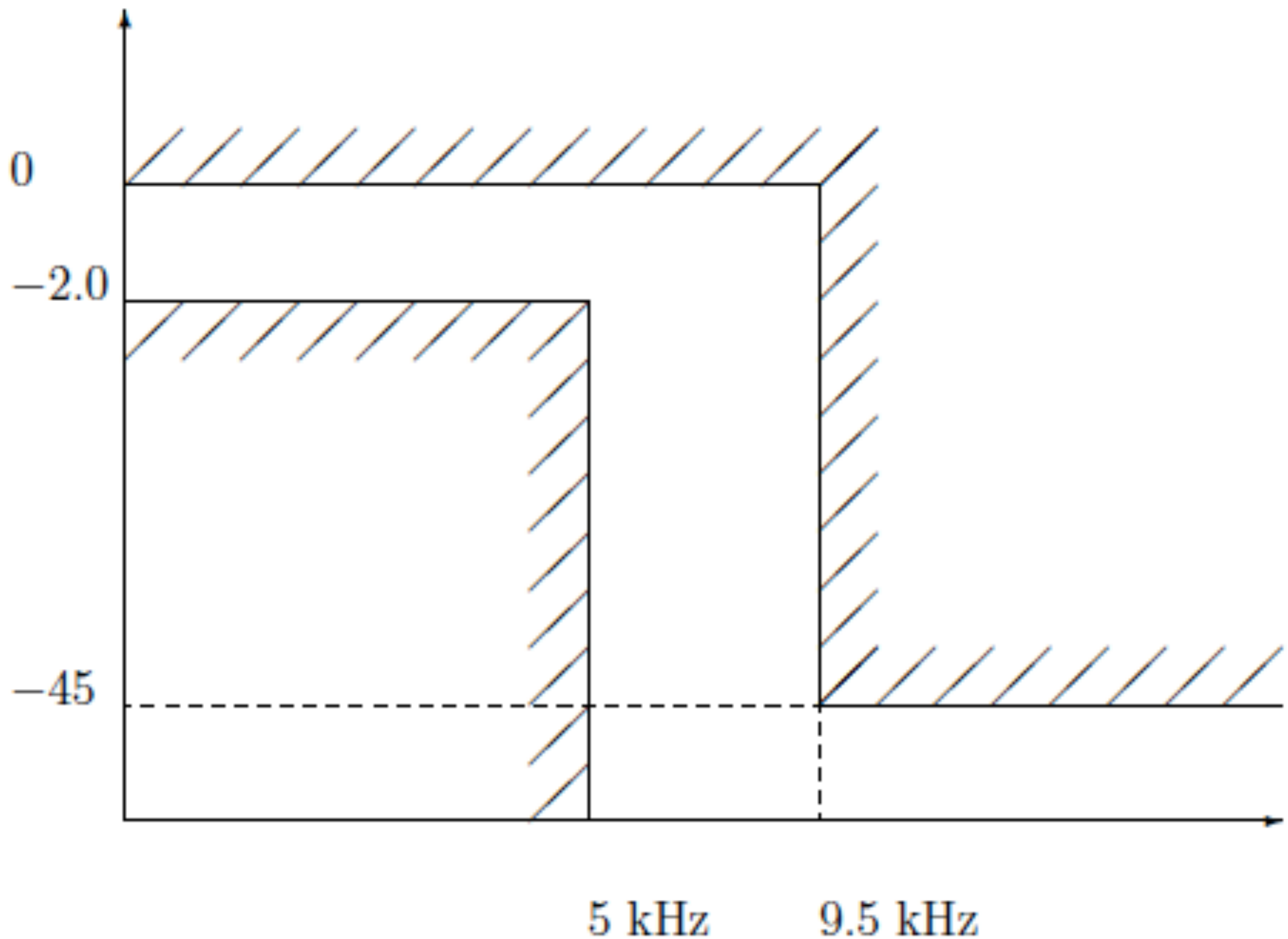
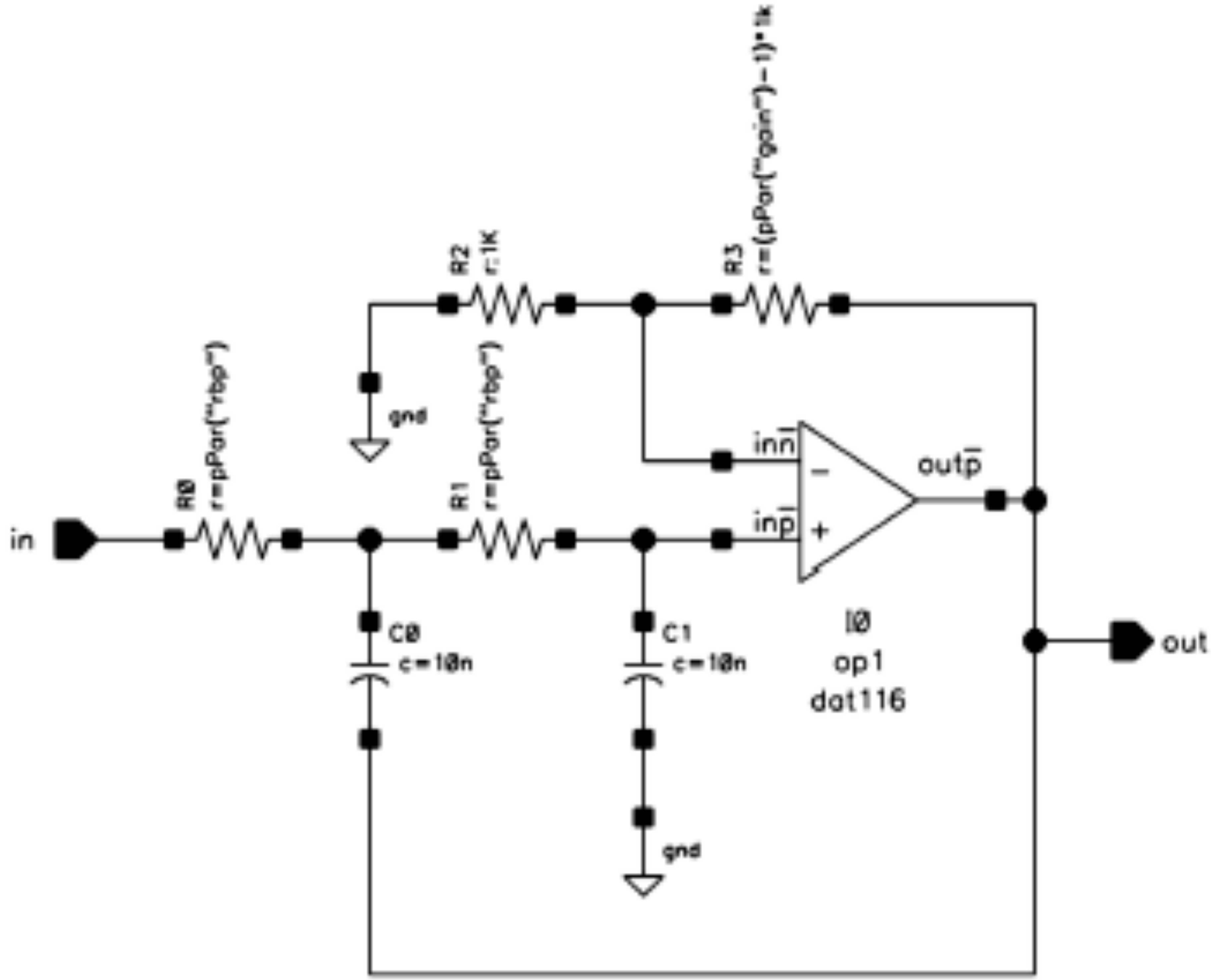


Figure 1: Low-pass filter specification (magnitude values in dB).



R3
r=(pPar("gain")-1)*1k
R2
r:1K
gnd
R0
r=pPar("rbp")
R1
r=pPar("rbp")
in
inn
inp
outp
C0
c=10n
C1
c=10n
I0
op1
dat116
out
gnd

Butterworth example - order

buttord(Wp, Ws, Rp, Rs, 's’) gives the required order of a continuous-time Butterworth filter with passband edge at Wp, stopband edge at Ws, passband ripple of Rp dB, and stopband ripple of Rs db. Example:

```
n = buttord(2 * pi * 5e3, 2* pi * 9.5e3, 2, 35, 's’) ;
n

n =

   7
```

Butterworth example - order 2

With two output arguments, also gives the nominal cutoff frequency Wn of the Butterworth filter that fulfills the spec:

```
[n , Wn] = buttord(2 * pi * 5e3, 9.5e3 * 2 * pi, 2, 35, 's') ;

n

n =

    7

Wn

Wn =

  3.3567e+04
```

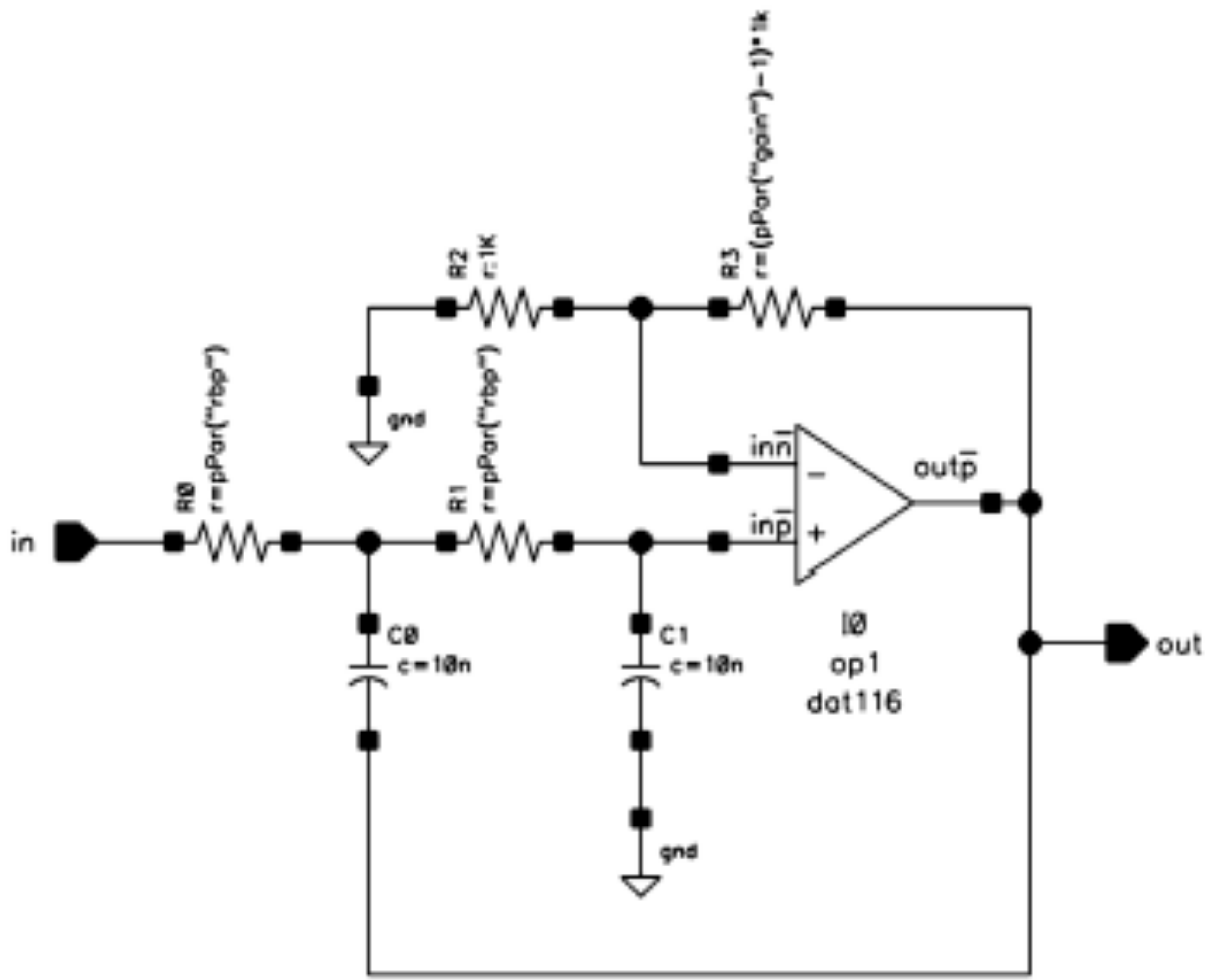
Butterworth example: poles

You may then use these parameters to let the function butter( ) actually calculate the pole positions for the filter:

```
[z, p, k] = butter(n, Wn, 'low', 's') ;
p

p =

  1.0e+04 *

 -0.7469 + 3.2725i
 -0.7469 - 3.2725i
 -2.0929 + 2.6244i
 -2.0929 - 2.6244i
 -3.0243 + 1.4564i
 -3.0243 - 1.4564i
 -3.3567 + 0.0000i
```

Analog Butterworth filters have no zeros:

```
z

z =

  Empty matrix: 0-by-1
```



R3
r=(pPar("gain")-1)*1k
R2
r:1K
R0
r=pPar("rbp")
R1
r=pPar("rbp")
gnd
in
inn
inp
outp
C0
c=10n
C1
c=10n
I0
op1
dat116
out
gnd

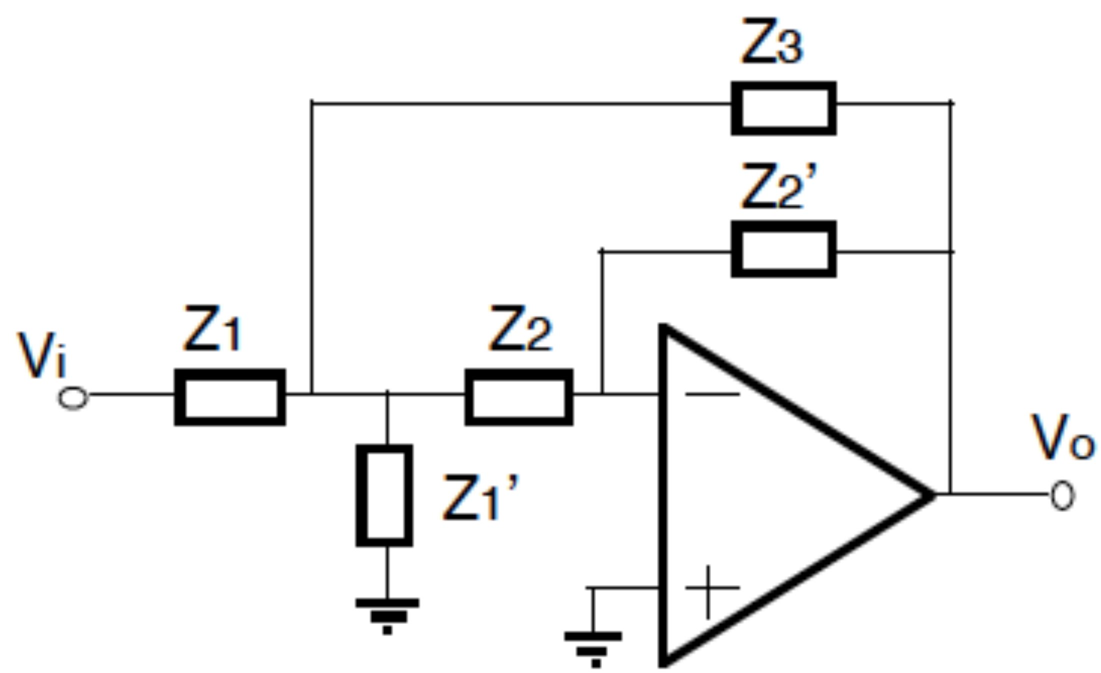
Poles -> R:s and C:s

- Each pole pair separately.
  - One biquad = biquadratic cell
- Identify R’s and C’s

For example (not Sallen-Key this time)

Single-Opamp Biquad

Rauch Biquadratic cell



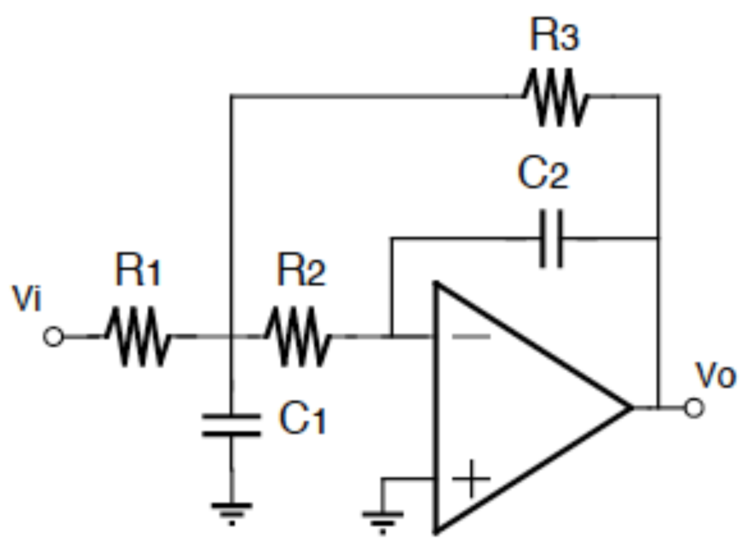
$$H(s) = \frac{v_O(s)}{v_i(s)} = -\frac{1}{\frac{Z_1}{Z_3} + \frac{Z_2}{Z_2'} \cdot \left(1 + \frac{Z_1}{Z_1'} + \frac{Z_1}{Z_2} + \frac{Z_1}{Z_3}\right)}$$

Figure from A. Baschirotto presentation : “Analog filters for Telecommunications”

Transfer function LP

Rauch Biquadratic cell

Lowpass Frequency Response



$Z_1 = R_1$

$Z_2 = R_2$

$Z_3 = R_3$

$Z_1' = 1/s \cdot C_1$

$Z_2' = 1/s \cdot C_2$

ı the passband, a current flows into the resistors

$$H(s) = \frac{1}{s^2 \cdot R_1 \cdot R_2 \cdot C_1 \cdot C_2 + s \cdot C_2 \cdot \left(R_1 + R_2 + \frac{R_1 \cdot R_2}{R_3}\right) + \frac{R_1}{R_3}}$$

Transfer function with two poles is:

$$H(s) = \frac{k}{s^2 + s(-p_1 - p_2) + p_1 p_2}$$

Identify terms!

Figure from A. Baschirotto presentation : “Analog filters for Telecommunications”

More freedom with more opamps

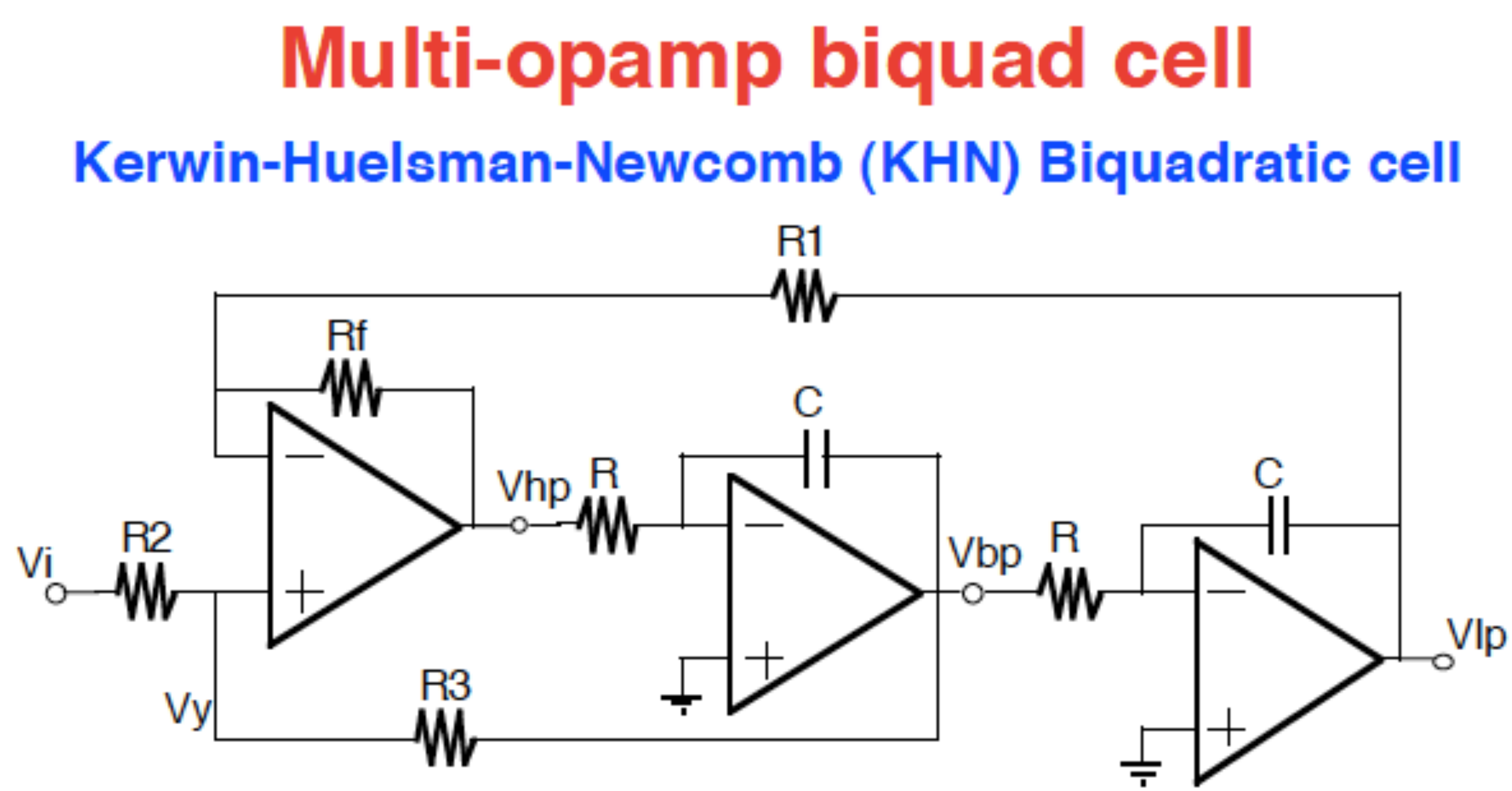
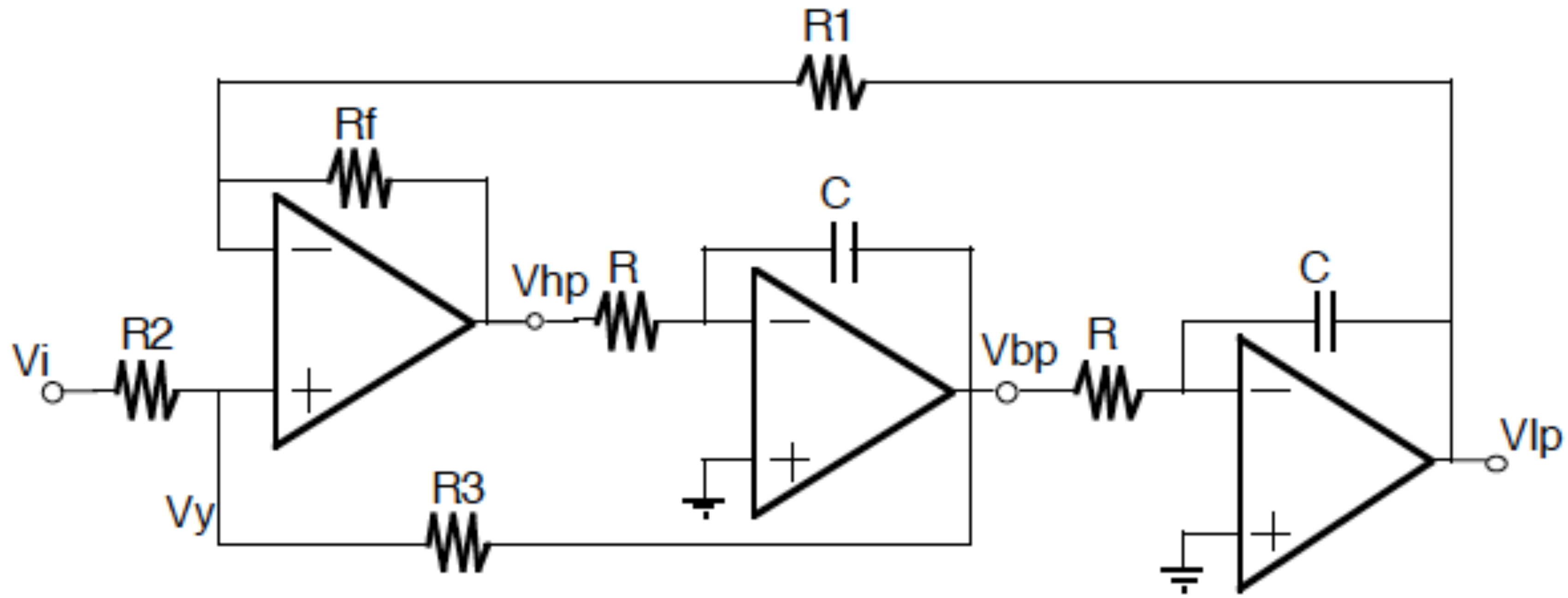


Figure from Baschirotto: “Analog filters for Telecommunications”

Multi-opamp biquad cell

Kerwin-Huelsman-Newcomb (KHN) Biquadratic cell



$$DEN(s) = s^2 + s \cdot \frac{R_2 \cdot (R_f + R_1)}{C \cdot R \cdot R_1 \cdot (R_2 + R_3)} + \frac{R_f}{C^2 \cdot R^2 \cdot R_1} = s^2 + s \cdot \frac{\omega_o}{Q} + \omega_o^2$$

$$\omega_o = \frac{1}{C \cdot R} \cdot \sqrt{\frac{R_f}{R_1}} \qquad Q = \left(1 + \frac{R_3}{R_2}\right) \cdot \frac{\sqrt{R_f \cdot R_1}}{R_f + R_1}$$

Figure from Baschirotto: “Analog filters for Telecommunications”

Topics for today

- More about OPamp-RC integrators
  - How good OPamps is required?
- LP. What about HP / BP / BS ?
- Passive filters
- Ladder filters
- Components
- Balanced implementations

Taxonomy of analog filters

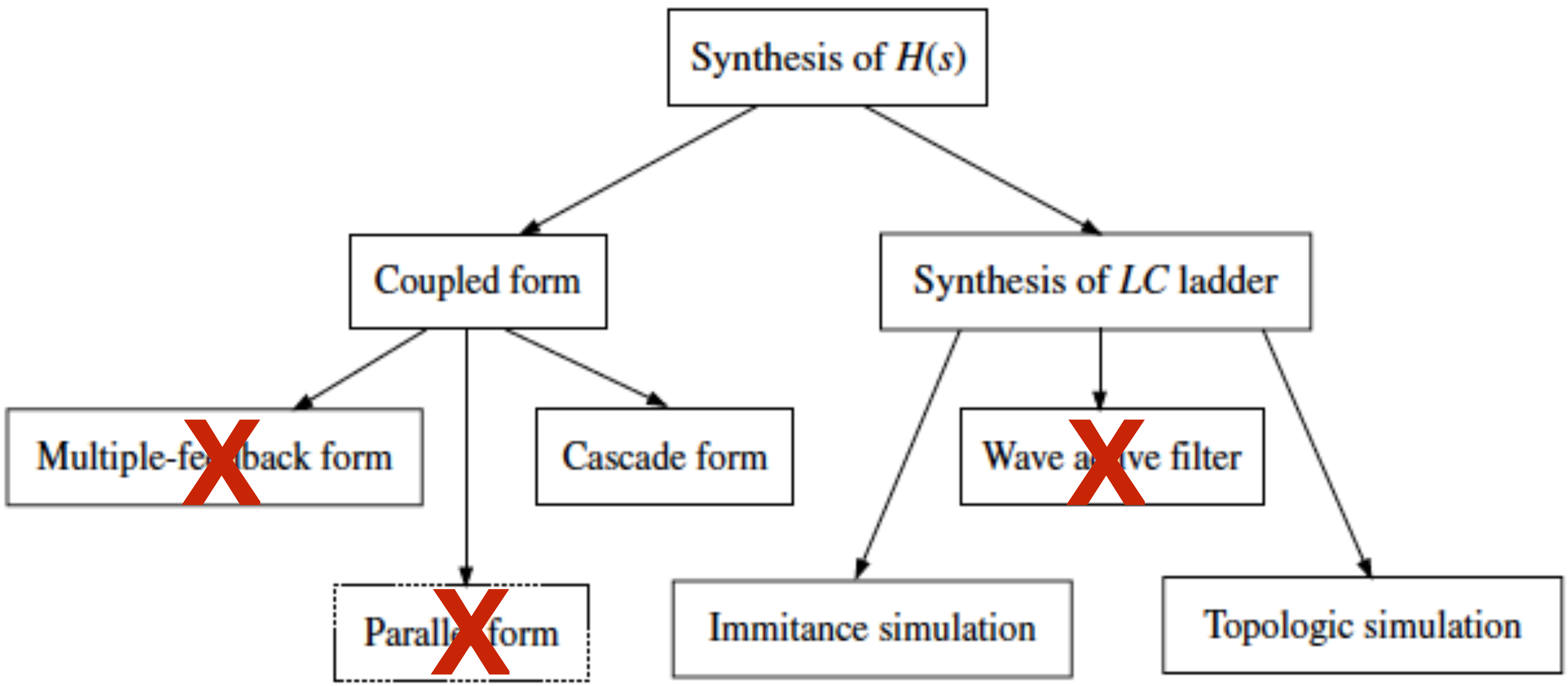


Fig. 7.1 Taxonomy for analog filter structures

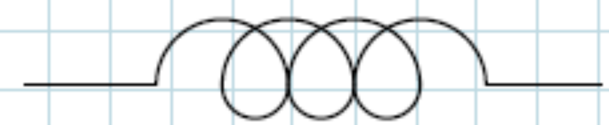
Wanhammar: Analog filters using MATLAB

Passive components

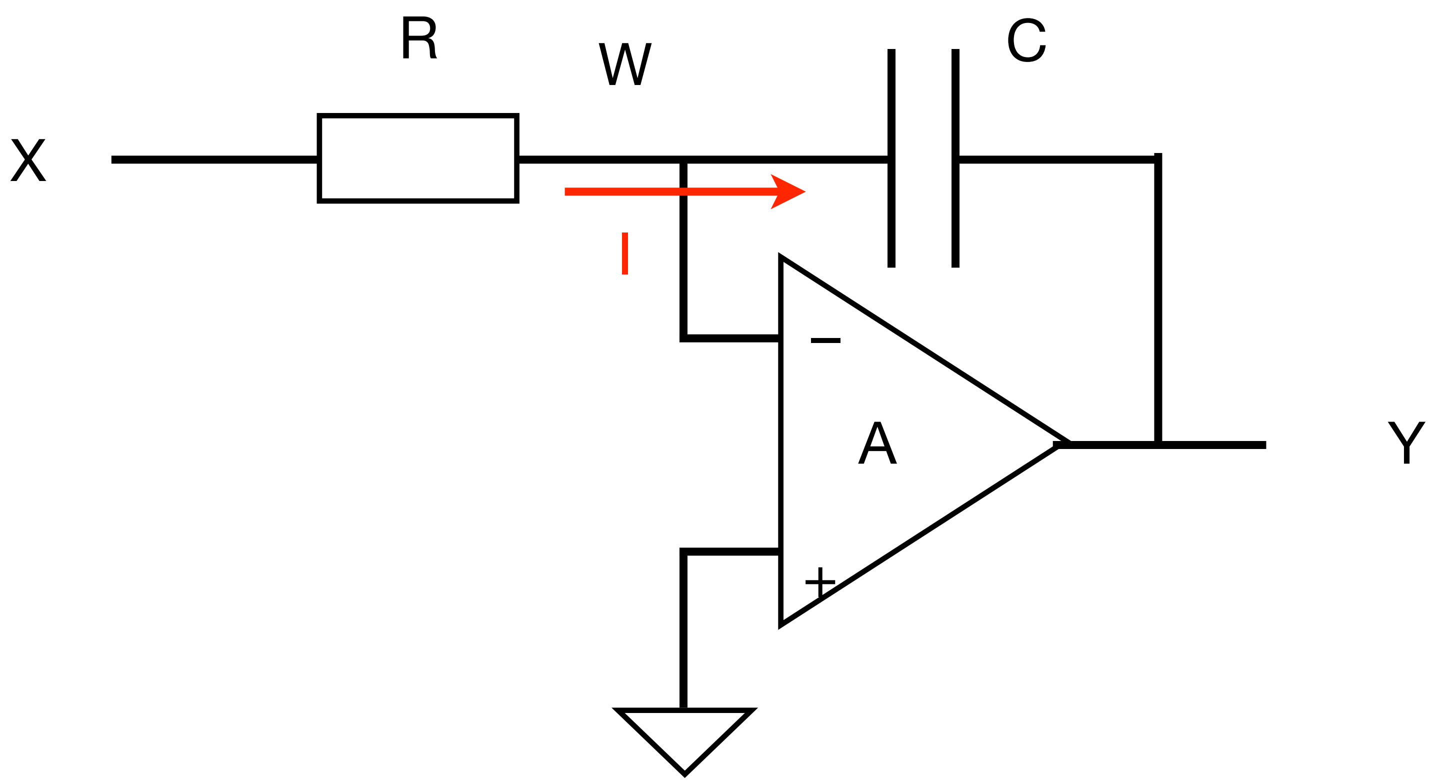
- Resistor
- Capacitor
- Inductor

$$I = C\frac{dV}{dt}$$

$$V = L\frac{dI}{dt}$$



OPamp-RC integrator
(Miller integrator)



OPamp-RC integrator
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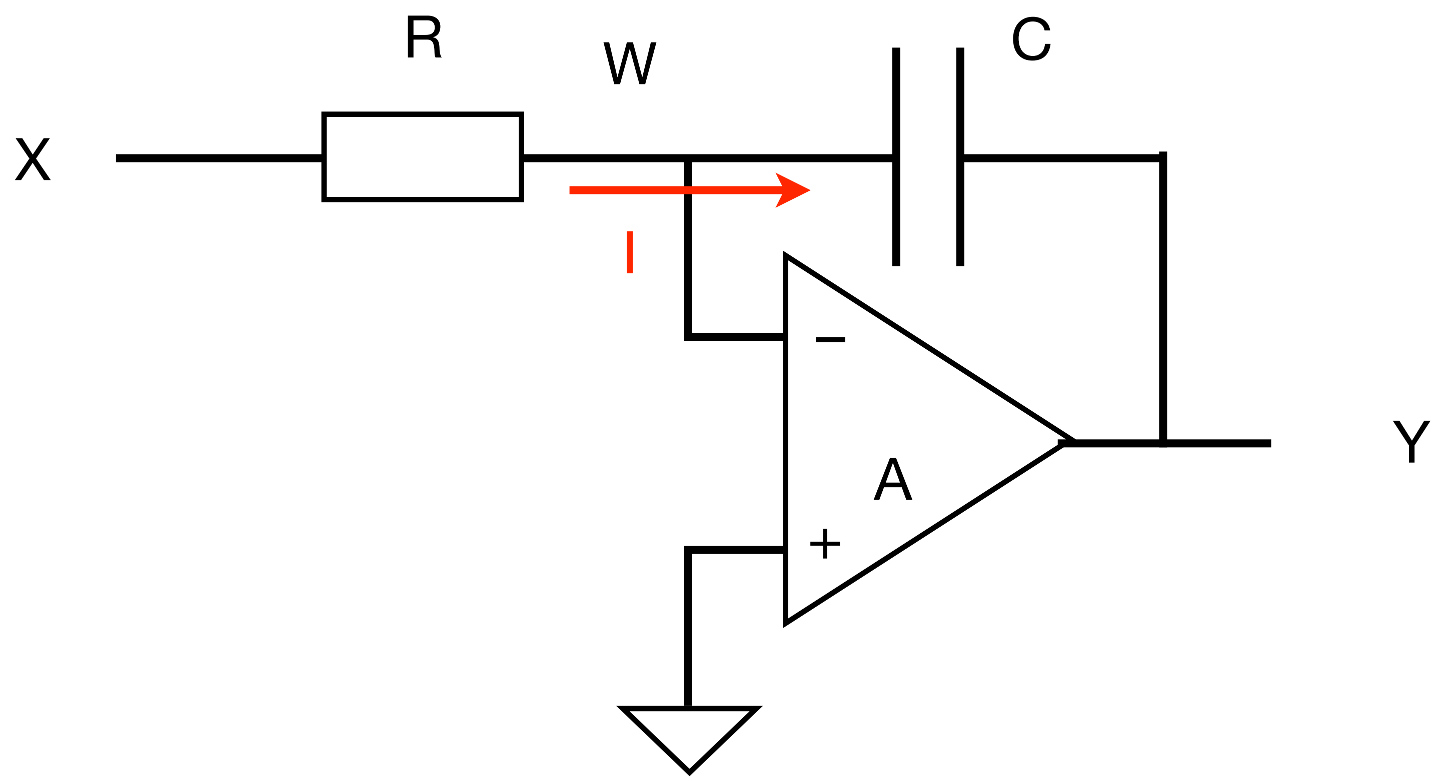
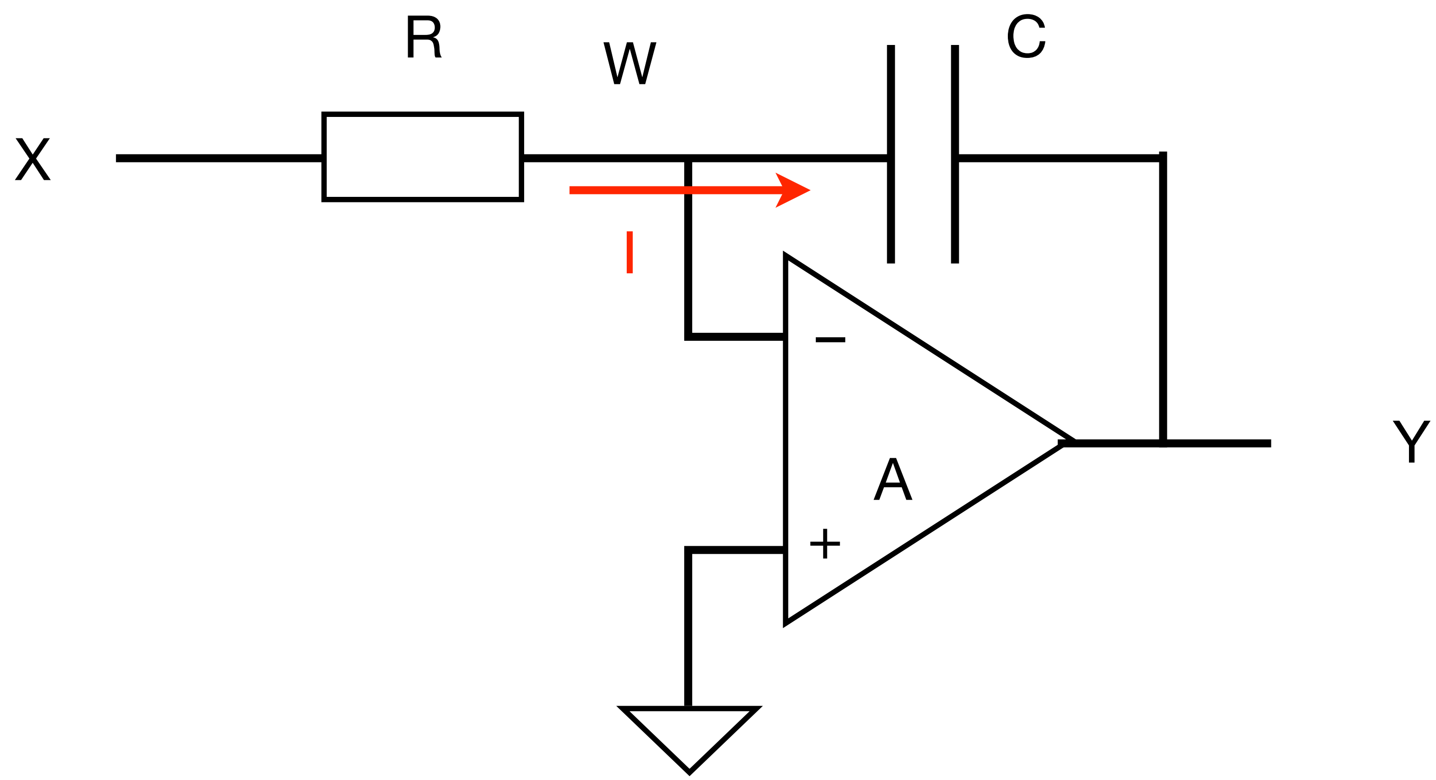


- $Y = \int I\, dt / C \approx - \int X\, dt / (R \cdot C)$
- Perfect integrator has infinite DC gain
- Real integrator limited by OPamp **gain** and **dominant pole**!

OPamp-RC integrator
(Miller integrator)



Perfect integrator: $H(s) = 1/s\tau$

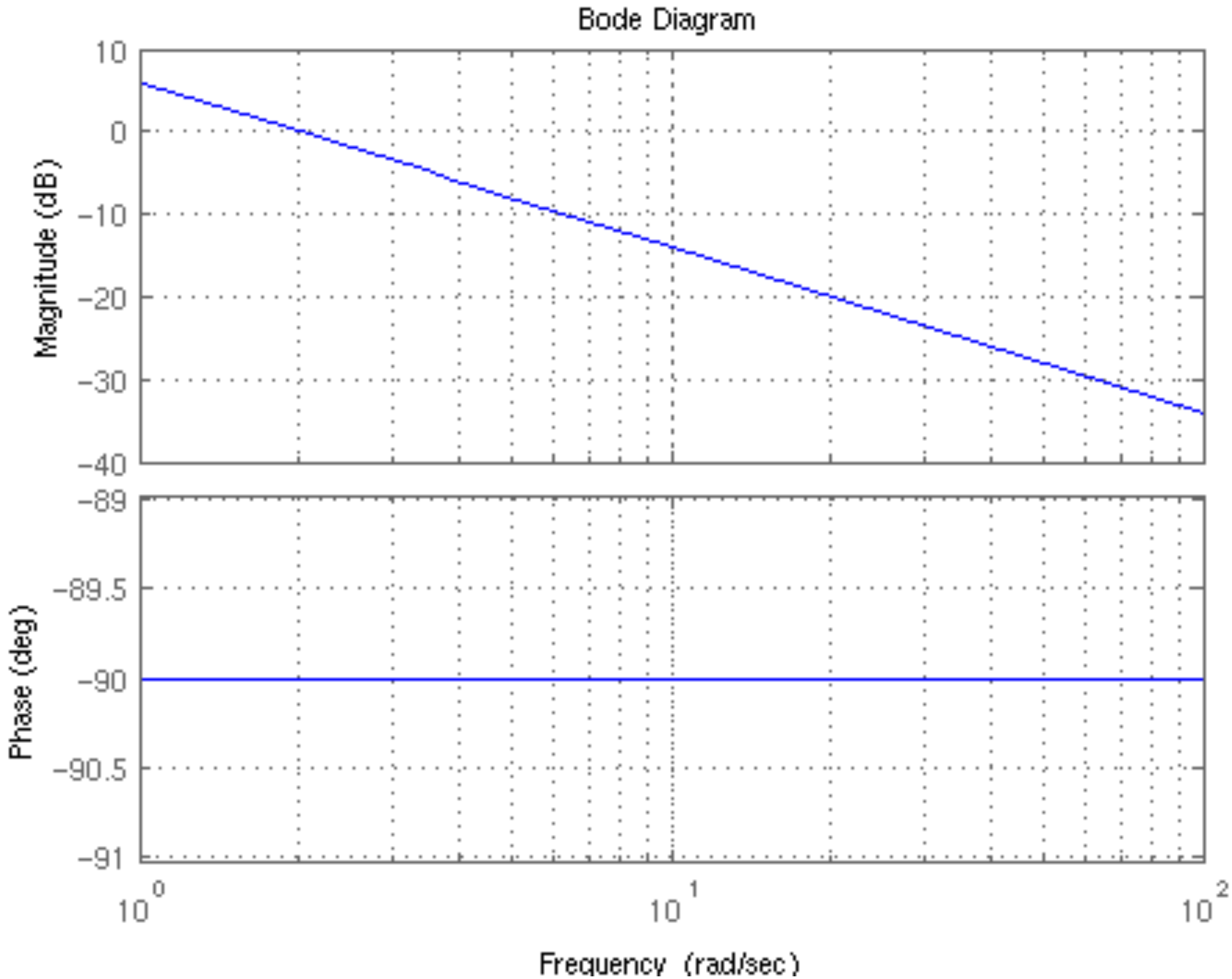
Imperfect integrator with loss q: $$H(s) = \frac{1}{s\tau + q}$$

Goals

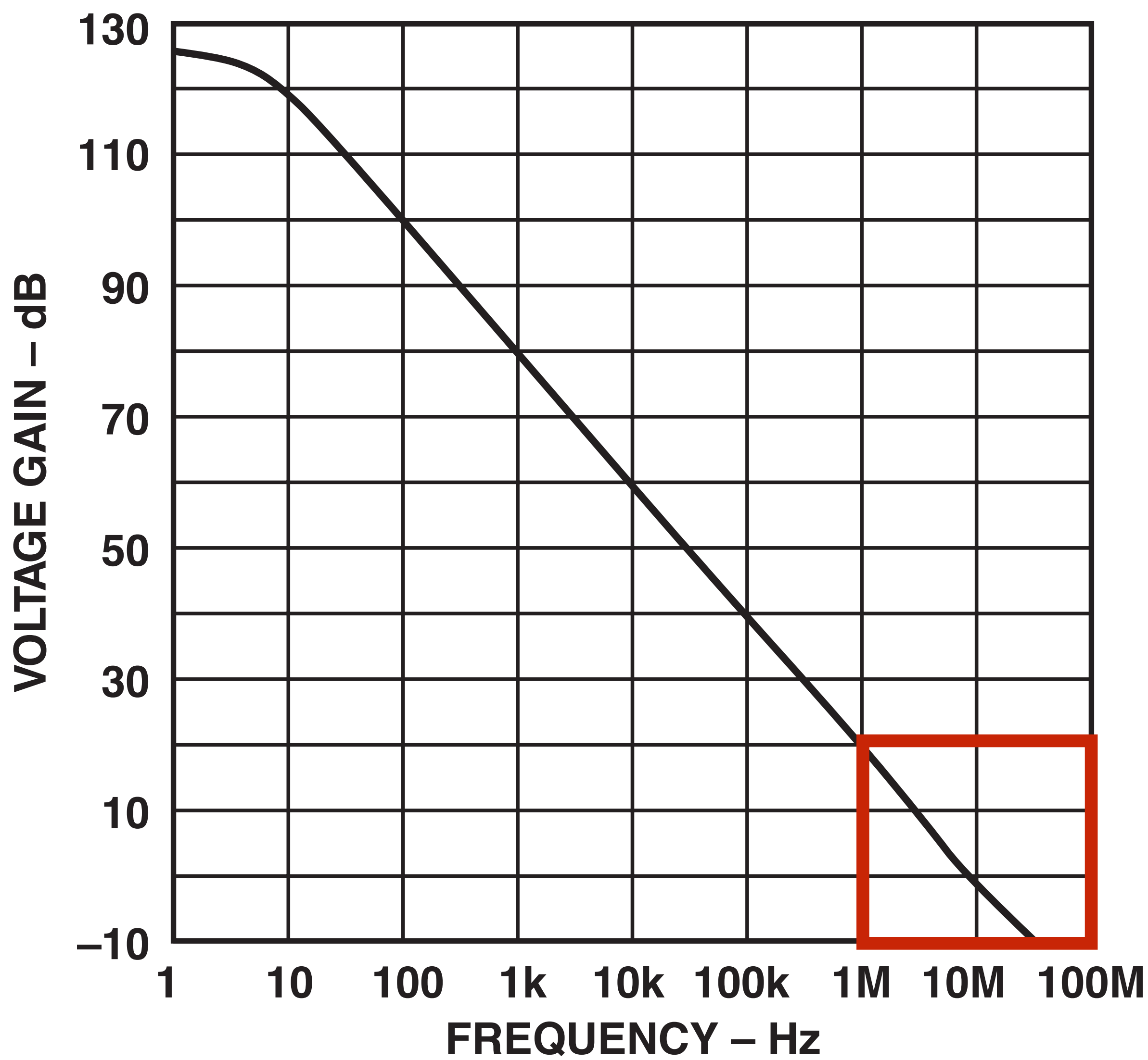
Show the effects of the **OP amplifier**
limited LF gain, A_0, and
dominant pole frequency (or GBW) on the
the **integrator** non-idealities (loss).

Show the requirement on the **integrator** quality (Q_{int})
from the desired **filter Q (Q_{ideal})**.

Perfect integrator Bode plot



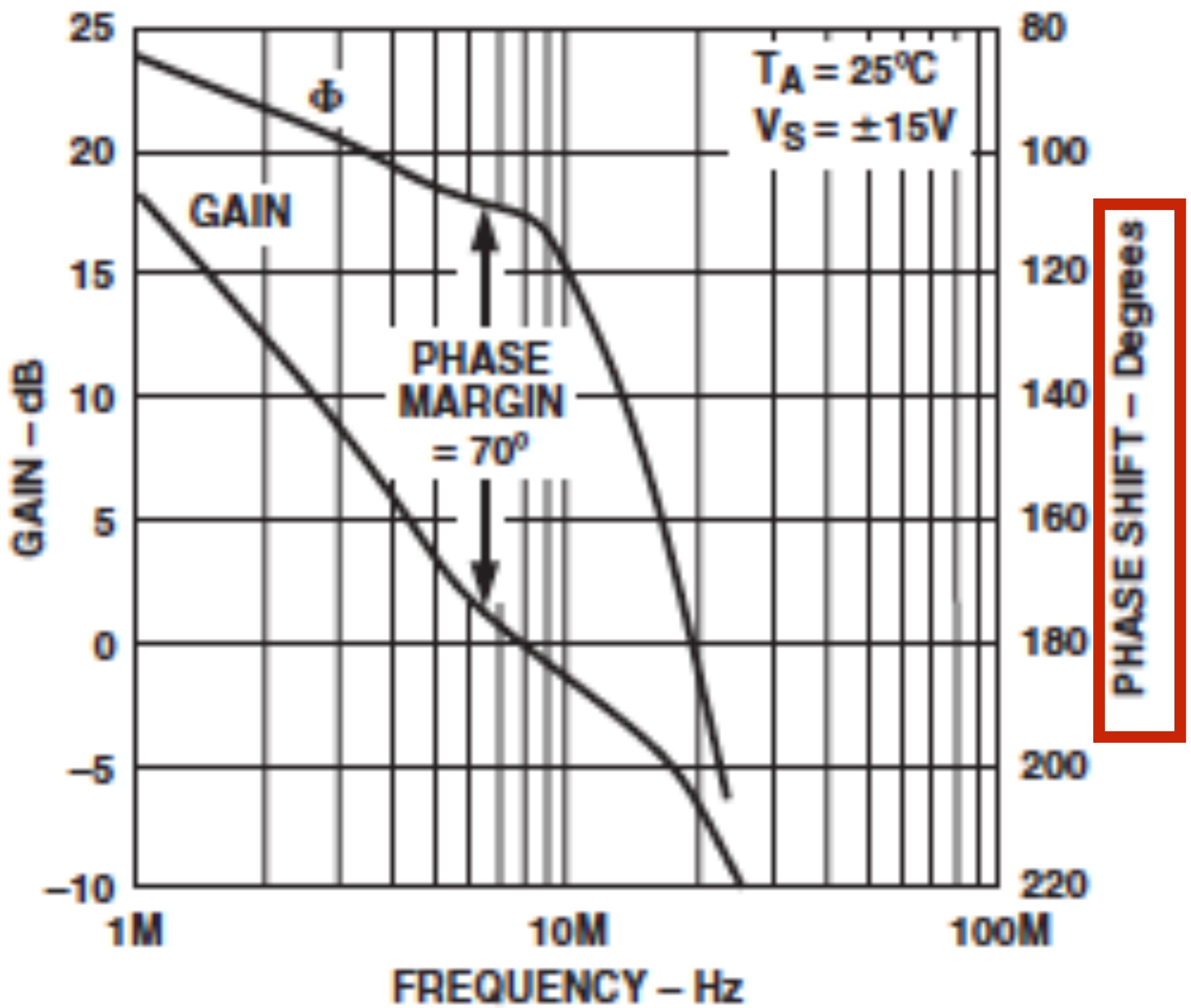
OP-amps from week 2...



TPC 16. Open-Loop Gain vs. Frequency

And what about the phase? How would that look?

Phaseshift closeup



TPC 18. Gain, Phase Shift vs. Frequency

OPamp RC integrator limitations

$$H_{ideal}(s) = -\frac{1}{sRC}$$

OPamp transfer function (assuming one dominant pole):

$$H_{OPamp}(s) = \frac{A_0}{1 + s\tau}$$

Closed-loop integrator transfer function?

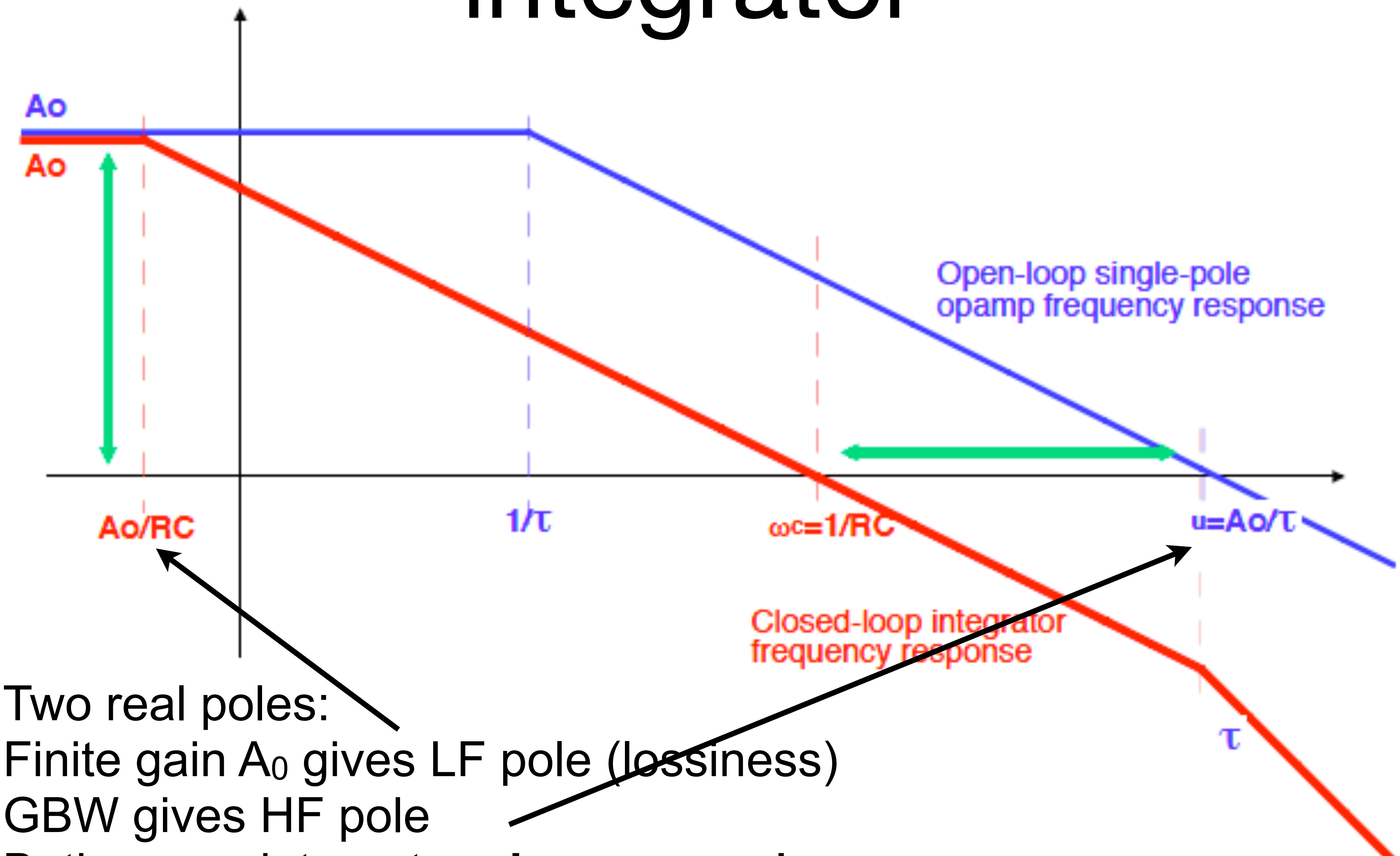
So good integrator requires: high LF gain (Ao) & high GBW

$$H_{CL_{real}}(s) = -\frac{A_0}{(1 + sRCA_0)(1 + s\tau/A_0)}$$

Intended pole! Extra pole

Note that Ao/τ = GBW!

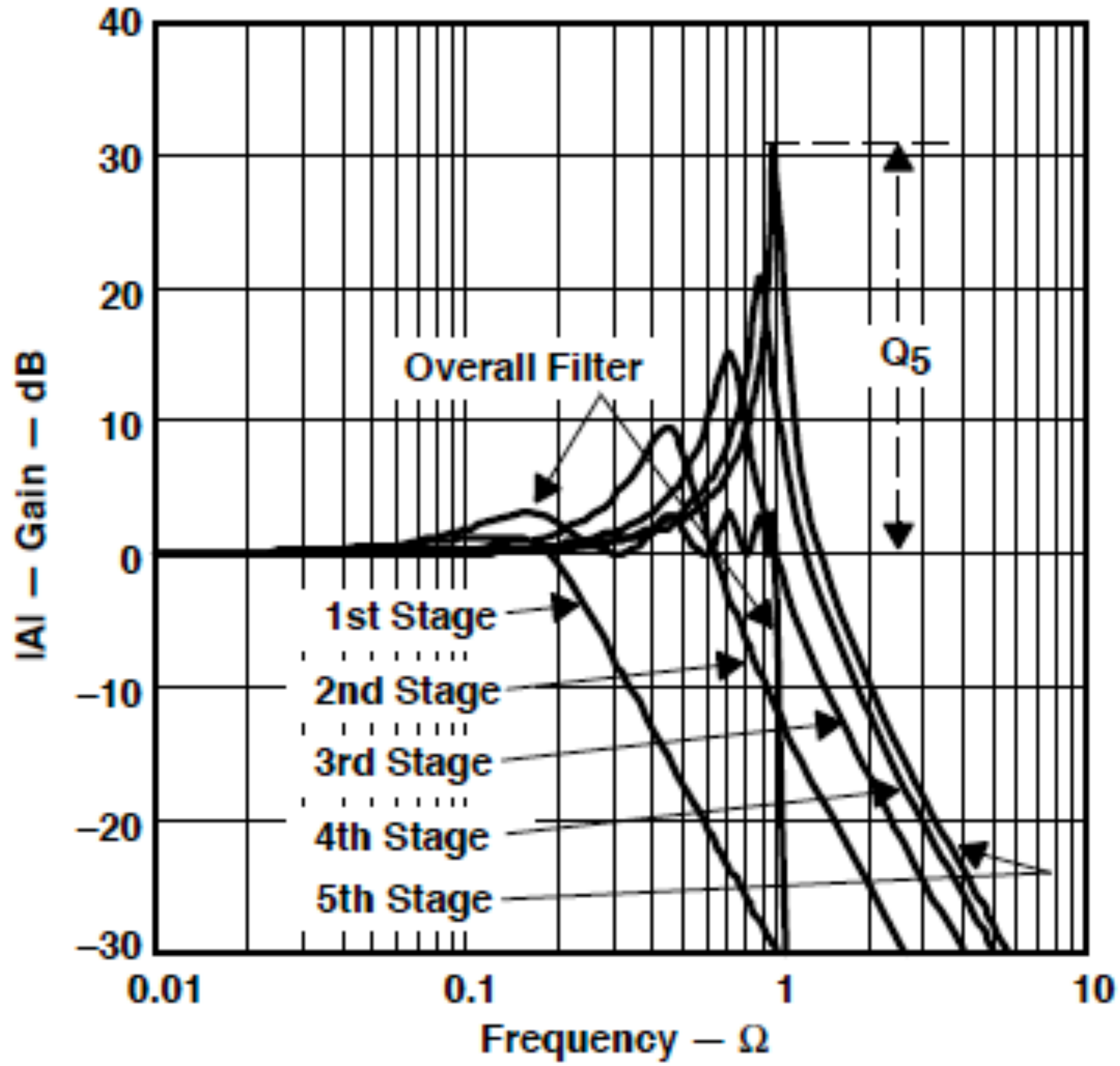
Limitations of OPamp-RC integrator



Two real poles:
Finite gain A_0 gives LF pole (lossiness)
GBW gives HF pole
Both cause integrator **phase errors**!

Figure from Baschirotto: “Analog filters for Telecommunications”

(recap) Higher Q for higher order filter



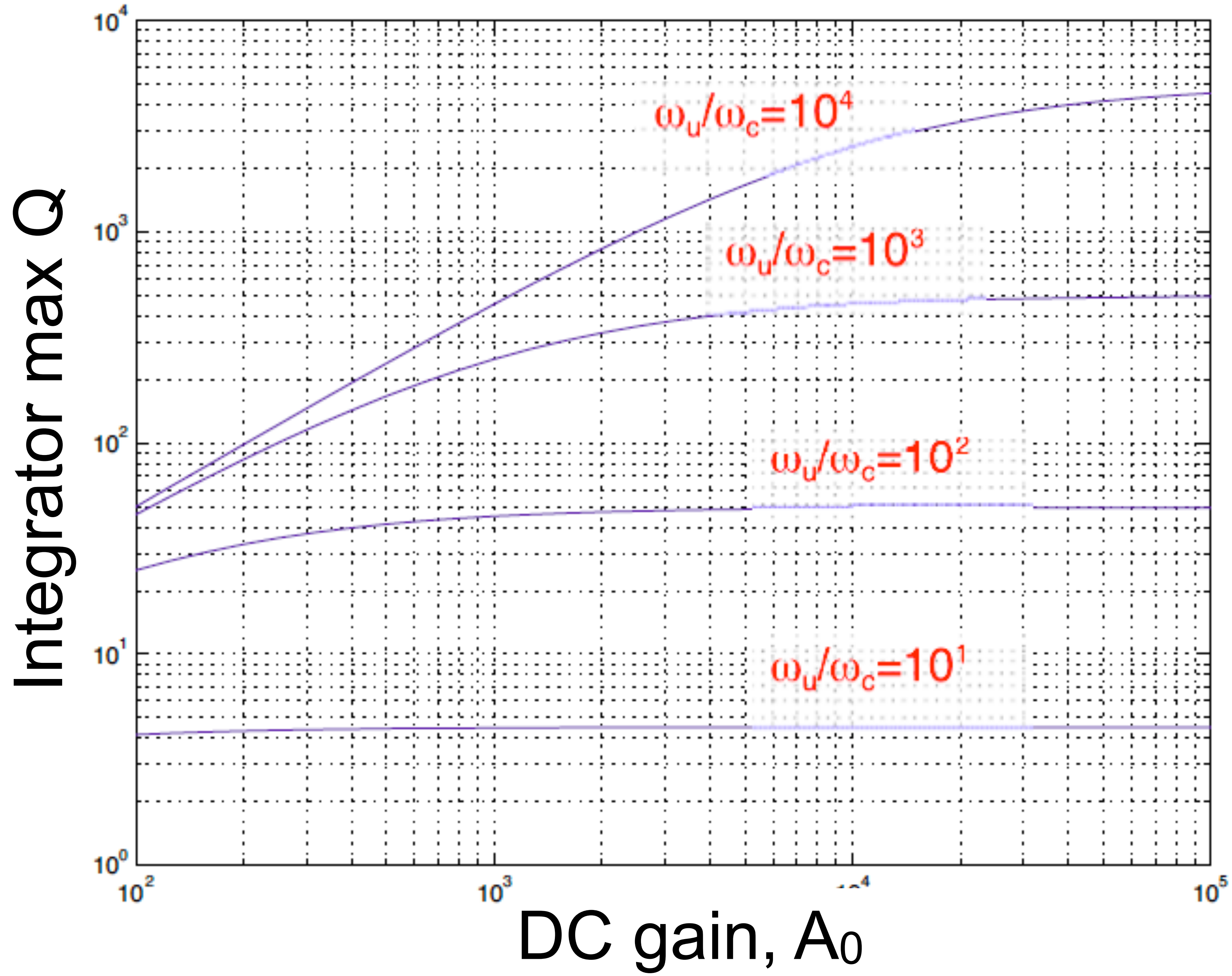
Example: 10th order Chebysjev filter.
How much phase error can we tolerate?

Q limitations

- If q is the loss in the integrator: $H(s) = \frac{1}{s\tau + q}$
- Then Q of integrator is $Q_{int} = \frac{\omega RC}{q}$
- The integrator Q: how close we can get to Q_{ideal}.
- The 2 Q:s are “in parallel”:

$$\frac{1}{Q_{real}} = \frac{1}{Q_{int}} + \frac{1}{Q_{ideal}}$$

OPamp parameters related to Q for integrator, A_0 and GBW (ω_u)



ω_c is integrator frequency = 1/RC

So, the higher integrator Q you want the higher the GBW has to be and Ao also has to be high

From Baschirotto: "Analog filters for Telecommunications"

Q limitations (cont)

Parallel connection of Q's:

$$\frac{1}{Q_{real}} = \frac{1}{Q_{int}} + \frac{1}{Q_{ideal}}$$

Can be expressed as a **requirement on integrator** to achieve (almost) ideal Q:

$$\frac{1}{Q_{int}} = (\frac{\Delta Q}{Q_{ideal} + \Delta Q})\frac{1}{Q_{ideal}}$$

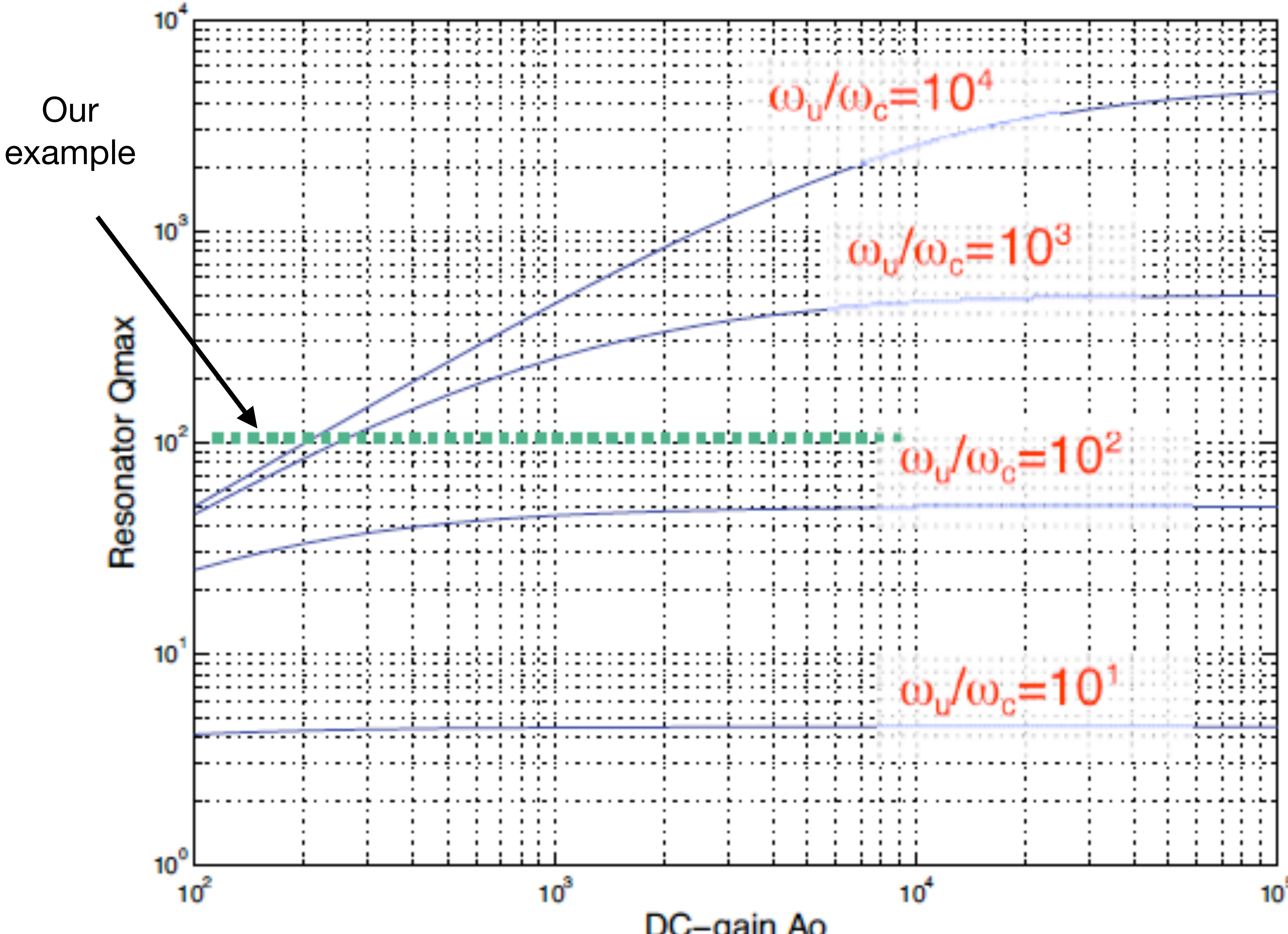
$$\frac{1}{Q_{int}} \approx (\frac{\Delta Q}{Q_{ideal}})\frac{1}{Q_{ideal}}$$

How good an integrator is required?

Conclusion

- You need really good OPamps to implement active filters (integrators)
  - High A_0 and high GBW
- In conclusion: Harder to implement higher cut-off frequencies and higher filter Q.

Requirements on integrator



Other filter types?

- LP filters dominate literature
  - Practically important in mixed-signal interfaces
- Other frequency selective filters may be designed by <u>transforming</u> LP filter
  - Poles/zeros or implementation
- All-pass filters are a special case…

Create HP from LP

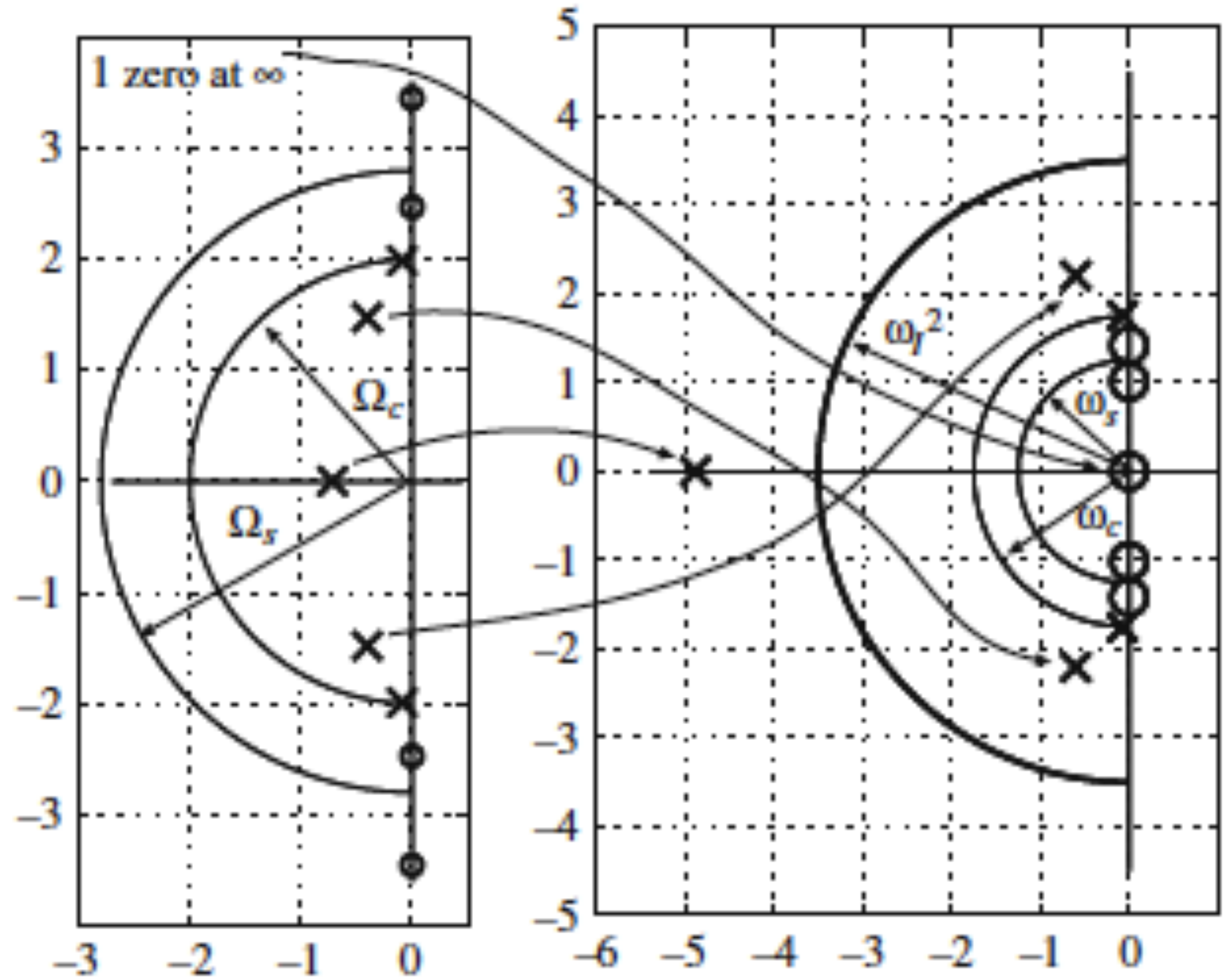


Fig. 2.50 LP-HP transformation of poles and zeros

- Invert s values for poles and zeros !
  - Will fit a “mirror image” of specification

BP from LP

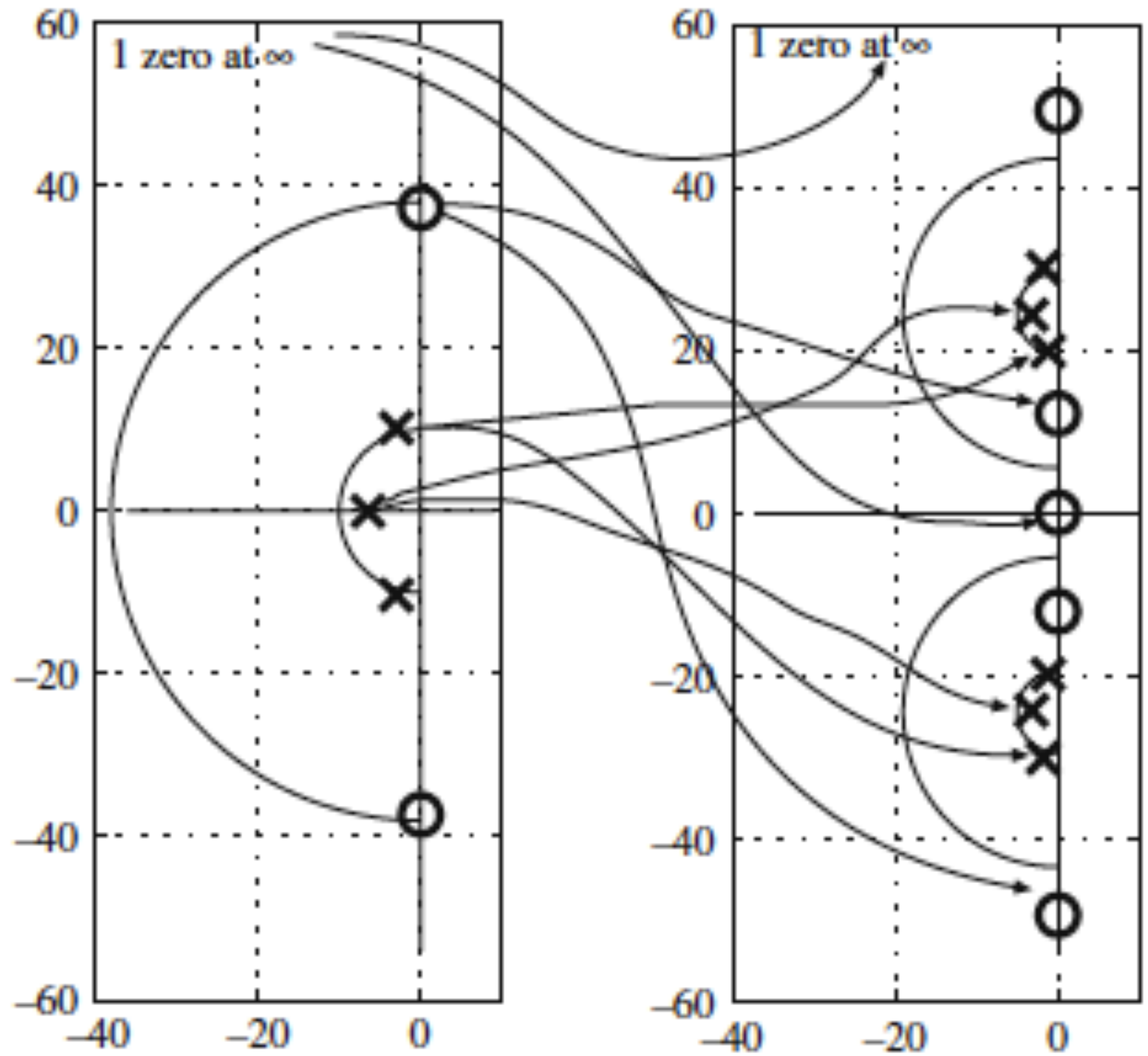


Fig. 2.56 LP-BP transformation of poles and zeros

- Similar idea (simple); not as general

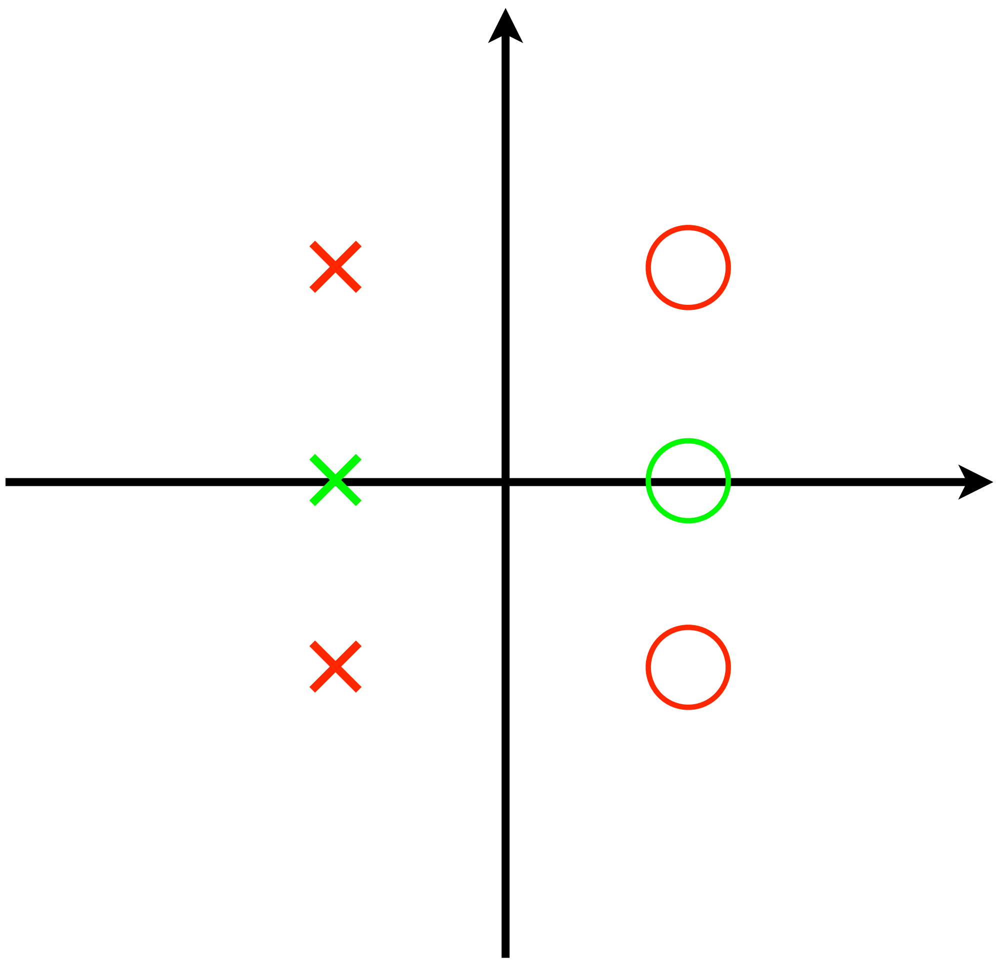
Implementation

- HP: can use “mirrors” of LP second-order links (such as Sallen-Key, etc)
- BP/BS: consider less sensitive structures
  - E.g. ladder filters; later today

Not shown in class

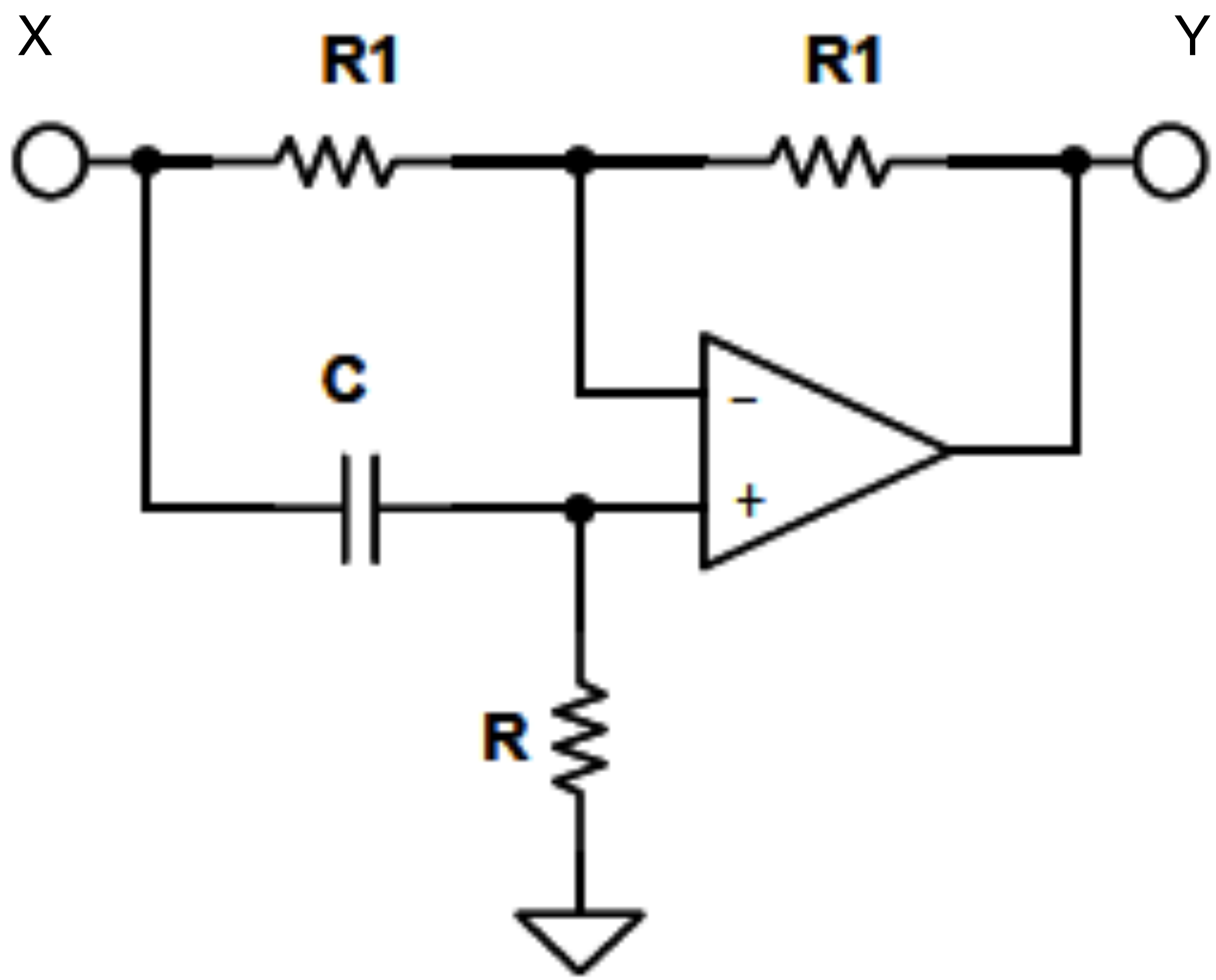
Allpass filters

- Amplitude function independent of frequency
    - Purpose is phase shift / delay
- Symmetric pole/ zero placement



Not shown in class

One implementation



pole at s = $-1/RC$

zero at s = $+1/RC$

Not shown in class

What is the result?

$$Y = A\left(\frac{XR}{R + \frac{1}{sC}} - \frac{X+Y}{2}\right)$$

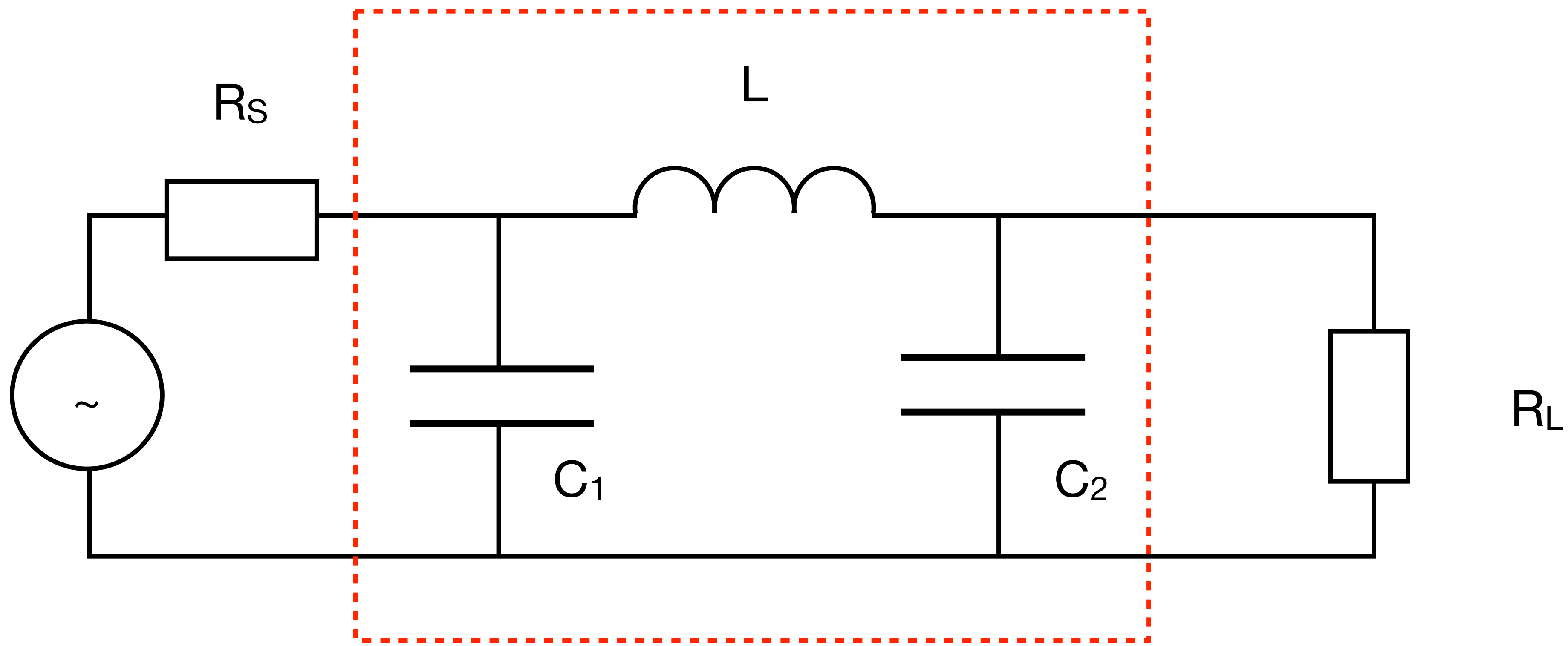
$$Y\left(1 + \frac{A}{2}\right) = XA\left(\frac{R}{R + \frac{1}{sC}} - \frac{1}{2}\right)$$

$$H(s) = \frac{Y}{X} = \frac{A\left(\frac{R}{R+\frac{1}{sC}} - \frac{1}{2}\right)}{\left(1 + \frac{A}{2}\right)} = \left(\frac{A}{2+A}\right)\frac{sRC - 1}{sRC + 1}$$

Passive filters

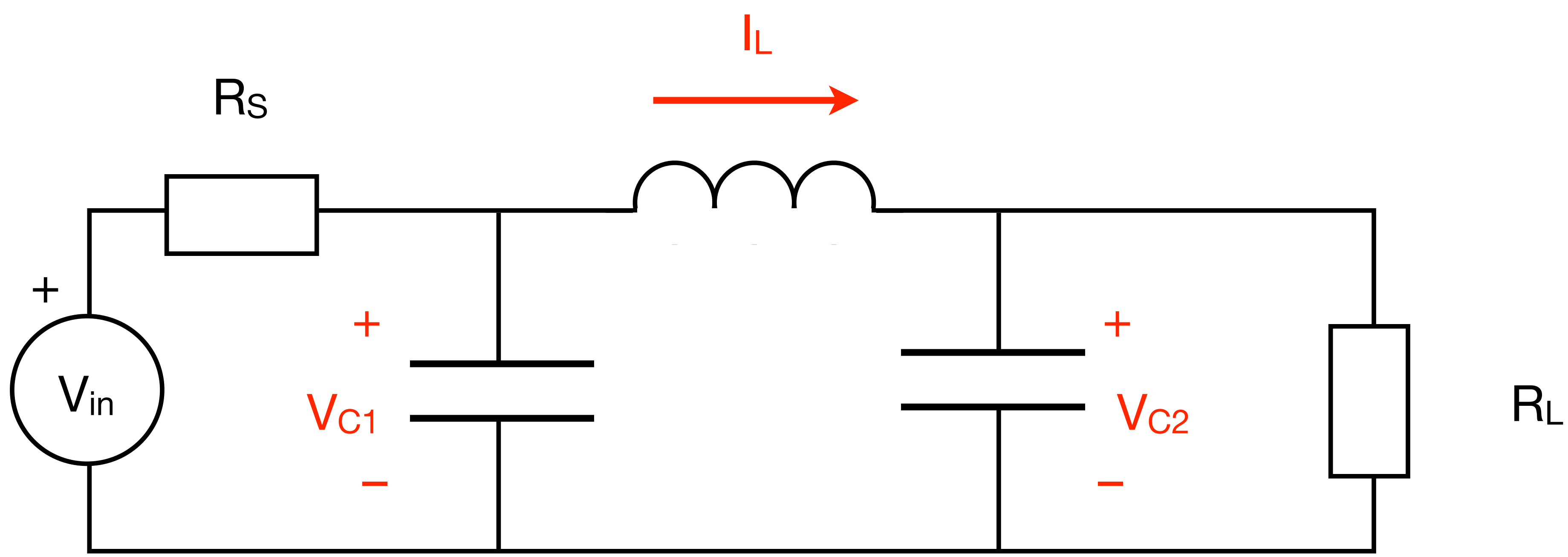
- In practice, used in high-frequency (on-chip) and high-power (off-chip) applications
- Large body of theory
- Many forms highly parameter-insensitive
  - Typically only in passband…
- Useful as “mental model” for active filters
  - “Prototype filter”

Example: LP LC filter



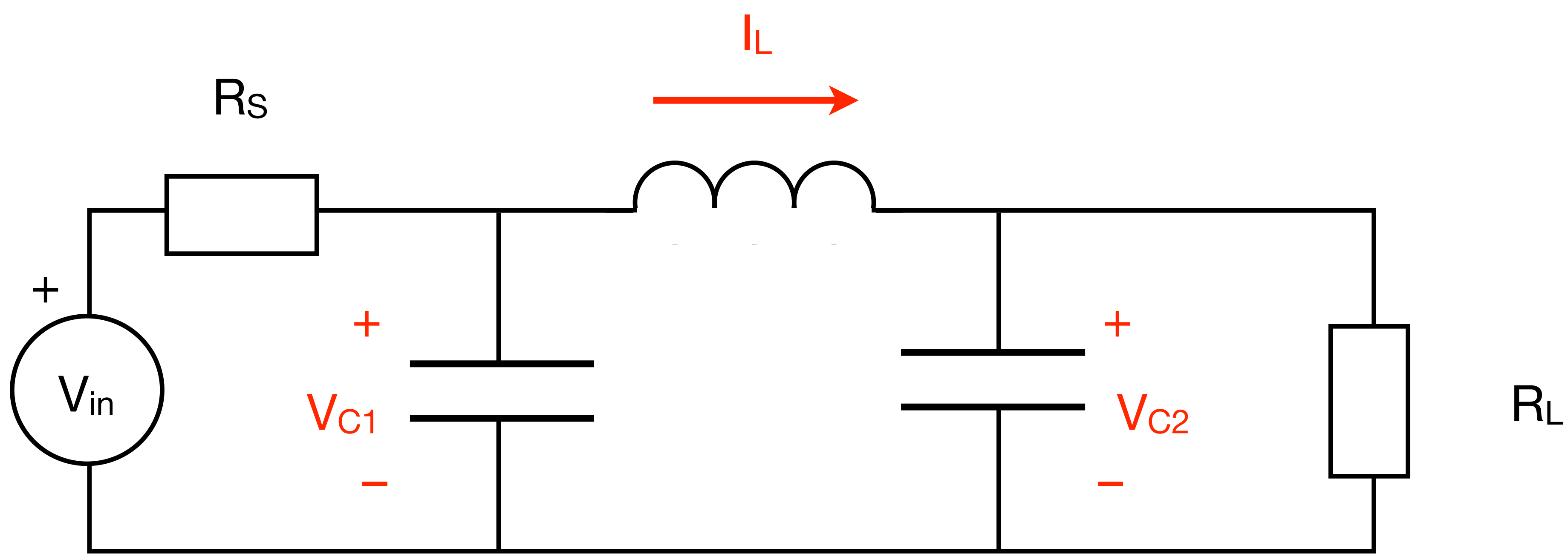
- 3rd order all-pole filter (zeros at ∞)
- Each reactive component is an integrator
- Zero attenuation at DC

State variables



- L currents, C voltages are state variables
- Write equation system for these

Equations



- $I_{RS} = (V_{in} - V_{C1}) / R_S$
- $V_{C1} = \int (I_{RS} - I_L)\, dt / C_1$
- $I_L = \int (V_{C1} - V_{C2})\, dt / L_1$
- $V_{C2} = \int (I_L - I_{RL})\, dt / C_2$
- $I_{RL} = V_{C2} / R_L$

Observations

- $I_{RS} = (V_{in} - V_{C1}) / R_S$
- $V_{C1} = \int (I_{RS} - I_L)\, dt / C_1$
- $I_L = \int (V_{C1} - V_{C2})\, dt / L_1$
- $V_{C2} = \int (I_L - I_{RL})\, dt / C_2$
- $I_{RL} = V_{C2} / R_L$

Observations

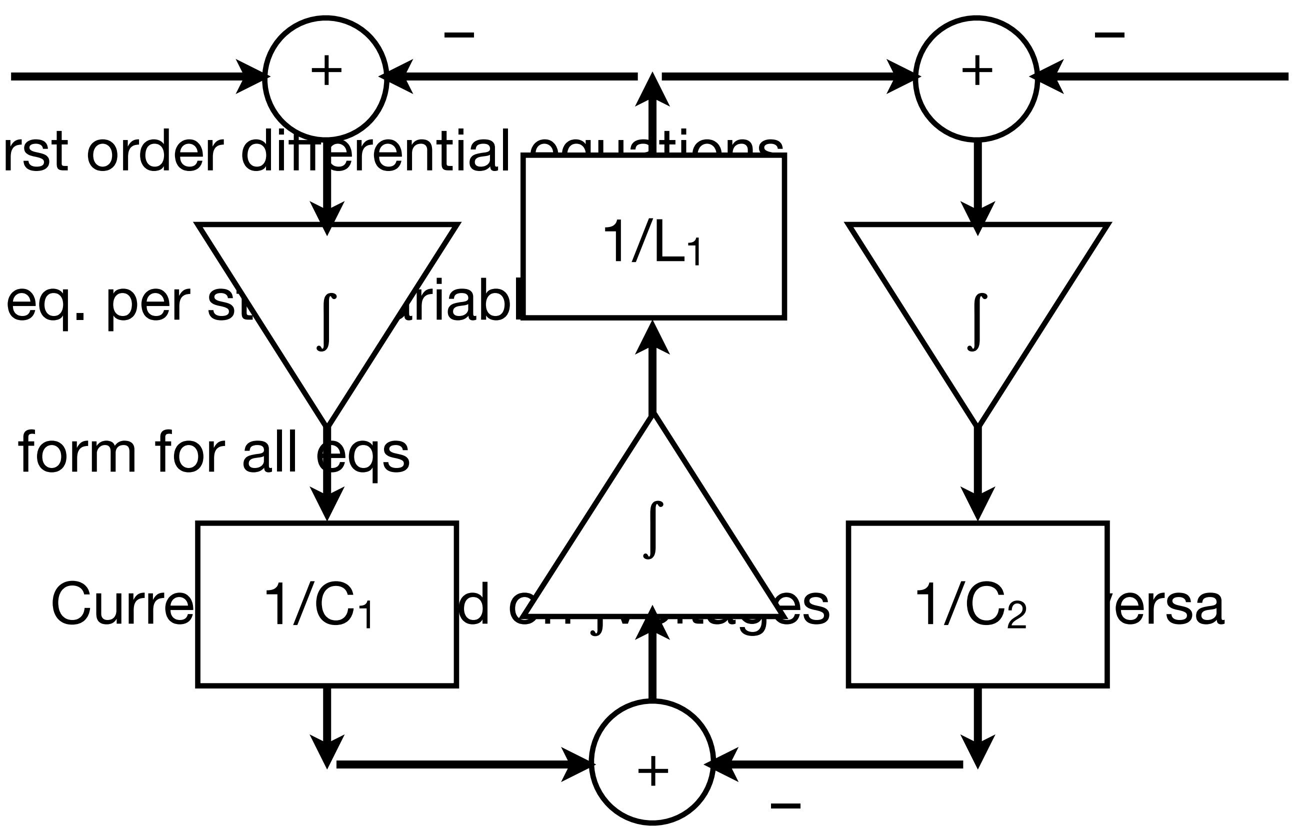
- $I_{RS} = (V_{in} - V_{C1}) / R_S$
- $V_{C1} = \int (I_{RS} - I_L)\, dt / C_1$
- $I_L = \int (V_{C1} - V_{C2})\, dt / L_1$
- $V_{C2} = \int (I_L - I_{RL})\, dt / C_2$
- $I_{RL} = V_{C2} / R_L$

First order differential equations

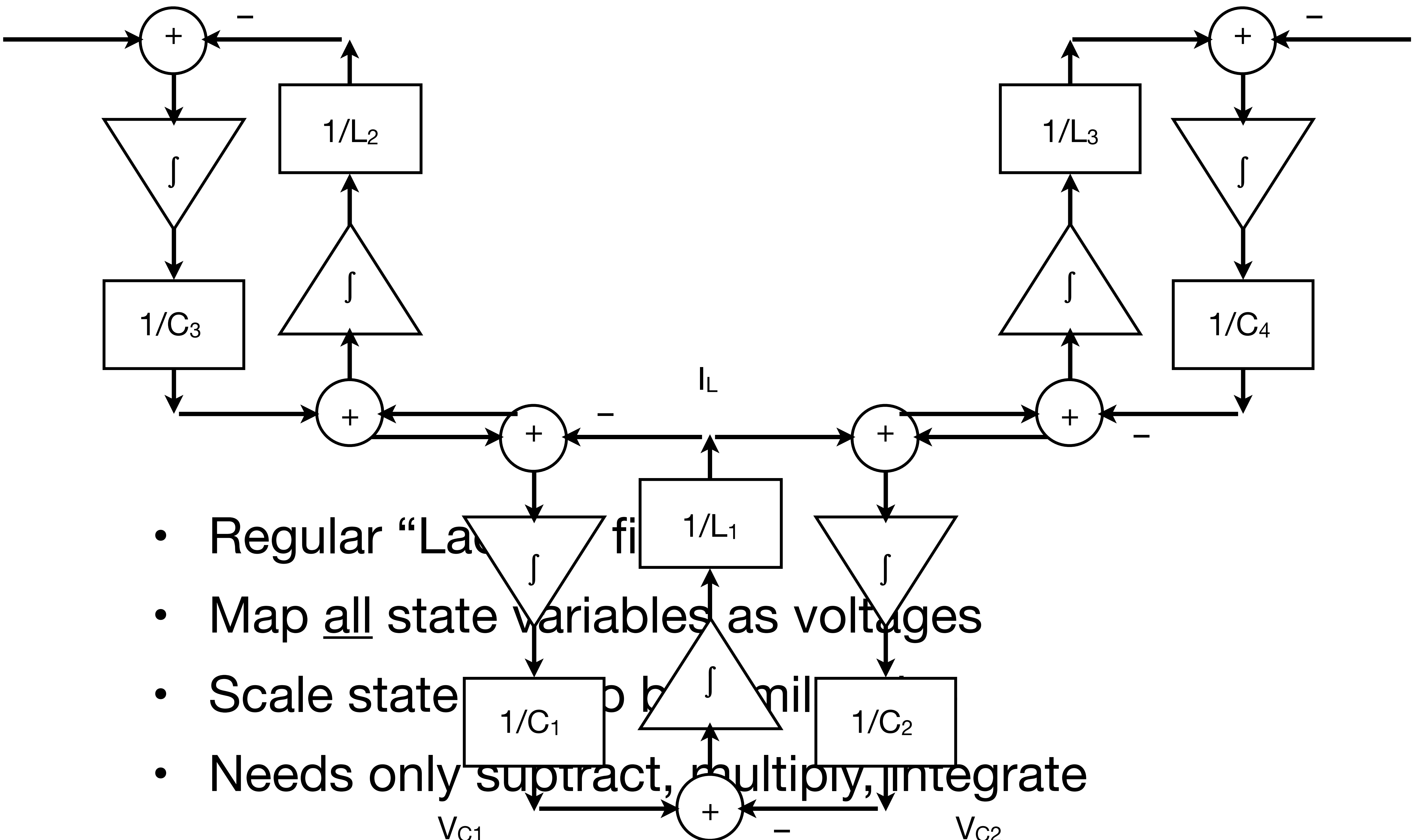
One eq. per st… …ariabl…

Same form for all eqs

Curre… …d on voltages… …ersa



Emulate!



- Regular “La[…] fi[…]
- Map <u>all</u> state variables as voltages
- Scale state[…]o b[…]mil[…]
- Needs only subtract, multiply, integrate

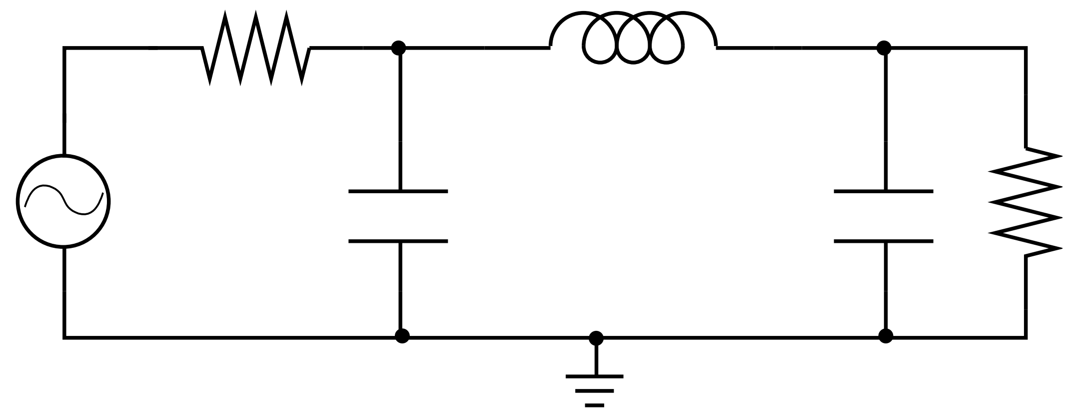
Ladder filters

- One of several “emulations” of different passive filters
- Only gains as design parameters
  - Good for matching!
- Low sensitivity to gain tolerances
  - Variations don’t bring instability
- Each pole and zero depends on all gains

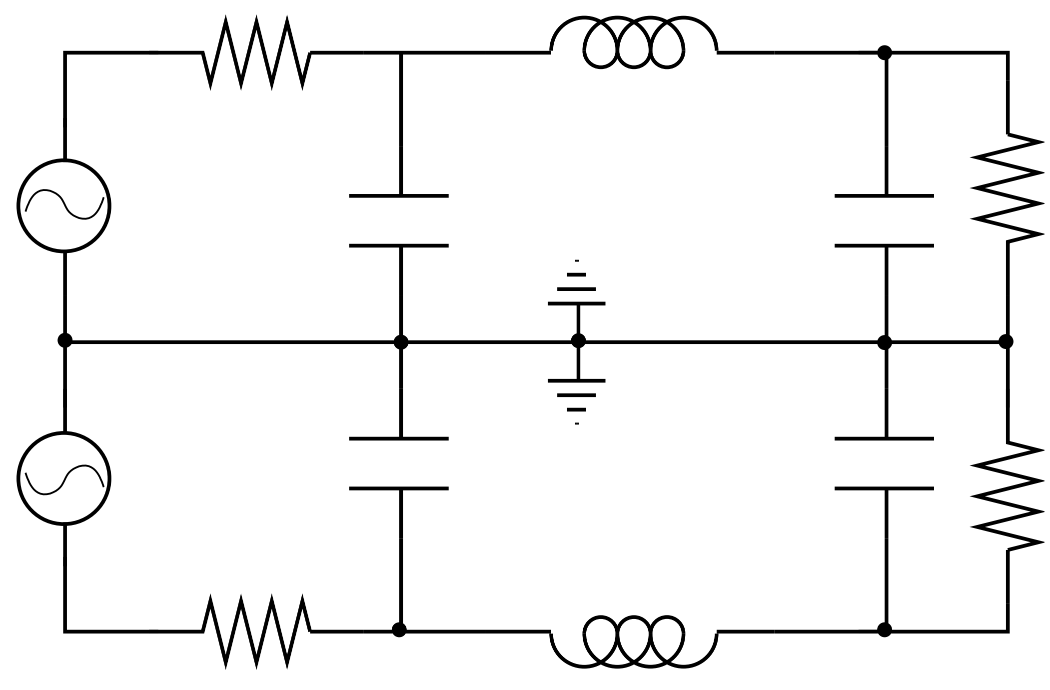
Operations

- Subtraction
  - Doable with invert + add
- Gain
  - Yes, we know how to do that
- Integrator?

In practice: differential implementation used (at least on chip)

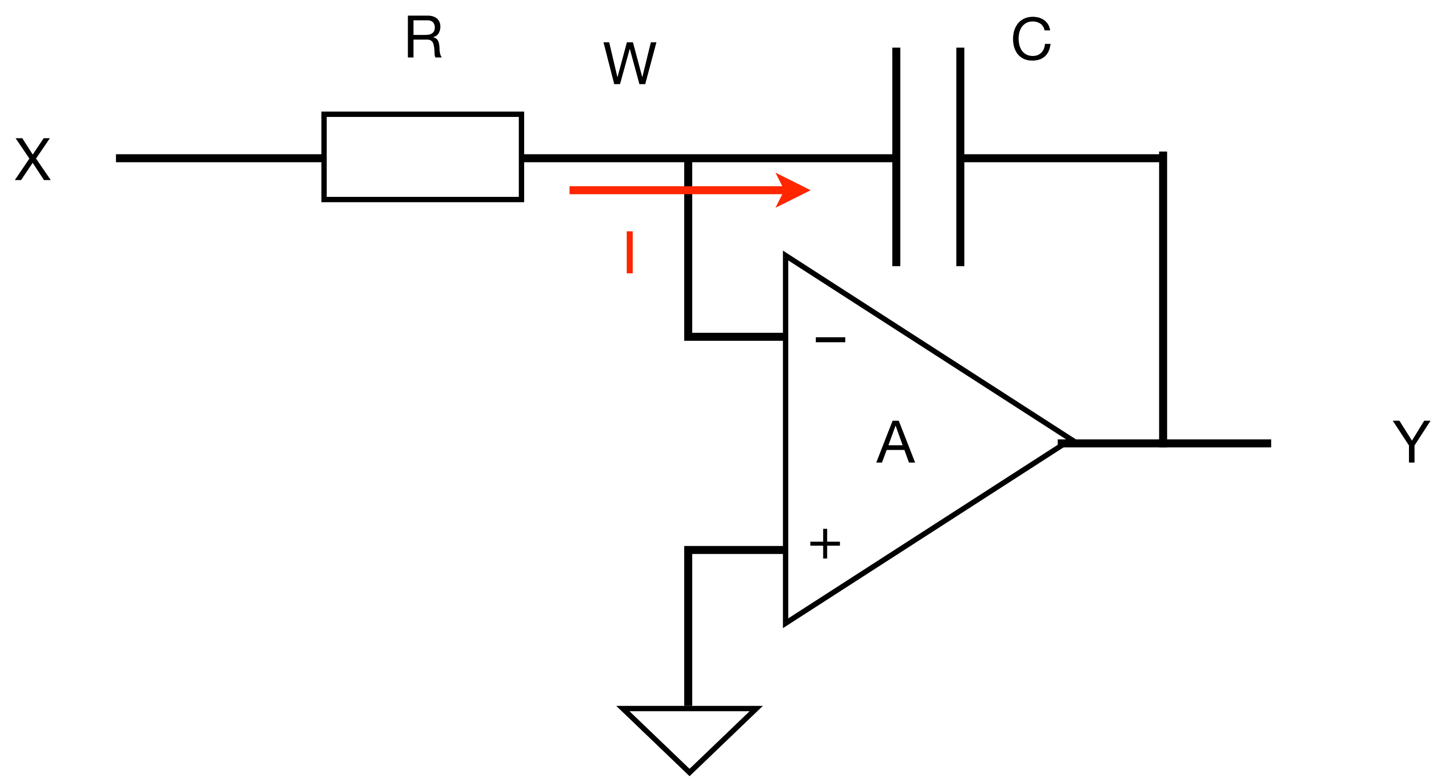


becomes



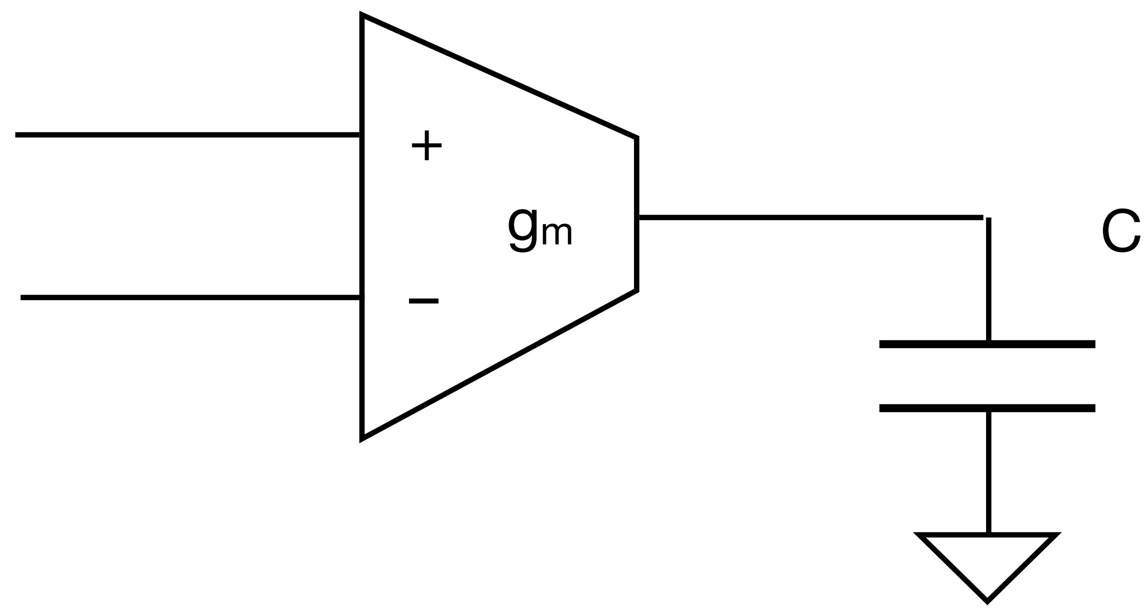
Subtraction trivial = switching two wires

Integrator #1



- $Y = \int I \, dt / C \approx - \int X \, dt / (R \cdot C)$
- Perfect integrator has infinite DC gain
- Real integrator limited by A and opamp pole

Integrator #2



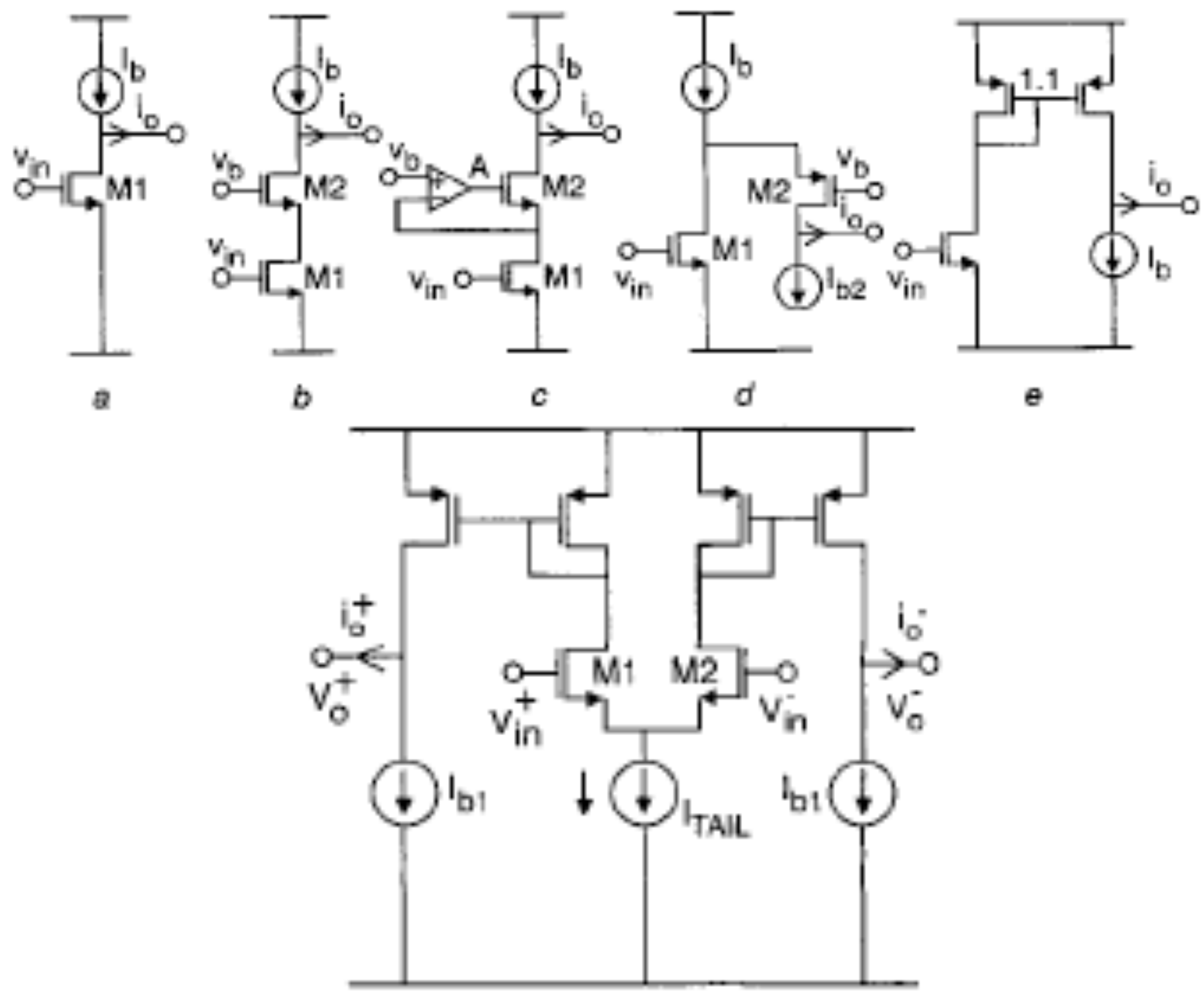
- Operational transconductance amp (OTA)
  - As opamp, but current output
  - $V_C = (g_m / C) \int (V_+ - V_-)\, dt$
- No R!
  - … but needs g_m / C to be well controlled

g_m-C filters

- g_m may be dynamically controlled!
  - g_m-C filters electronically tunable!
- C can track g_m quite well if built using transistor gate oxides
- Very low power level achievable
- No feedback!

Not shown in class

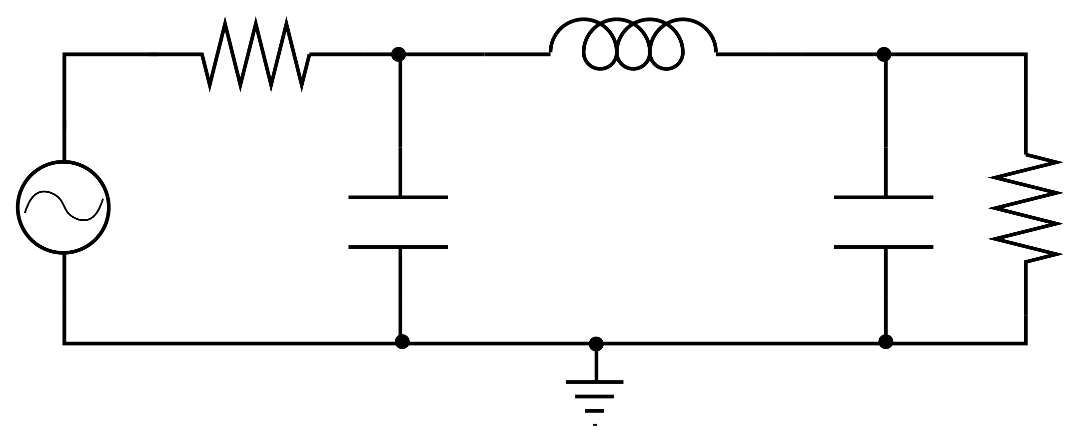
Transconductor examples



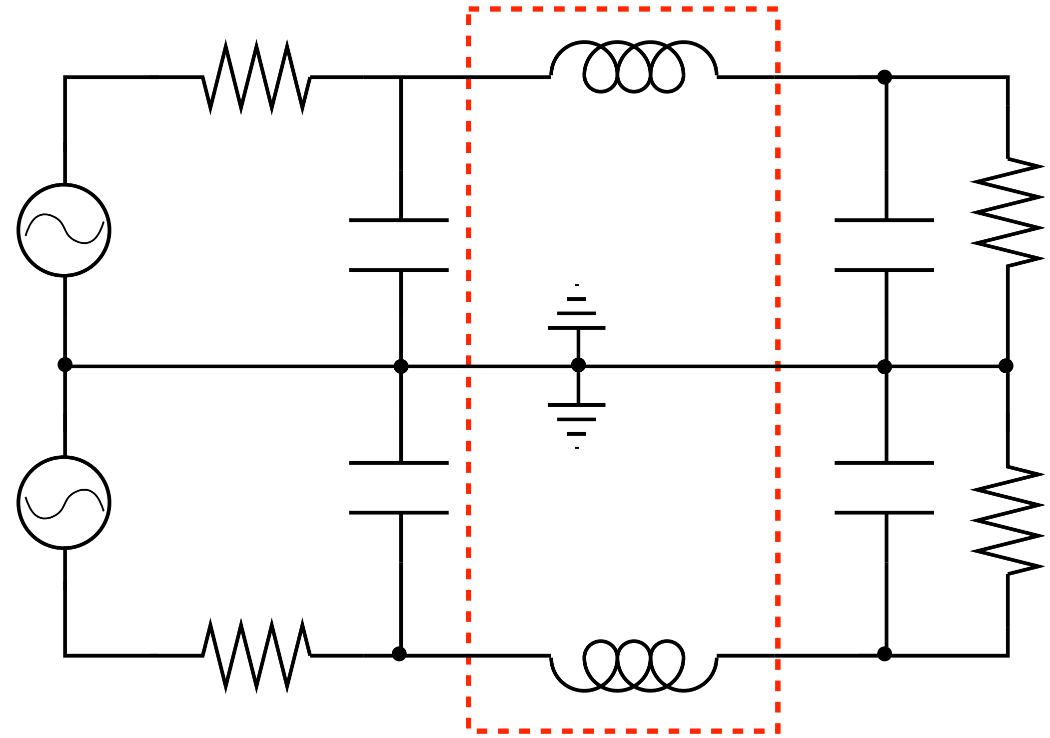
Figures from: Sanchez-Sinencio & Silva Martinez: CMOS transconductance amplifiers, architectures and active filters: a tutorial, IEE Proceedings 2000

Not shown in class

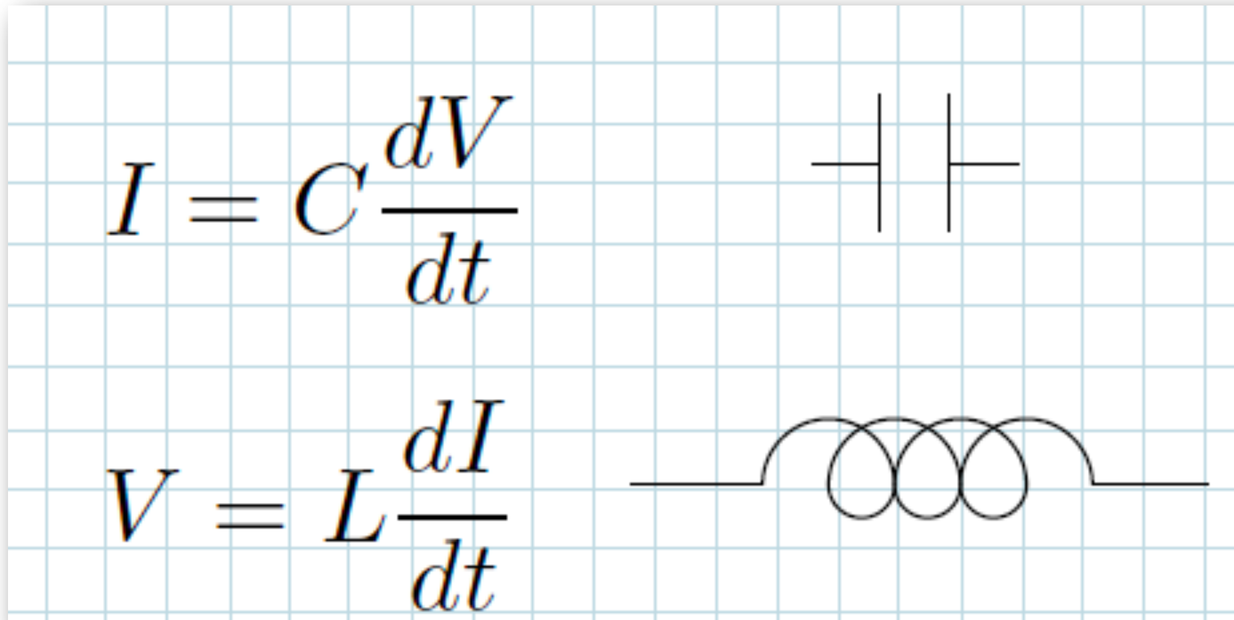
Implement passive filters by element replacement



= Another way of building gm-C filters



Simulated inductor



$$I = C\frac{dV}{dt}$$

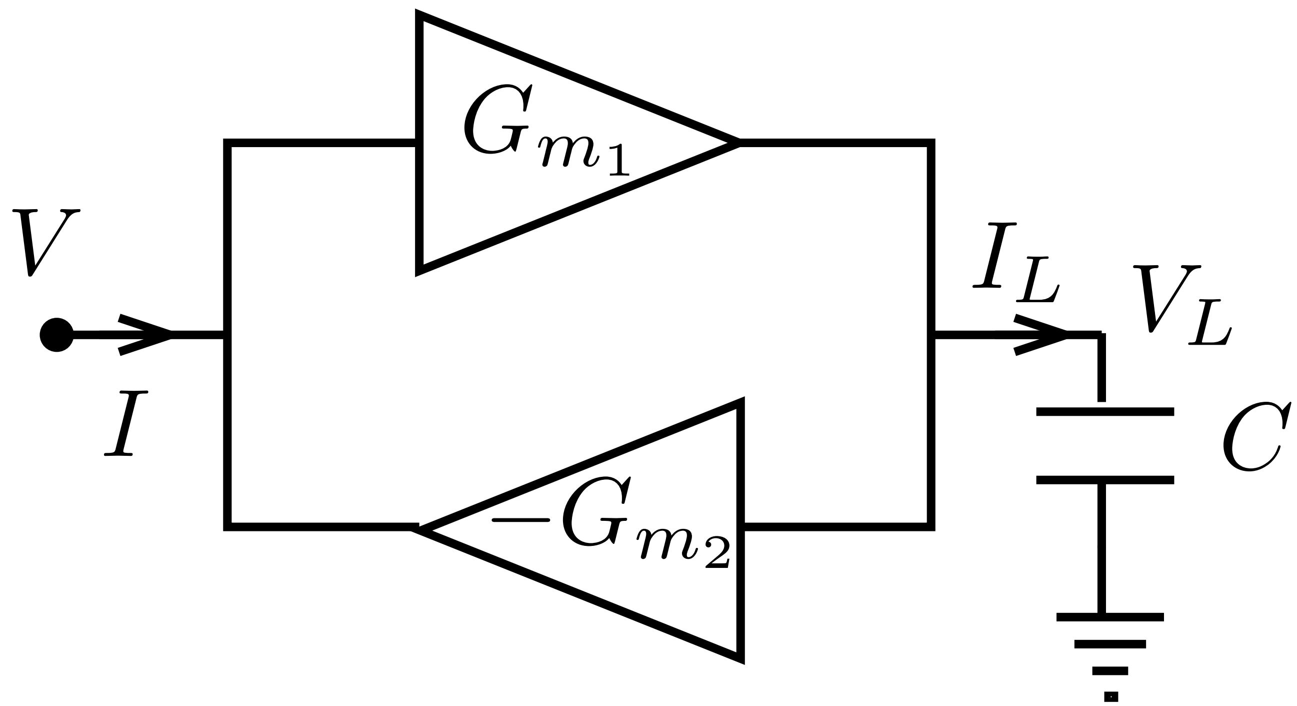
$$V = L\frac{dI}{dt}$$

Idea: If we could switch current and voltage a capacitor could act as an inductor!

A transconductor does this one way =>
Use two!

Not shown in class

Active grounded inductor



$$I_L = G_{m_1} V$$

$$V_L = \frac{I_L}{sC}$$

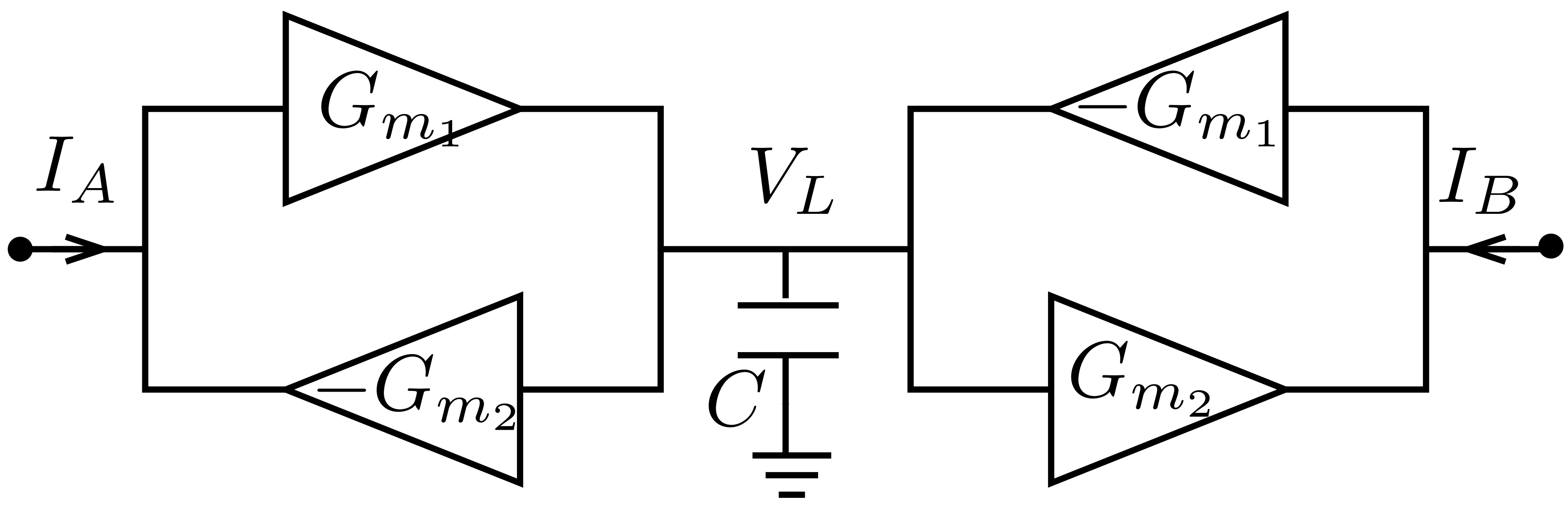
$$I = G_{m_2} V_L$$

$$Z = \frac{V}{I} = \frac{sC}{G_{m_1} G_{m_2}}$$

$$L_{gyr} = \frac{C}{G_{m_1} G_{m_2}}$$

Not shown in class

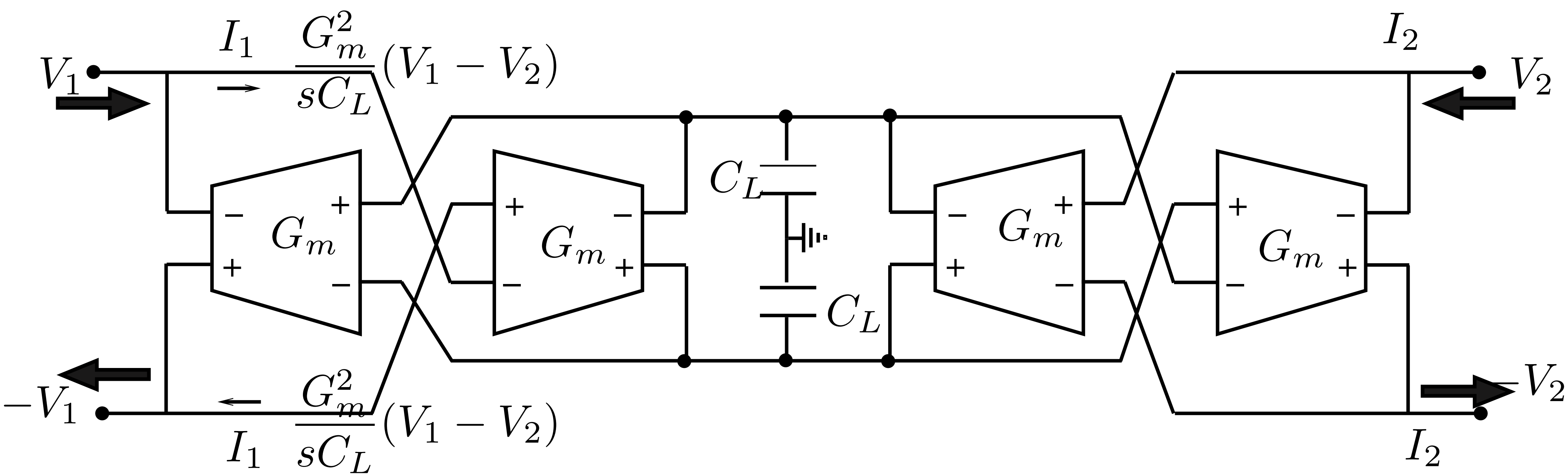
Active floating inductor



Same idea applied to both ends of inductor

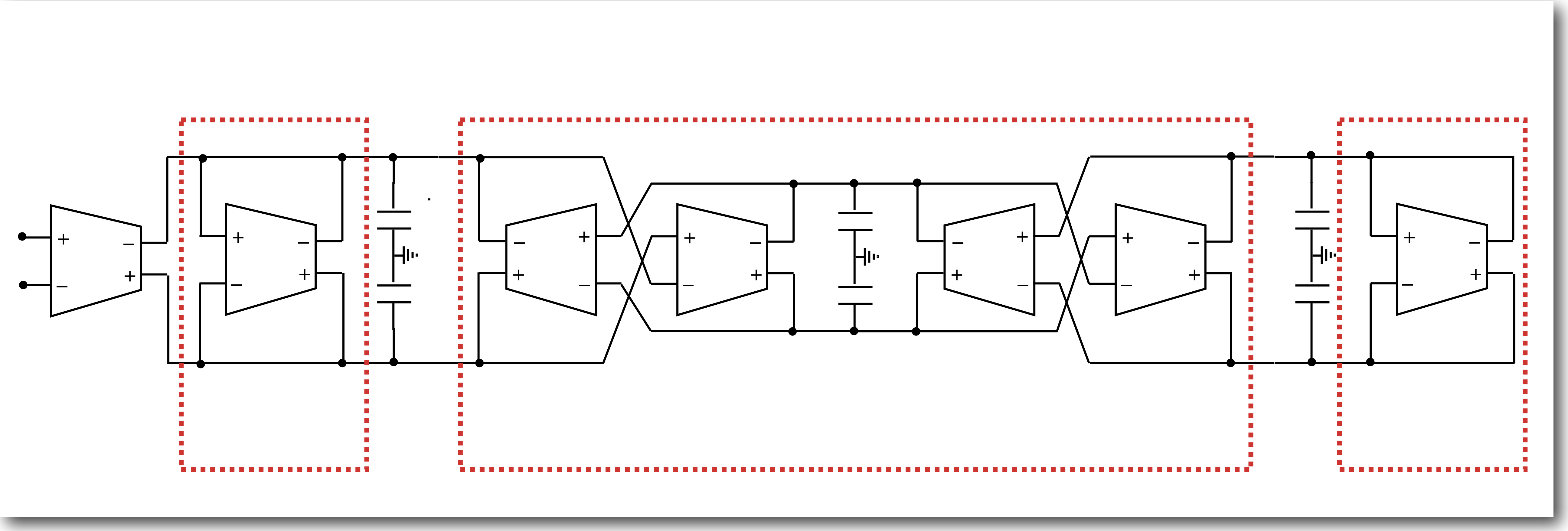
Not shown in class

Symmetric active floating inductor



Not shown in class

3rd-order differential filter w. active inductor



Integration issues

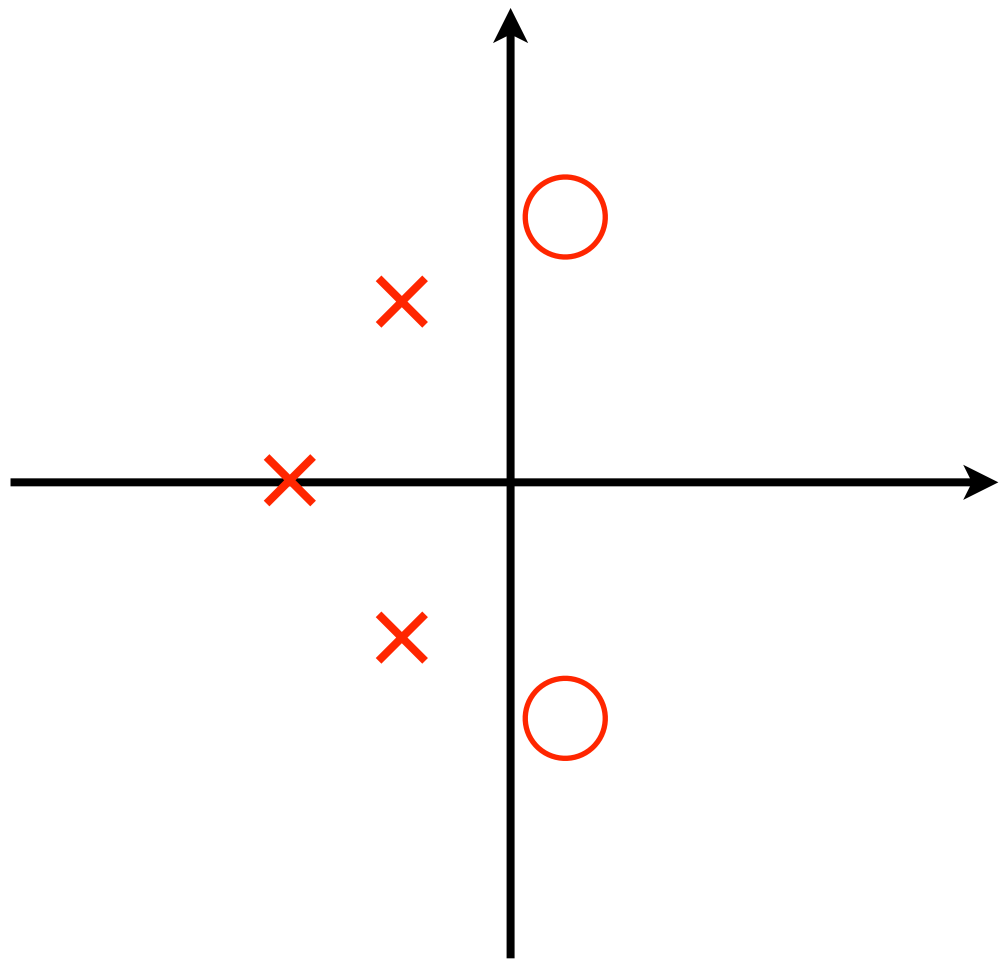
- Circuit variations move poles and zeros
  - May break specifications!
- Ladder filters: good matching properties
  - Design parameters are all gains

Matching

- Best matching reached with small-integer parameter ratios (5 : 4 : 2, etc)
    - Unit resistances / capacitances
    - Restrictions for pole / zero placement
- Thus, may reach certain accuracy with smaller capacitances
    - Lower power per stage
    - Higher-order filter <u>may</u> offer lower power!

Tuning

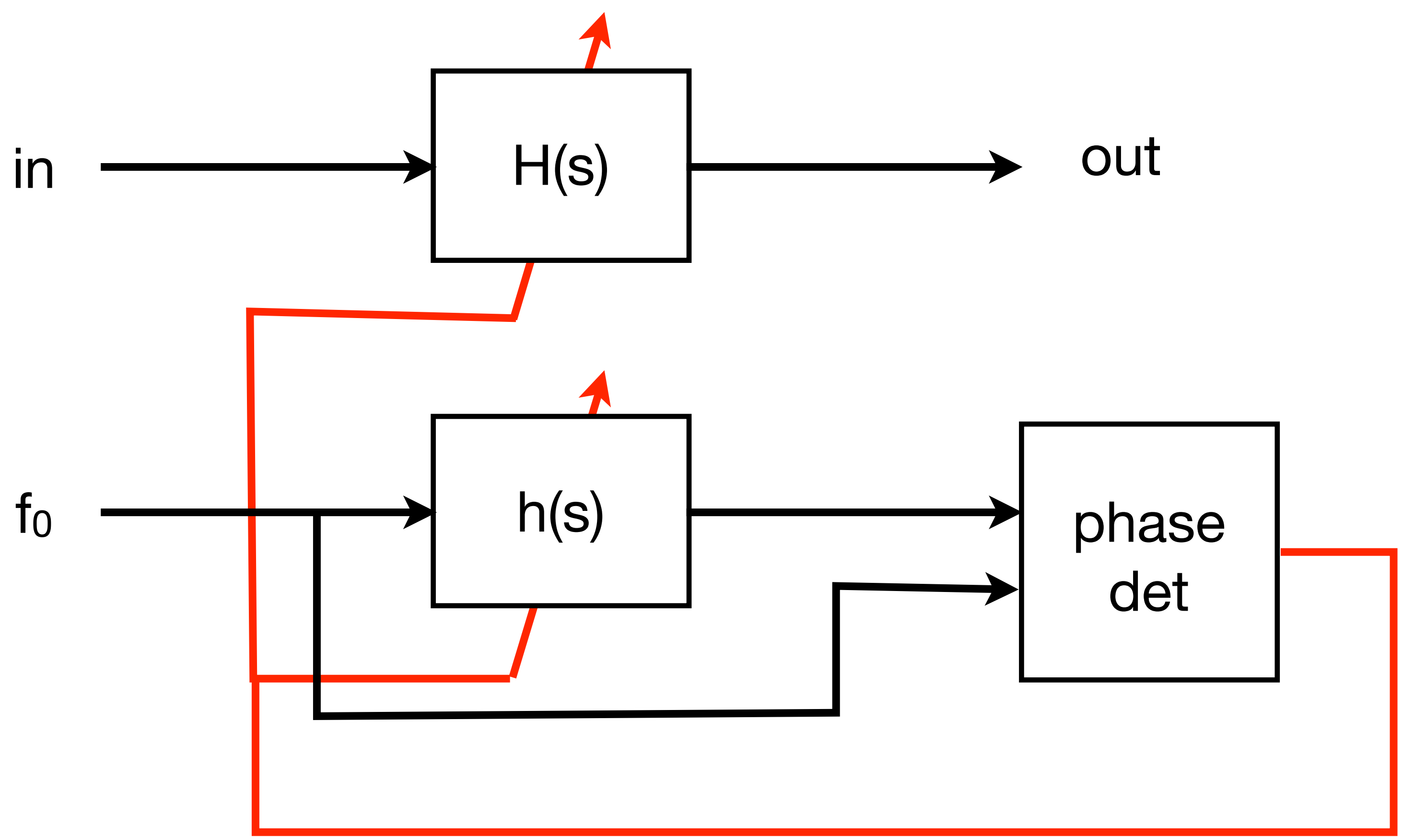
- Relative component matching can be quite good
- Absolute accuracy troublesome
  - Time/frequency scale for filter may be off



Tuning strategy outline

- Include small auxiliary filter which tracks main filter parameters
  - Control both filters with same voltage
- Aux filter designed to have phase $\pi/2$ at some f_0
- Apply f_0 to aux filter; steer V_{ctrl} s.t. phase is $\pi/2$

Tuning example



- Control feedback loop needs consideration…

Summary

- High requirements on OPamps for integrators
  - Especially if the desired filter Q is high
- Transformations useful in filter design
  - Re-use LP results for HP, BP, …
  - Re-use passive results for active filters
- Two main design styles
  - Cascade of second-order sections
  - Ladder / Lattice / etc
- Sensitivity influenced by design style!