Time-continuous filters: an overview (I)

DAT116, Nov 26, 2018

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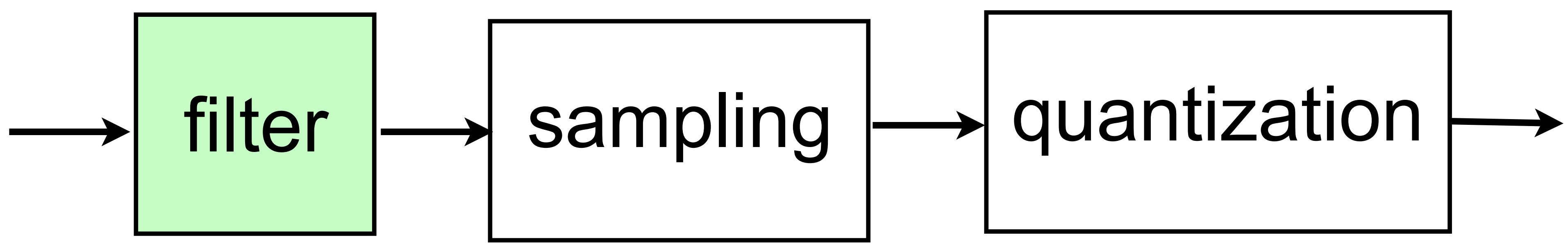
Administration

- Do not forget to sign up for oral examination - doodle will be made available this week
- Jan 14, 16 & 17 - Mon & Wednesday-Thursday
- Anyone needs to do it before holidays?

Why?

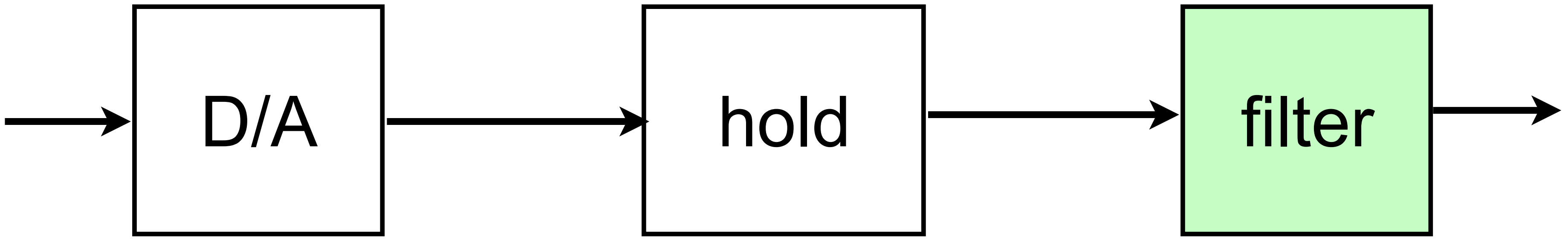
- With amplifiers, the most common analog building block
- Necessary in all data-conversion systems
- Co-specified with converter
  - May limit performance if incompetently selected!
  - Best filter design depends on **signal** and **noise** (= undesired signals) in application!

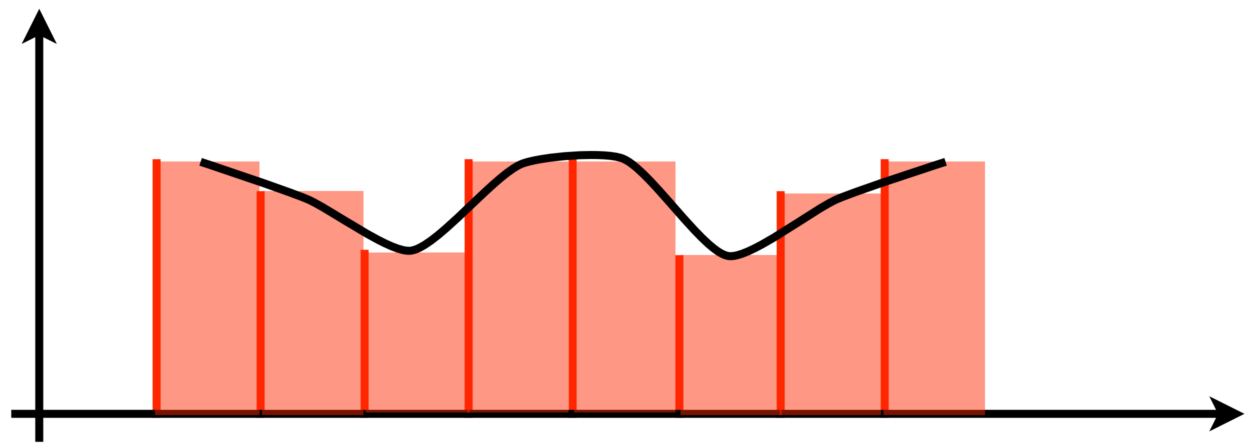
Example: A/D path



- Filter band-limits signal before sampling
  - Prevents frequency aliasing
  - “Anti-alias filter”
    - Knowledge about noise (= undesired signals) in **application** helps when selecting filter

Example: D/A path





- With “hold” function, filter band-limits output signal
- = Removes folding artifacts (steps) in signal
  - **Application** sets a limit for tolerated artifacts.

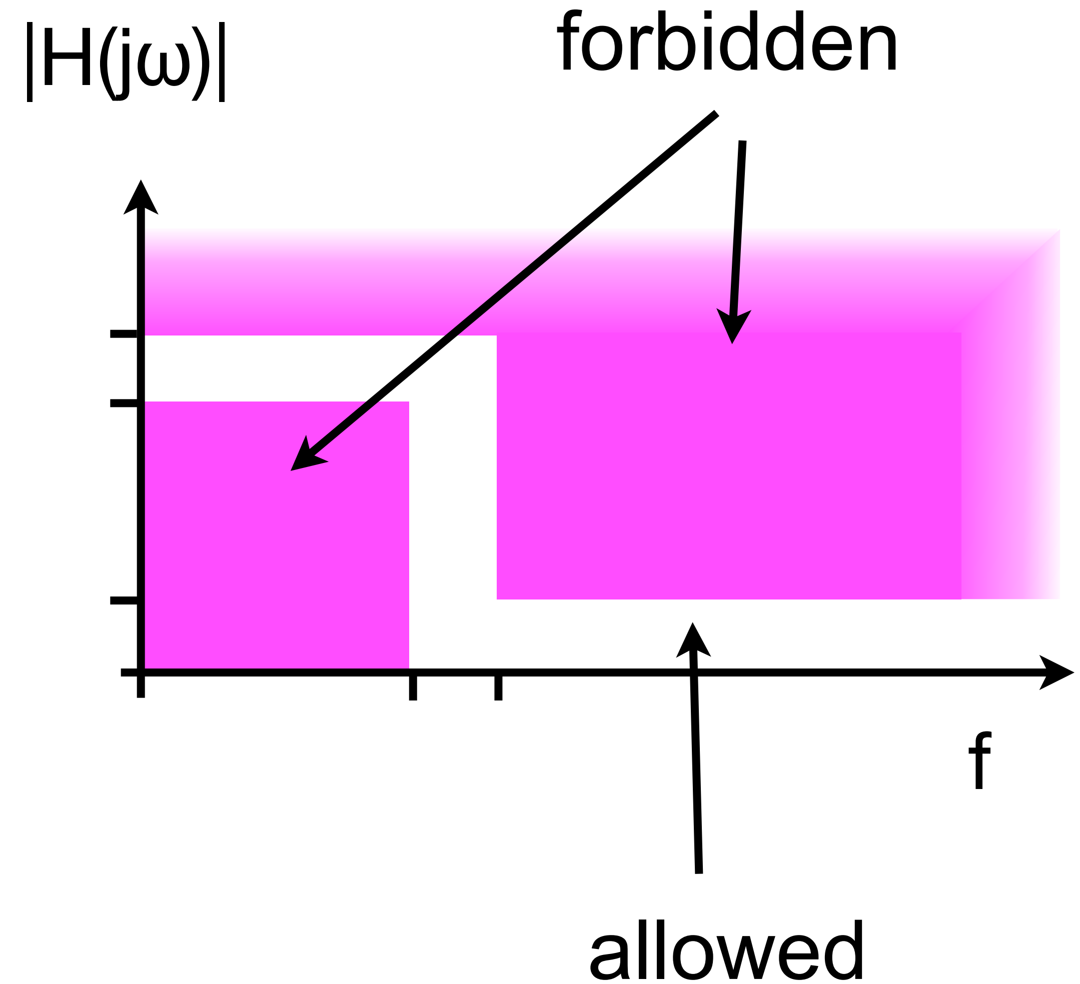
What?

- **Specifications**
  - Box specification
- **Transfer function - pole placement**
  - Classic types
- **Topology selection**
  - Cascade or coupled form
- **Implementation(s)**

2 lectures

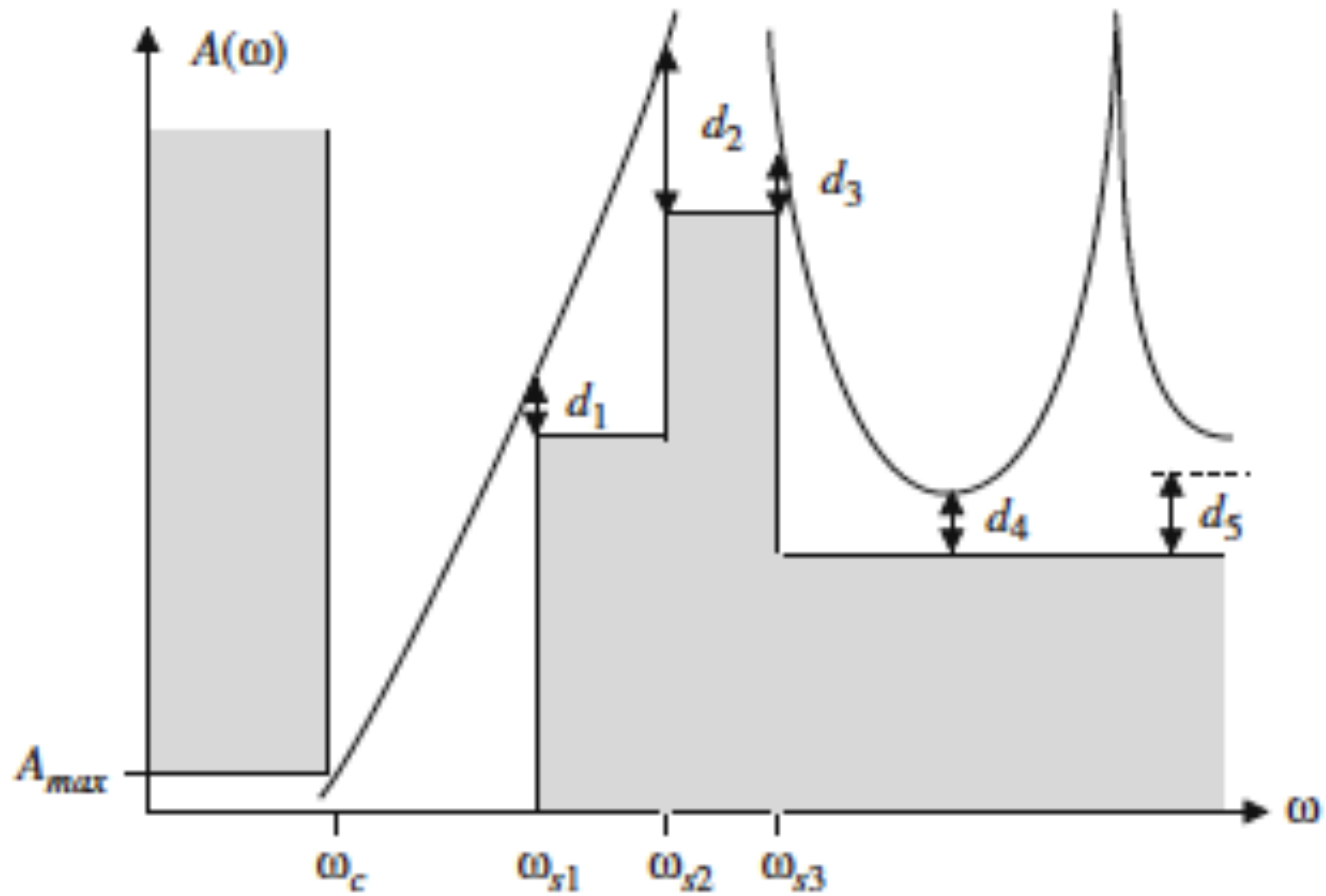
Specs

1. Amplitude / magnitude / attenuation



2. Phase
3. Phase delay
4. Group delay

1. More complex example



- Attenuation function rather than magnitude
- Low-pass, complex stopband spec

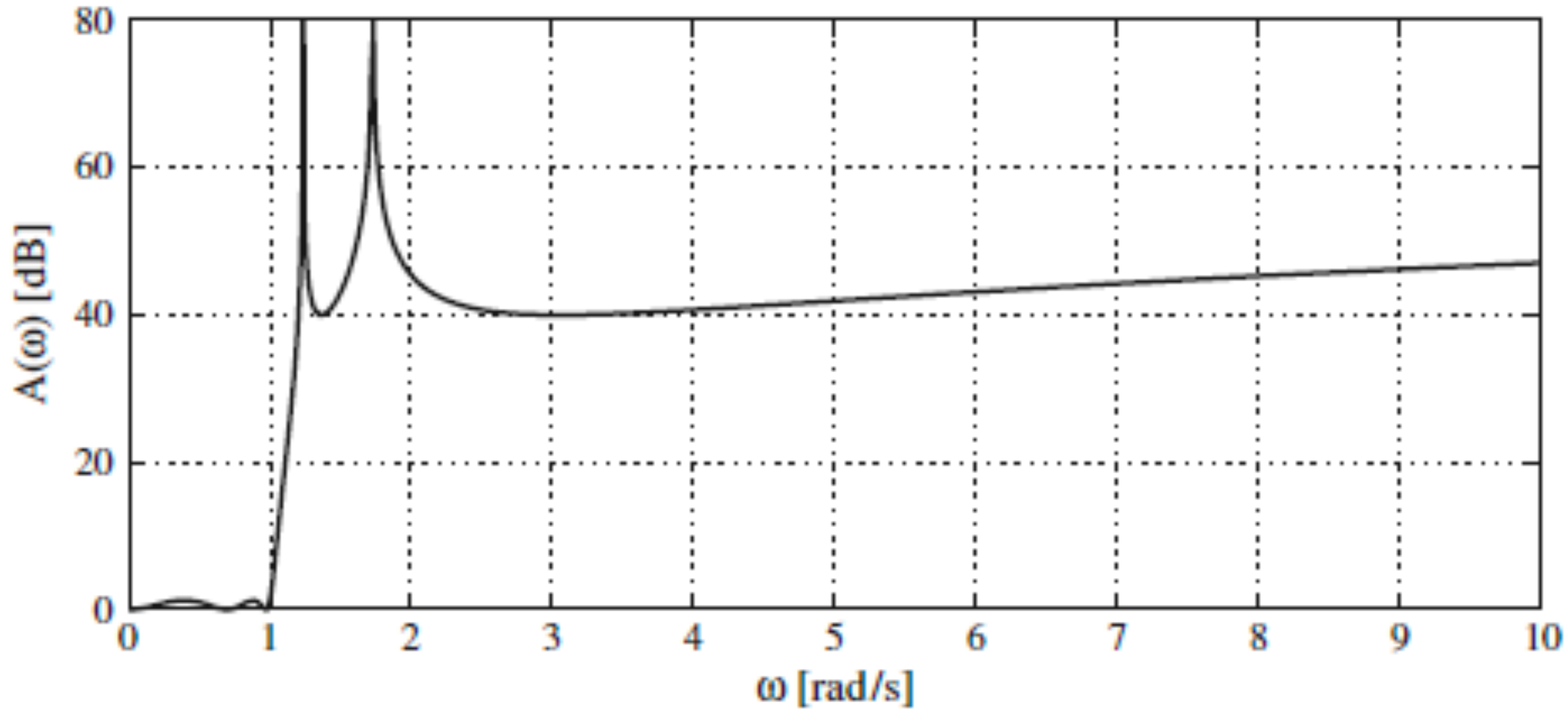
Definitions 1.-4.

- Transfer function: $H(j\omega) = Y(j\omega) / X(j\omega)$
- **Magnitude**: $|H(j\omega)|$, often in dB

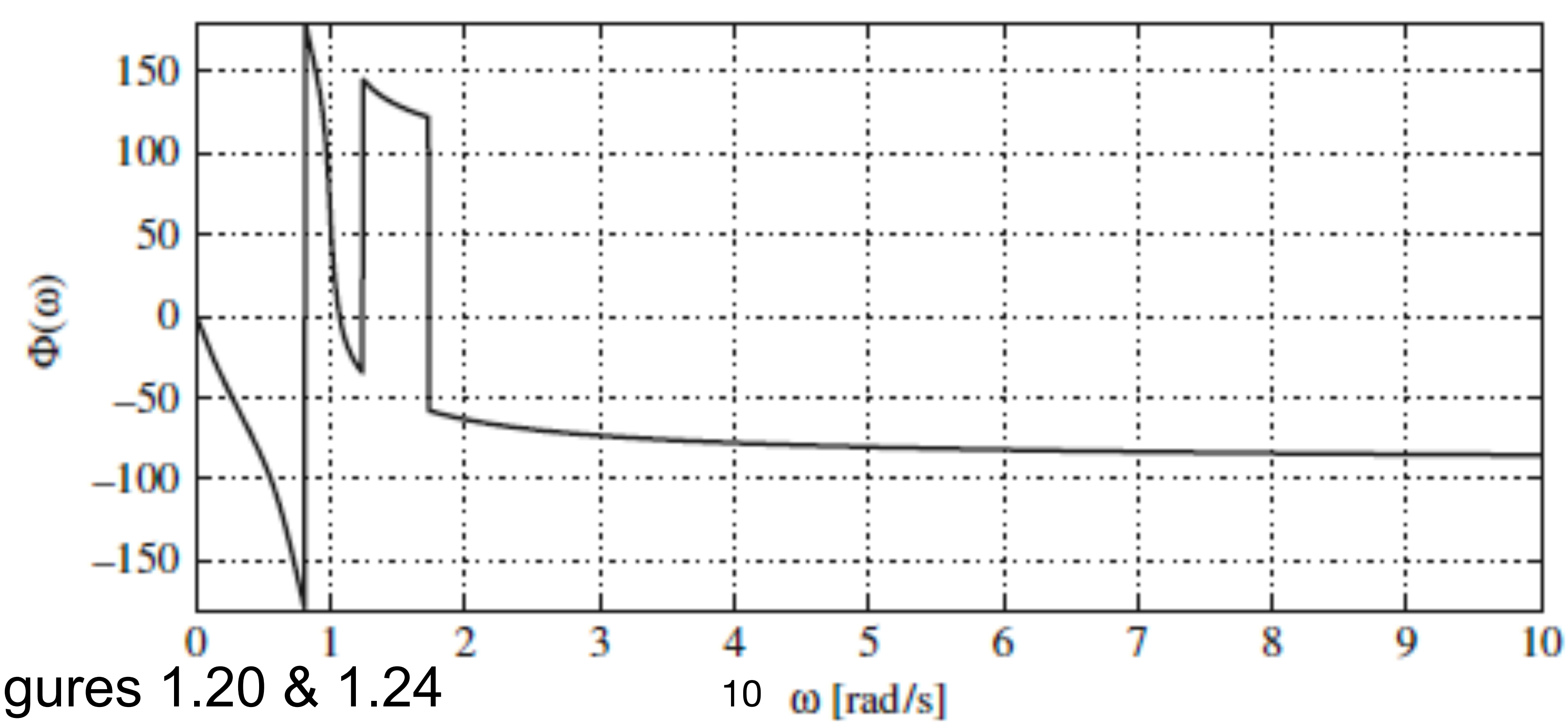
  Attenuation = Damping
  - **Attenuation** often used instead: $1/|H(j\omega)|$
- **Phase**: $Ø(\omega) = \arg(H(j\omega))$
- Impulse response: $h(t) = \text{Four}^{-1}(H)$

Example 5th order elliptic filter (Cauer)

Attenu-
ation

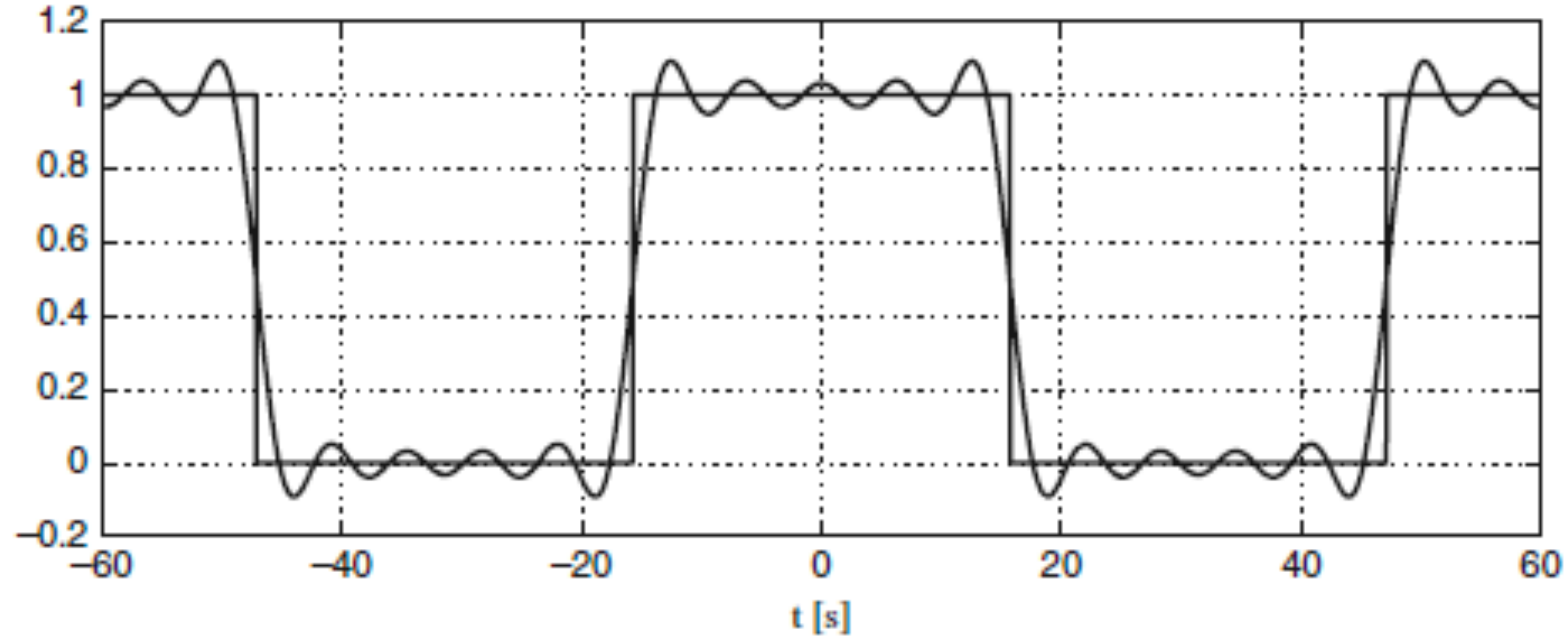


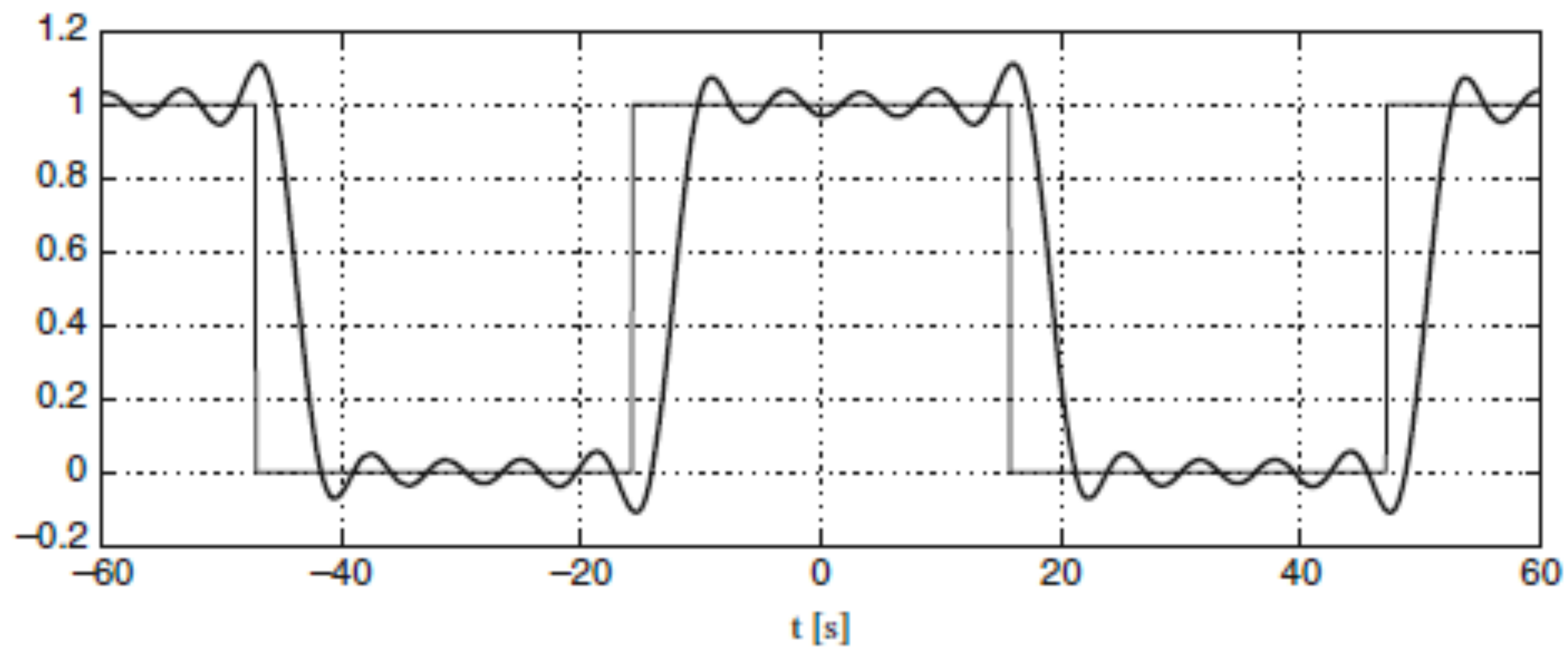
Phase



Wanhammar: figures 1.20 & 1.24

Ideal and real filter w. square-wave input

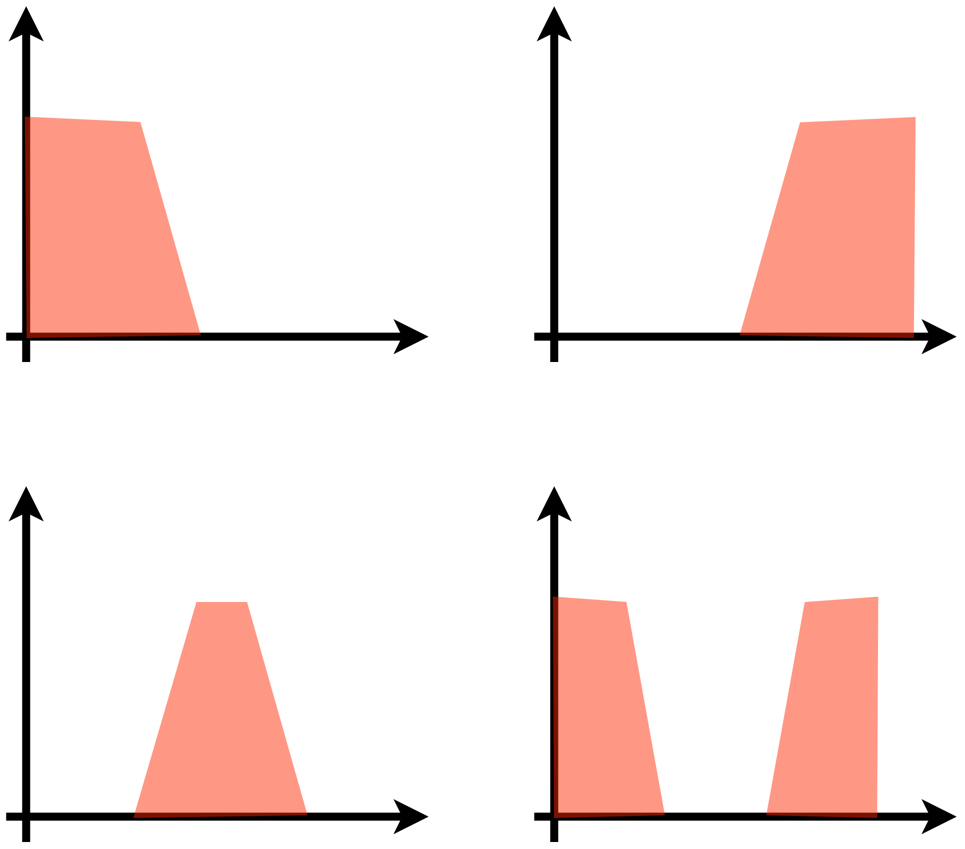




Wanhammar: figures 1.22 and 1.23

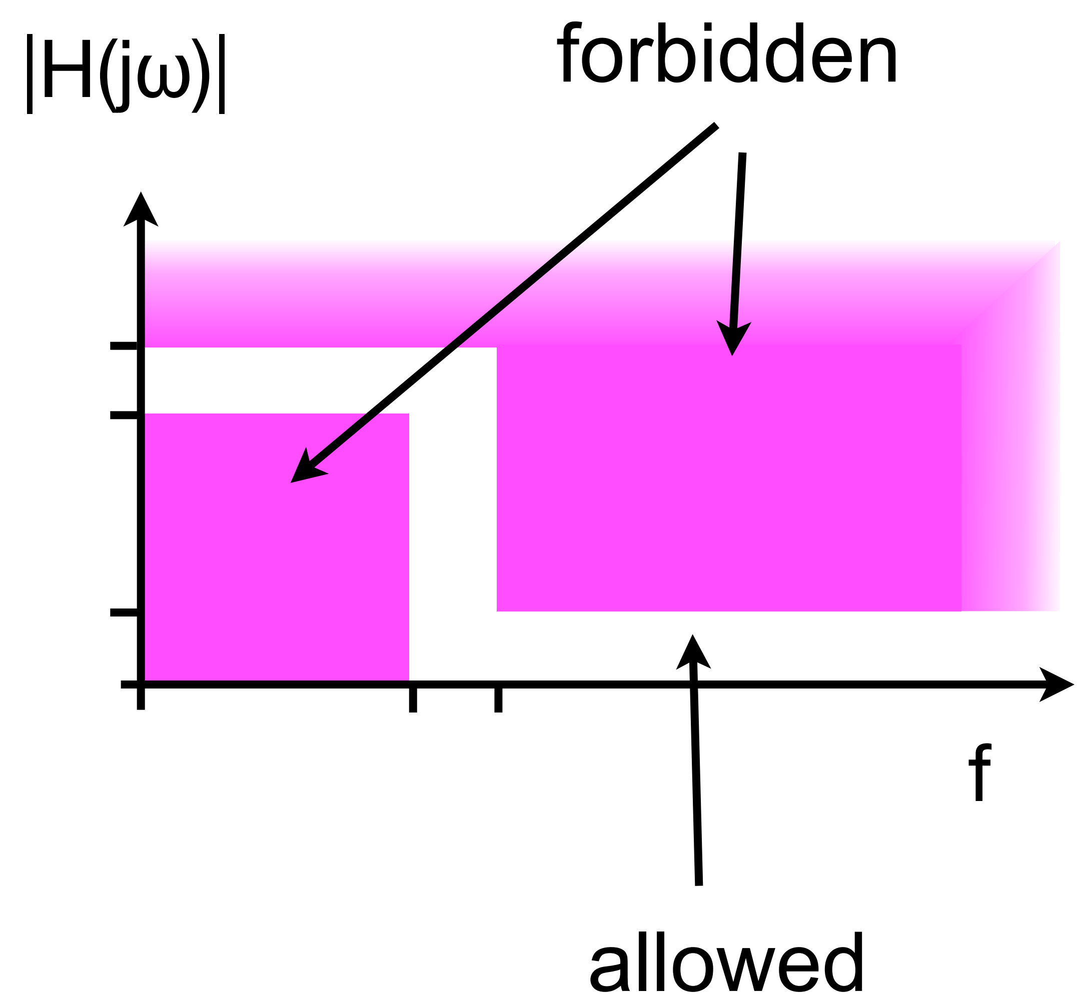
LP / HP / BP / BS / AP

- The 5 classic **categories of specifications**
    - Low-pass
    - High-pass
    - Band-pass
    - Band-stop
    - All-pass



Spec given- what next?

1. Amplitude / magnitude / attenuation



2. Phase
3. Phase delay
4. Group delay

Find transfer function = poles and zeros!

Transfer (xfer) function

- Rational function in s (Laplace domain) or jω (Fourier domain)
  - Ratio of two polynomials
  - Zeros of denominator polynomial are poles of xfer function
- Denominator order ≥ numerator order
- Filter order is denominator order

Higher order means more hardware and more power!

Poles in 2nd order systems

From P & Z to xfer function?

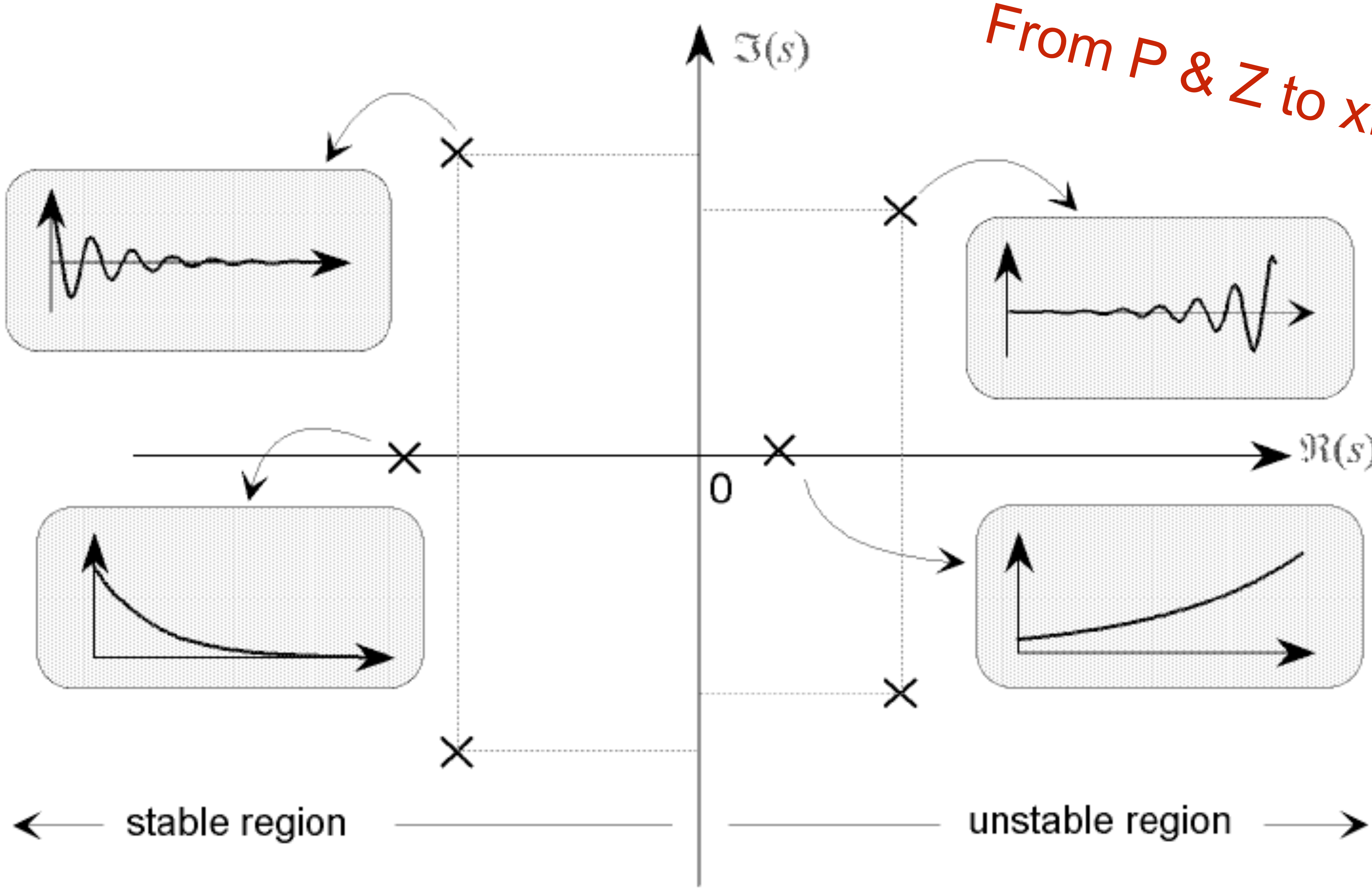


Figure 2: The specification of the form of components of the homogeneous response from the system pole locations on the pole-zero plot.

From: MIT: Understanding poles and zeros
web.mit.edu/2.14/www/Handouts/**PoleZero**.pdf

2nd order system

From P & Z to xfer function?

The pole locations of the classical second-order homogeneous system

$$\frac{d^2y}{dt^2} + 2\zeta\omega_n\frac{dy}{dt} + \omega_n^2 y = 0, \quad (13)$$

described in Section 9.3 are given by

$$p_1, p_2 = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}. \quad (14)$$

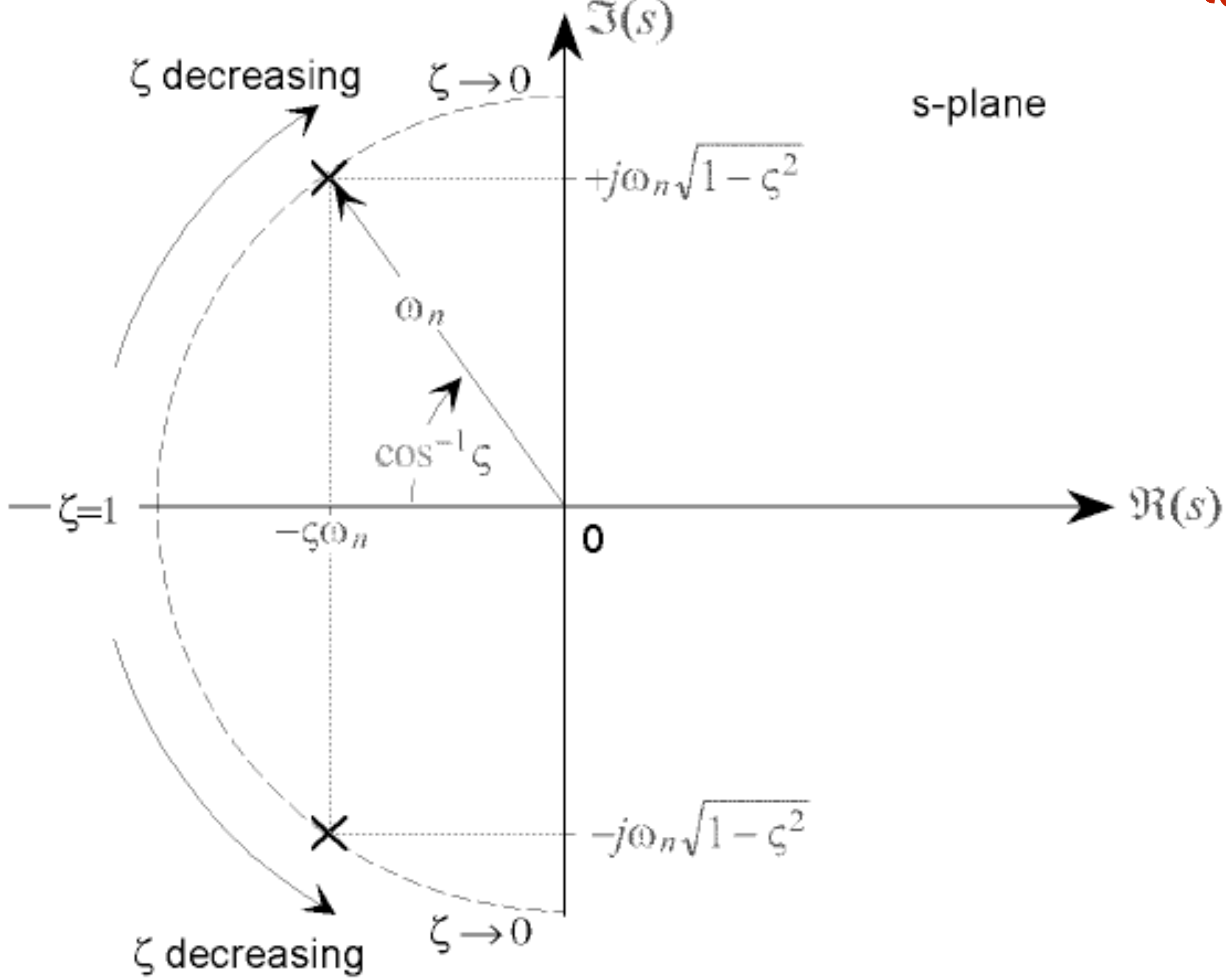
If $\zeta \geq 1$, corresponding to an overdamped system, the two poles are real and lie in the left-half plane. For an underdamped system, $0 \leq \zeta < 1$, the poles form a complex conjugate pair,

$$p_1, p_2 = -\zeta\omega_n \pm j\omega_n\sqrt{1 - \zeta^2} \quad (15)$$

and are located in the left-half plane, as shown in Fig. 4. From this figure it can be seen that the poles lie at a distance ω_n from the origin, and at an angle $\pm\cos^{-1}(\zeta)$ from the negative real axis. The poles for an underdamped second-order system therefore lie on a semi-circle with a radius defined by ω_n, at an angle defined by the value of the damping ratio ζ.

2nd order underdamped system

From P & Z to xfer function?



From poles zeros to transfer function

From P & Z to xfer function?

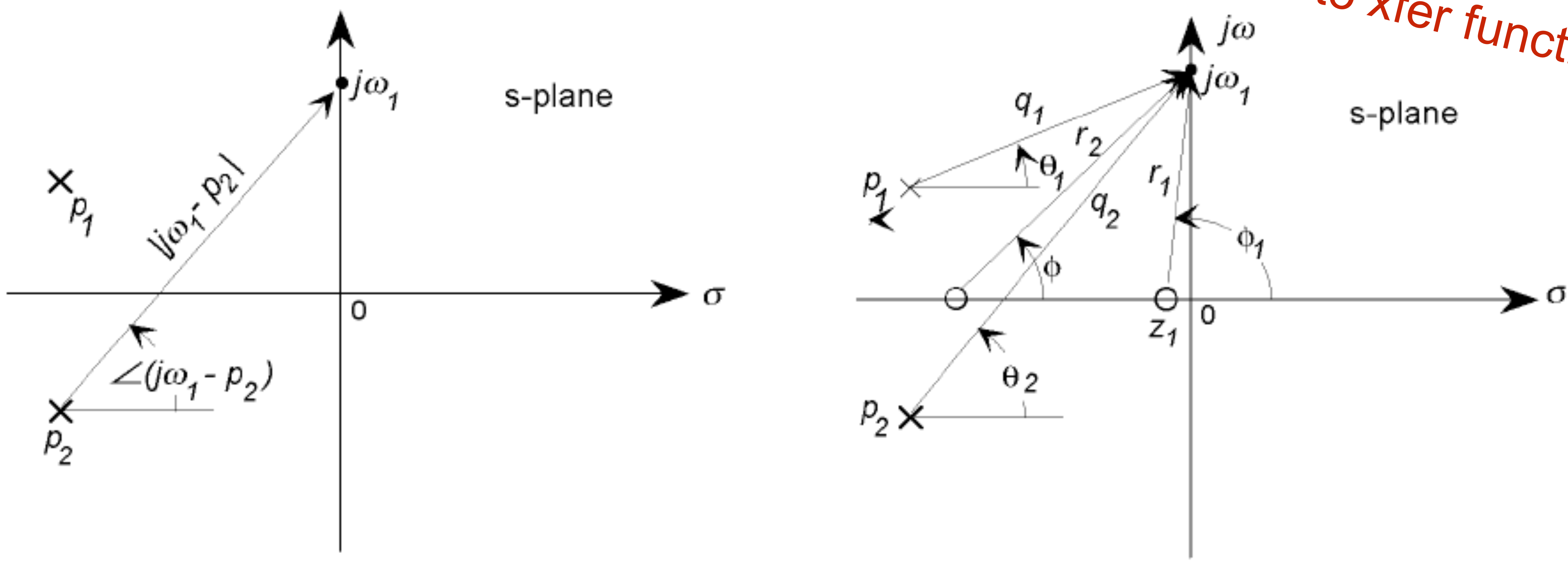
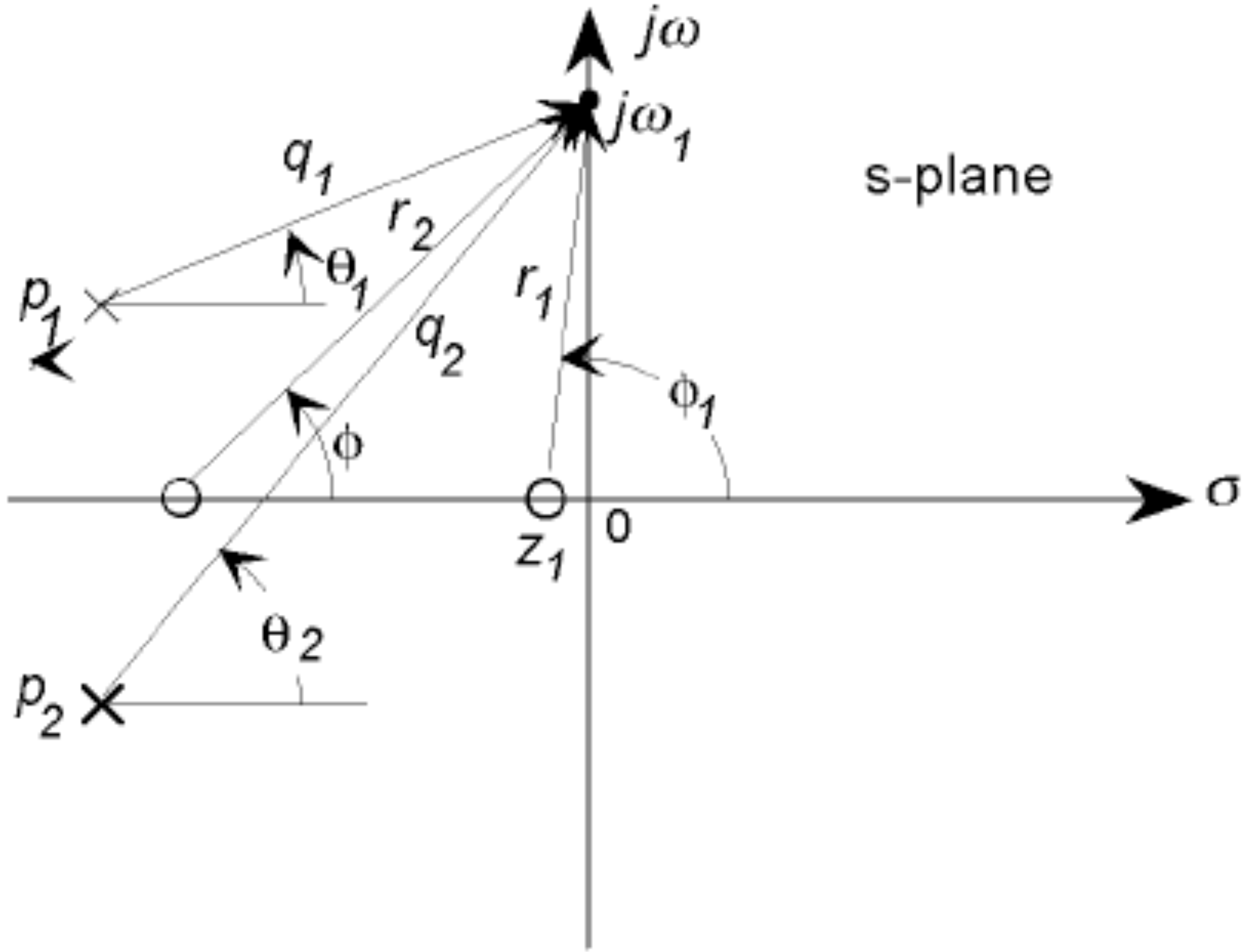


Figure 7: Definition of the vector quantities used in defining the frequency response function from the pole-zero plot. In (a) the vector from a pole (or zero) is defined, in (b) the vectors from all poles and zeros in a typical system are shown.

From: MIT: Understanding poles and zeros
[web.mit.edu/2.14/www/Handouts/**PoleZero**.pdf](web.mit.edu/2.14/www/Handouts/PoleZero.pdf)

Closeup



Transfer function

From P & Z to xfer function?

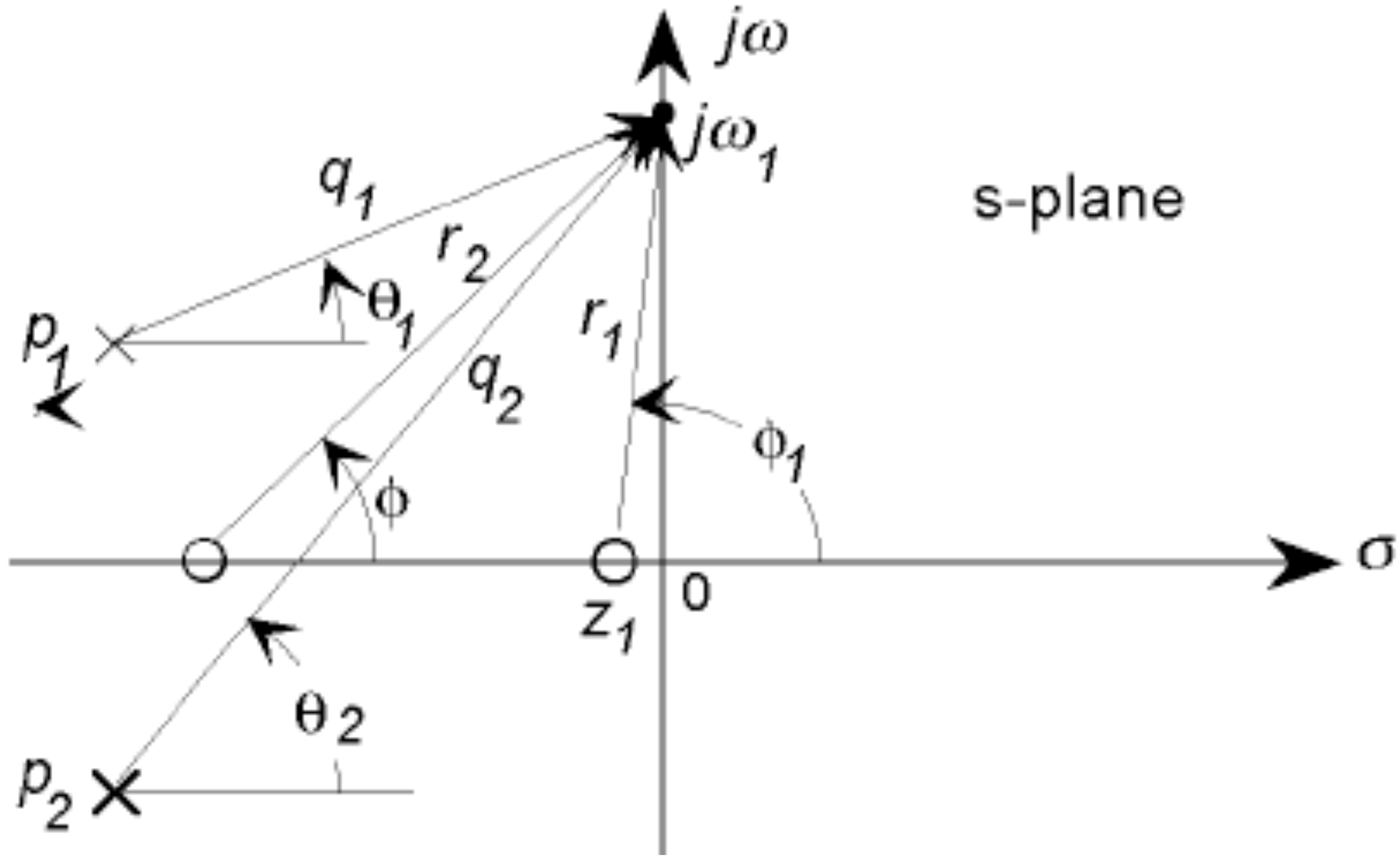
As defined above, if the vector from the pole p_i to the point $s = j\omega$ has length q_i and an angle θ_i from the horizontal, and the vector from the zero z_i to the point $j\omega$ has a length r_i and an angle ϕ_i, as shown in Fig. 7b, the value of the frequency response at the point $j\omega$ is

$$|H(j\omega)| = K\frac{r_1 \ldots r_m}{q_1 \ldots q_n} \quad (35)$$

$$\angle H(j\omega) = (\phi_1 + \ldots + \phi_m) - (\theta_1 + \ldots + \theta_n) \quad (36)$$

From: MIT: Understanding poles and zeros
web.mit.edu/2.14/www/Handouts/**PoleZero**.pdf

Closeup again



$$|H(j\omega)| = K\frac{r_1 \ldots r_m}{q_1 \ldots q_n}$$

$$\angle H(j\omega) = (\phi_1 + \ldots + \phi_m) - (\theta_1 + \ldots + \theta_n)$$

What is Q?

Stands for **Quality factor**

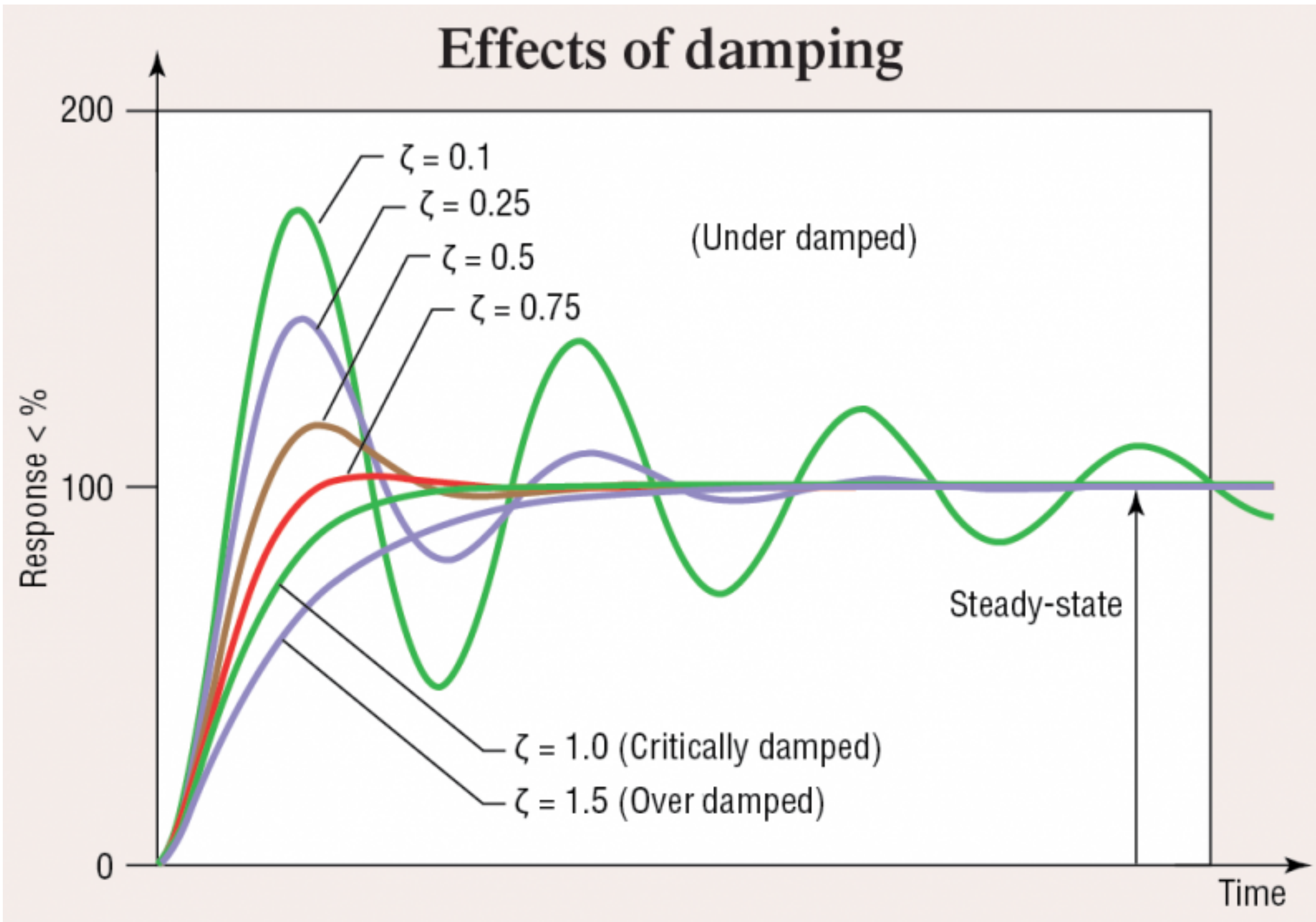
A dimensionless parameter which says how **under-damped** an oscillator is

High Q (= high quality) in oscillator when the oscillation continues for a long time

$$\zeta = \frac{1}{2Q}$$

Second-order pole pair step response

ζ is damping ratio



Characterization of second-order system

Behavior of second-order system	ζ	Q
Overdamped (= real poles)		
Critically damped (at the point between real and complex)	ζ =1	
Underdamped (= complex poles)		

Characterization of second-order system

Behavior of second-order system	ζ	Q
Overdamped (= real poles)	$\zeta > 1$	
Critically damped (at the point between real and complex)	$\zeta = 1$	
Underdamped (= complex poles)	$0 \leq \zeta < 1$	

Characterization of second-order system

Behavior of second-order system	ζ	Q
Overdamped (= real poles)	$\zeta > 1$	
Critically damped (at the point between real and complex)	$\zeta = 1$	$Q = 0.5$
Underdamped (= complex poles)	$0 \le \zeta < 1$	

Characterization of second-order system

Behavior of second-order system	ζ	Q
Overdamped (= real poles)	$\zeta > 1$	$Q < 0.5$
Critically damped (at the point between real and complex)	$\zeta = 1$	$Q = 0.5$
Underdamped (= complex poles)	$0 \leq \zeta < 1$	$Q > 0.5$

Q for the pole pairs

- Q for filters/poles : A measure of the angle that describes the complex pole pair:
  - $Q = 1/(2\cos\psi)$ where ψ is the pole angle
- Characterization (as we saw):
  - $Q < 0.5$ => overdamped
  - $Q = 0.5$ => critically damped
  - $Q > 0.5$ => underdamped

Q in general

- For resonators:

$$Q = 2\pi \times \frac{\text{Energy Stored}}{\text{Energy dissipated per cycle}} = 2\pi f_r \times \frac{\text{Energy Stored}}{\text{Power Loss}}.$$

- For high-Q systems

$$Q \approx \frac{f_r}{\Delta f} = \frac{\omega_r}{\Delta\omega},$$

- For reactive componts etc:

$$Q(\omega) = \omega \times \frac{\text{Maximum Energy Stored}}{\text{Power Loss}},$$

Example higher order

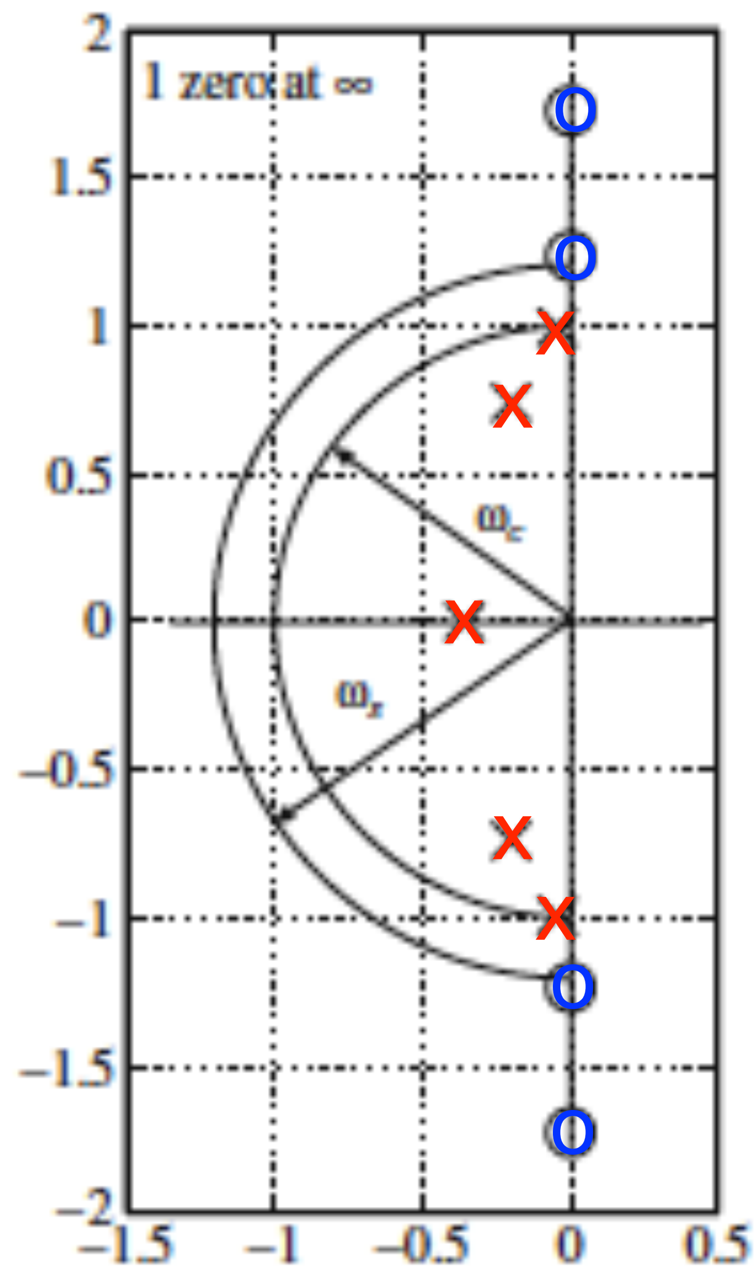
$$\frac{s + 248}{s^3 + 12.5s^2 + 79s + 248}$$

$$\frac{s + 248}{(s + 3.14 + 5.44i)(s + 3.14 - 5.44i)(s + 6.28)}$$

- 3rd order filter
- Two equivalent formulations

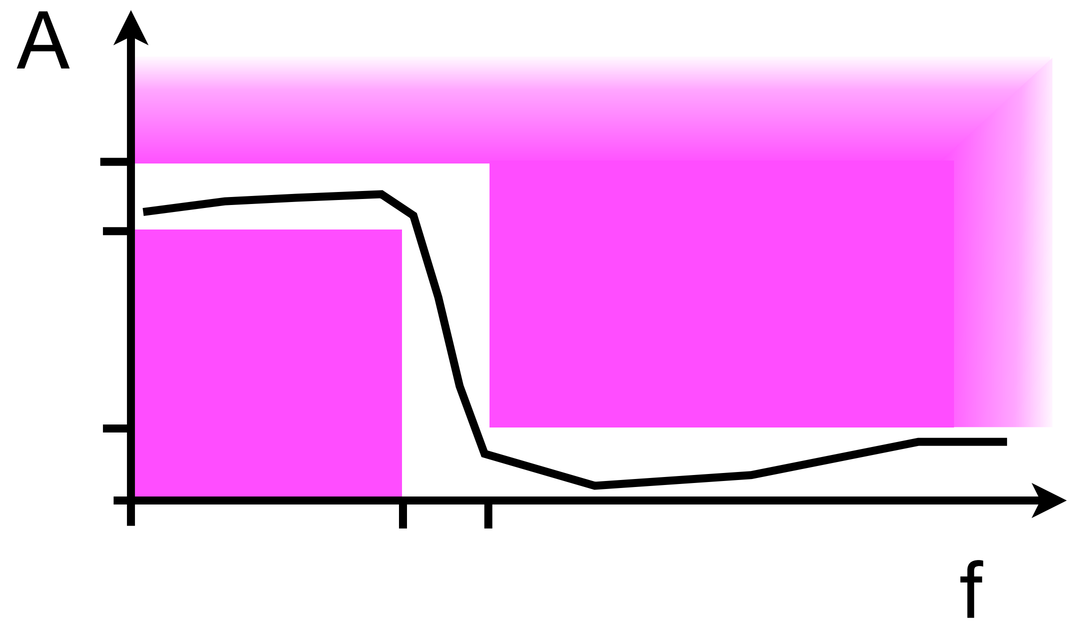
s-plane view

- 5th order filter
- jω on positive imaginary axis
- Poles in left half plane
- Symmetry around real axis



Wanhammar: figure 1.26

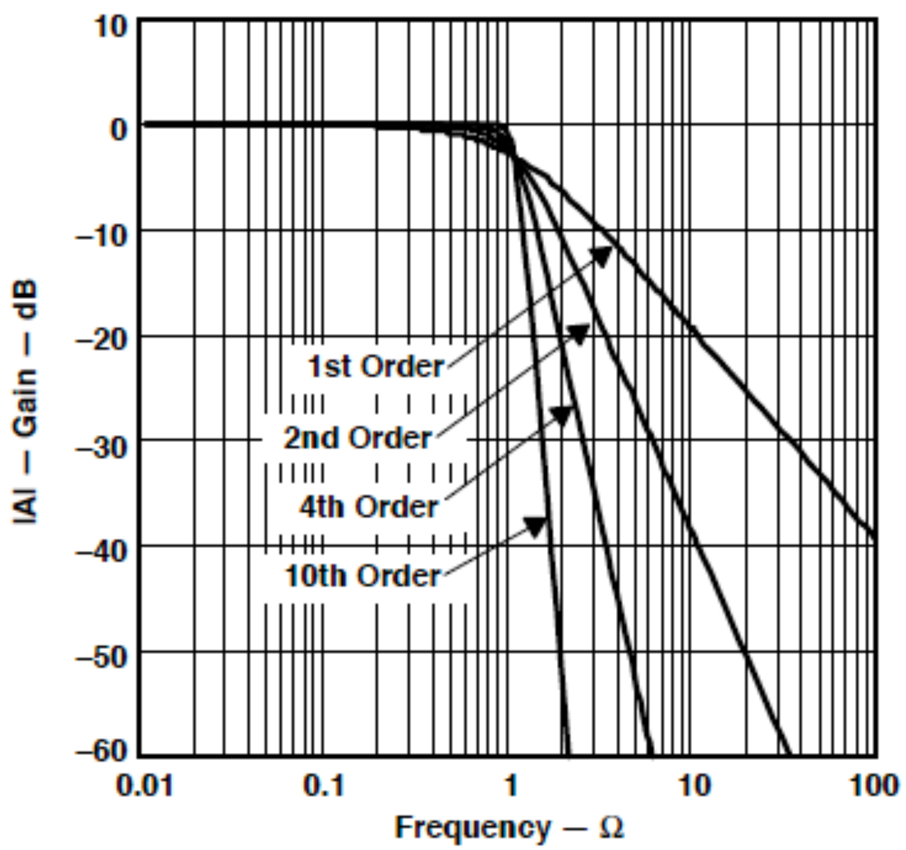
Find xfer function to fit spec



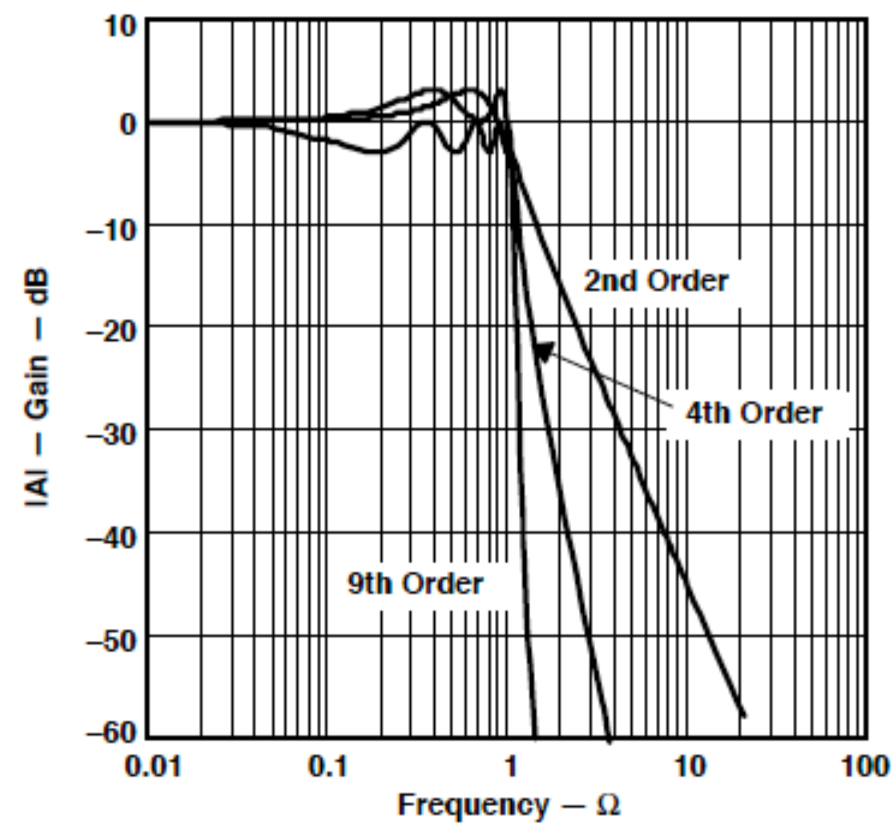
- Many xfer functions will fit a certain spec!
  - A set of classics to choose from.
- Select based on additional criteria
  - Typically, order (which determines cost)

Classic xfer function classes

Butterworth



Chebysjev

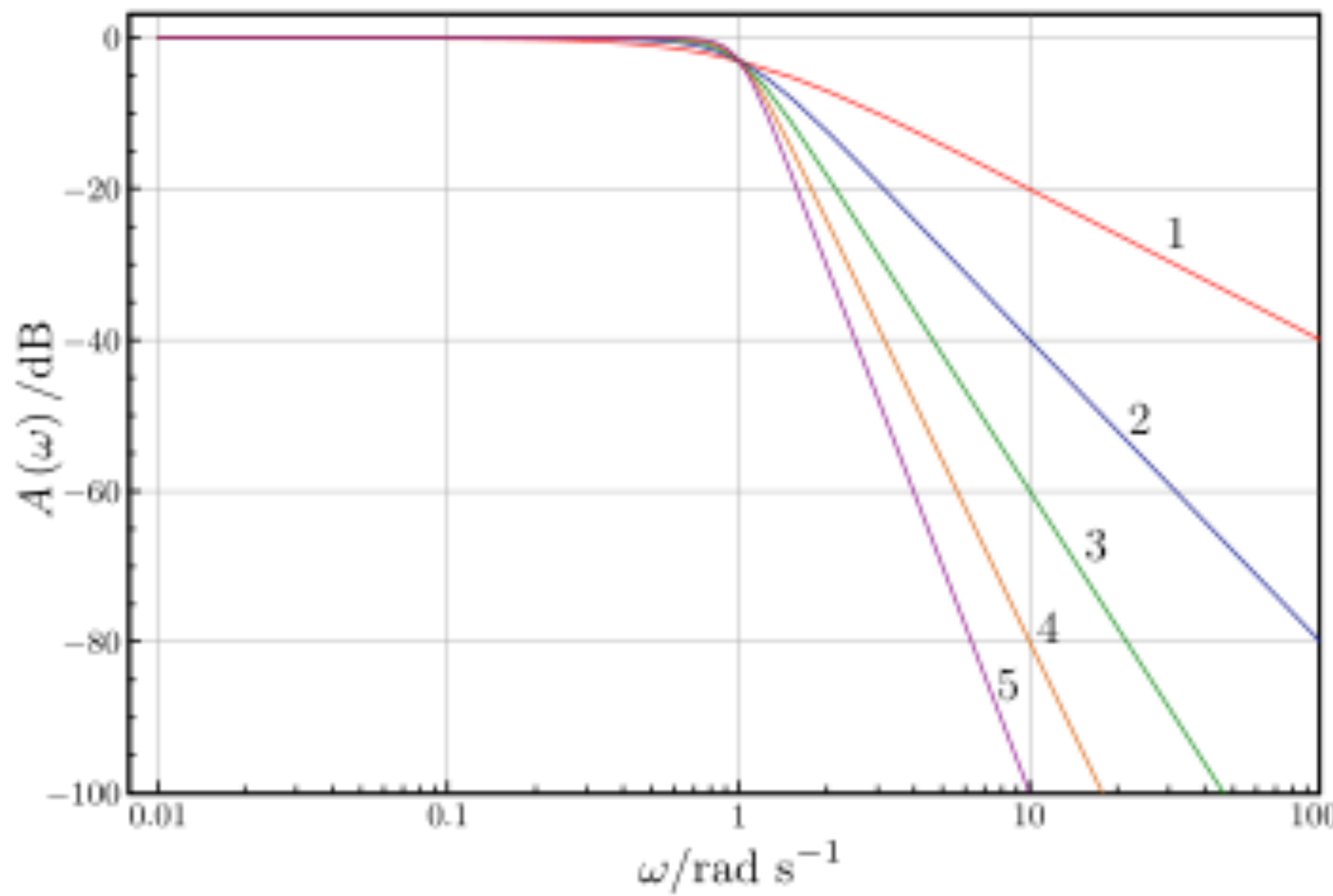


Different well-known ways to place poles and zeros to achieve certain characteristics

Other classes are: Inverse Chebysjev, Elliptic (Cauer), Bessel

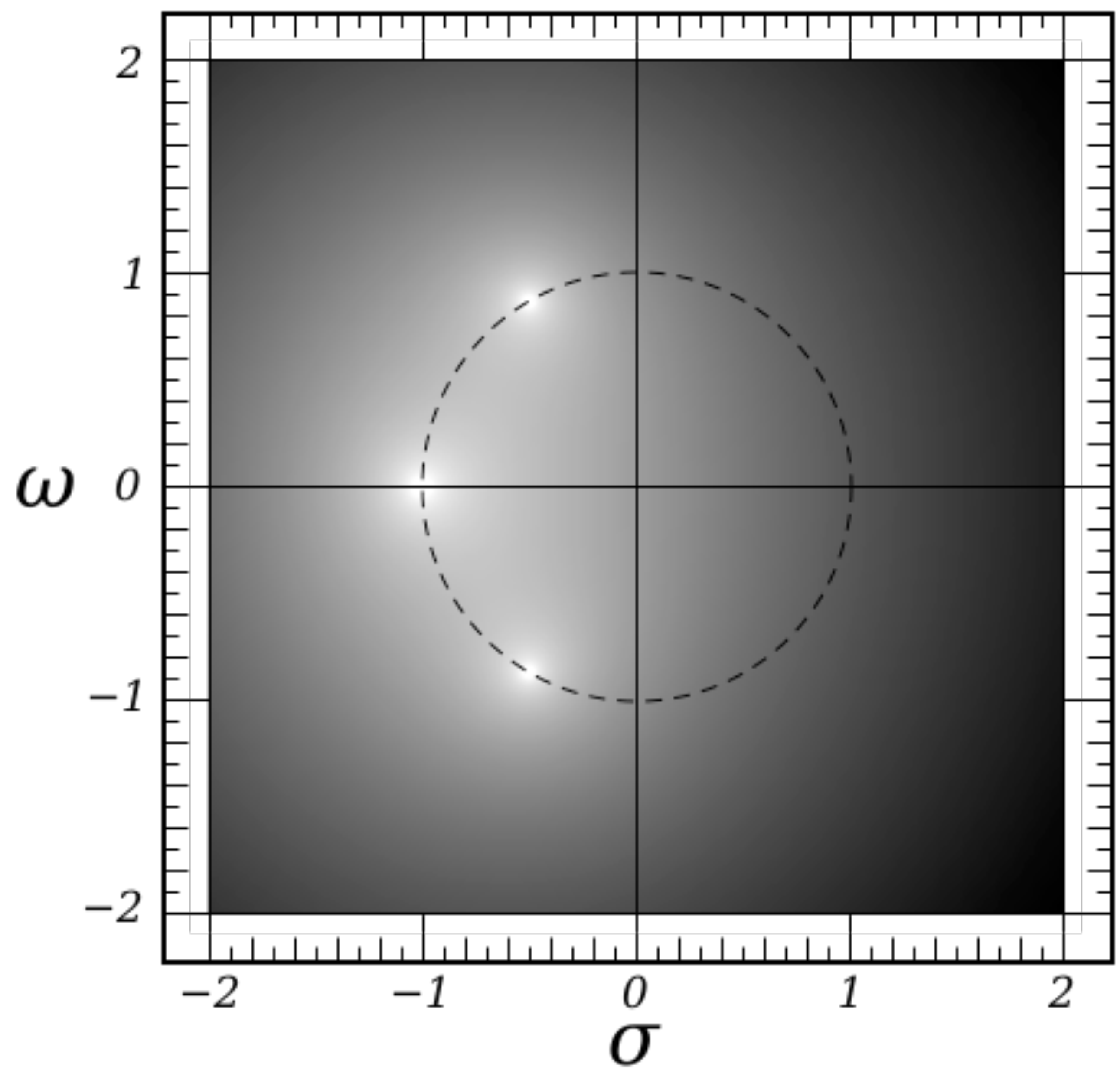
TI report: "Active filter techniques" http://www.ti.com/lit/ml/sloa088/sloa088.pdf

Butterworth gain curves
orders 1-5



Place poles with constraint that gain function is monotonic

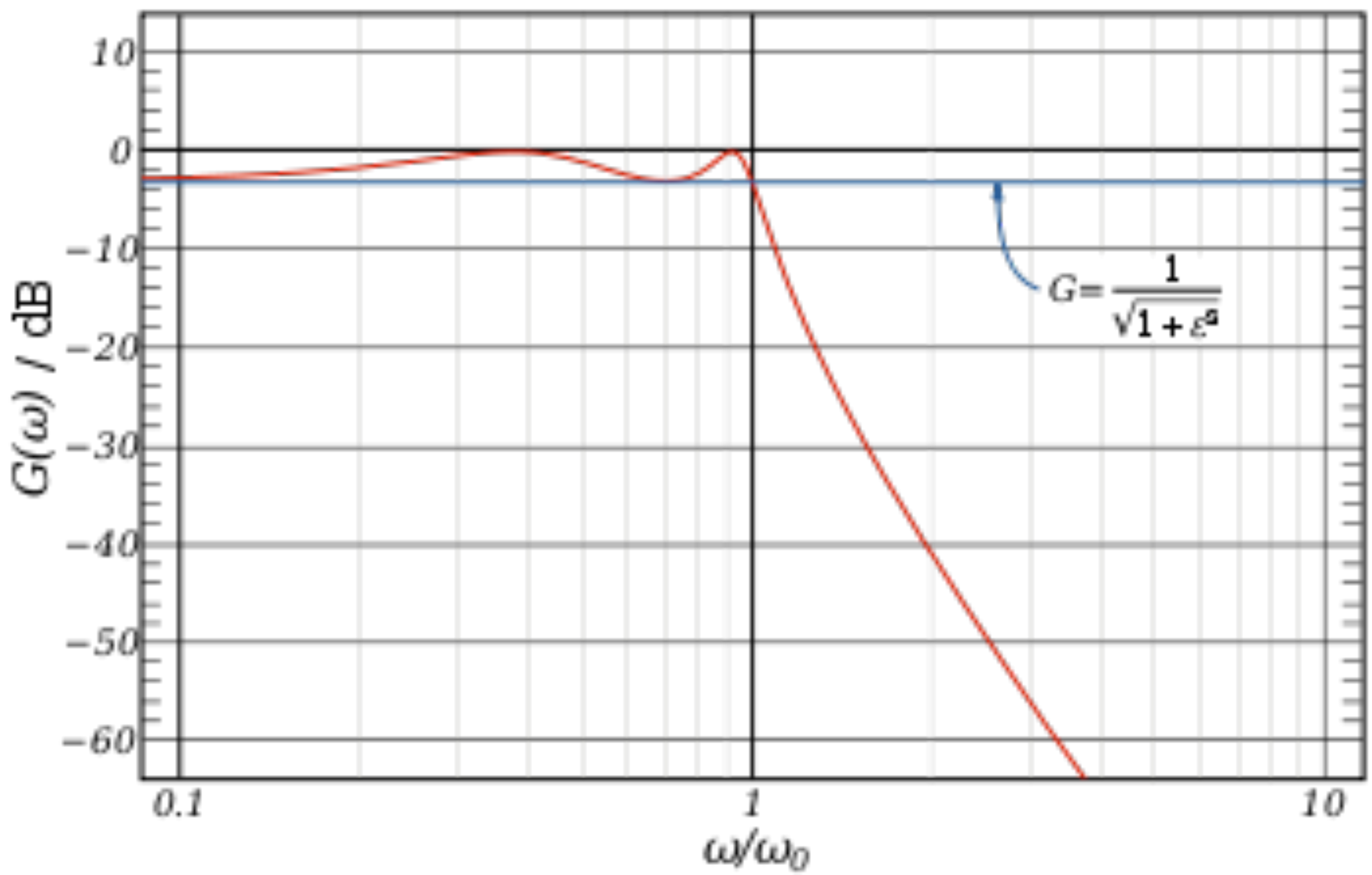
Butterworth poles



Example:
3rd-order
filter

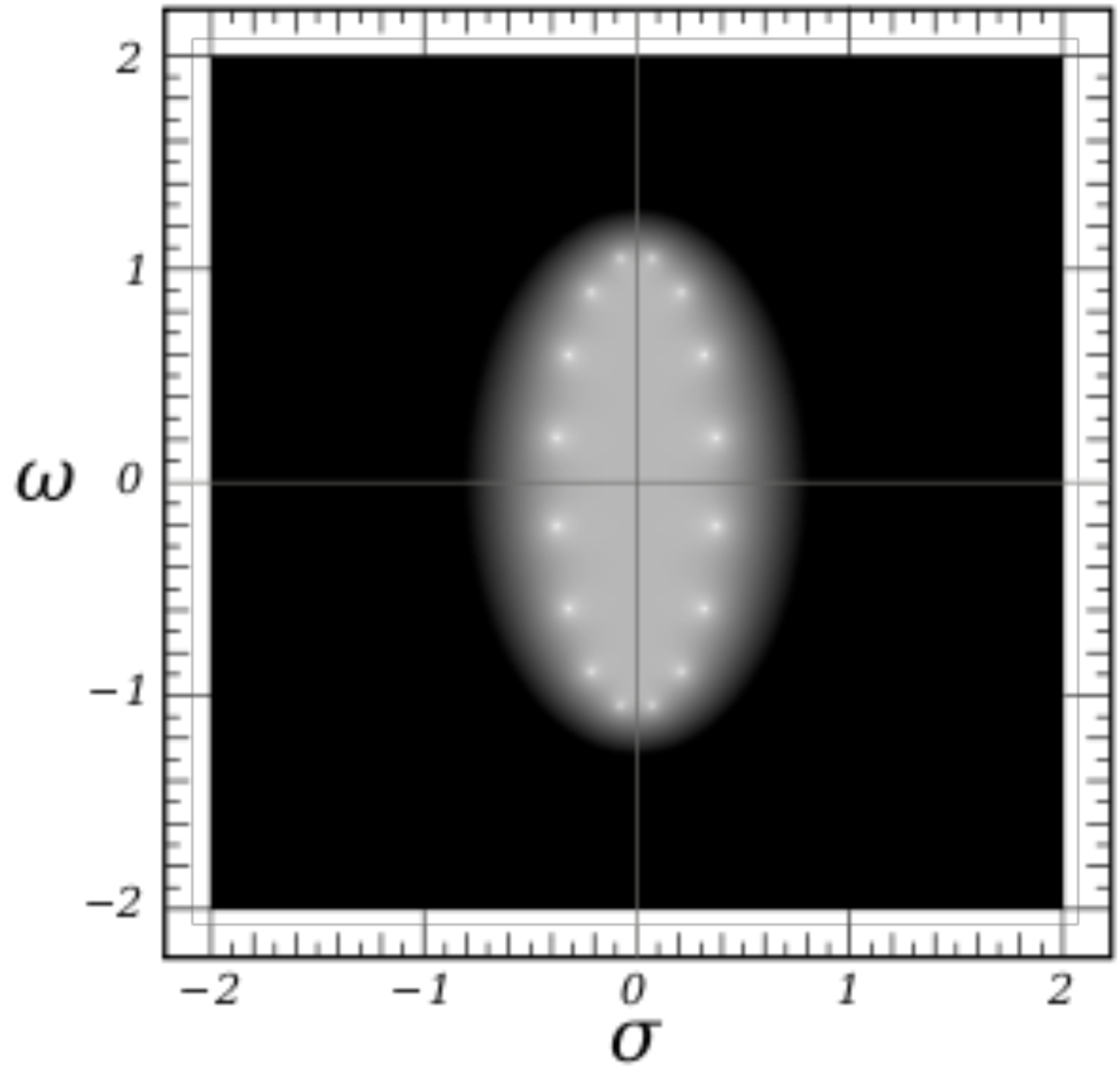
Butterworth poles: equidistant on a circle
(Note: equidistant true with poles also in right half plane)

Chebyshev



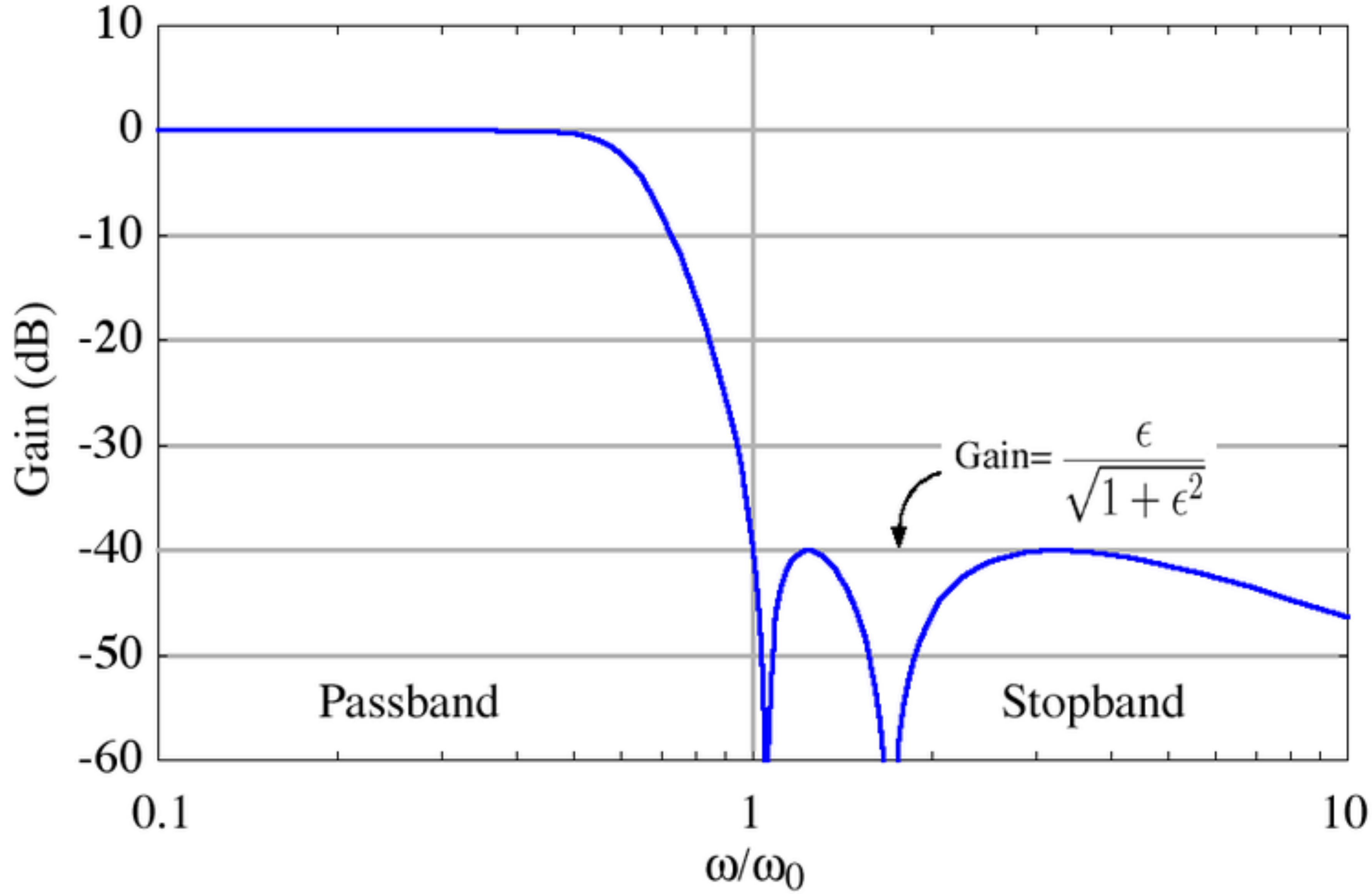
Steeper close to passband for same order,
but has ripple in passband
Constraints: certain ripple in passband; monotonic in stopband
Parameter: ripple factor

Chebysjev poles



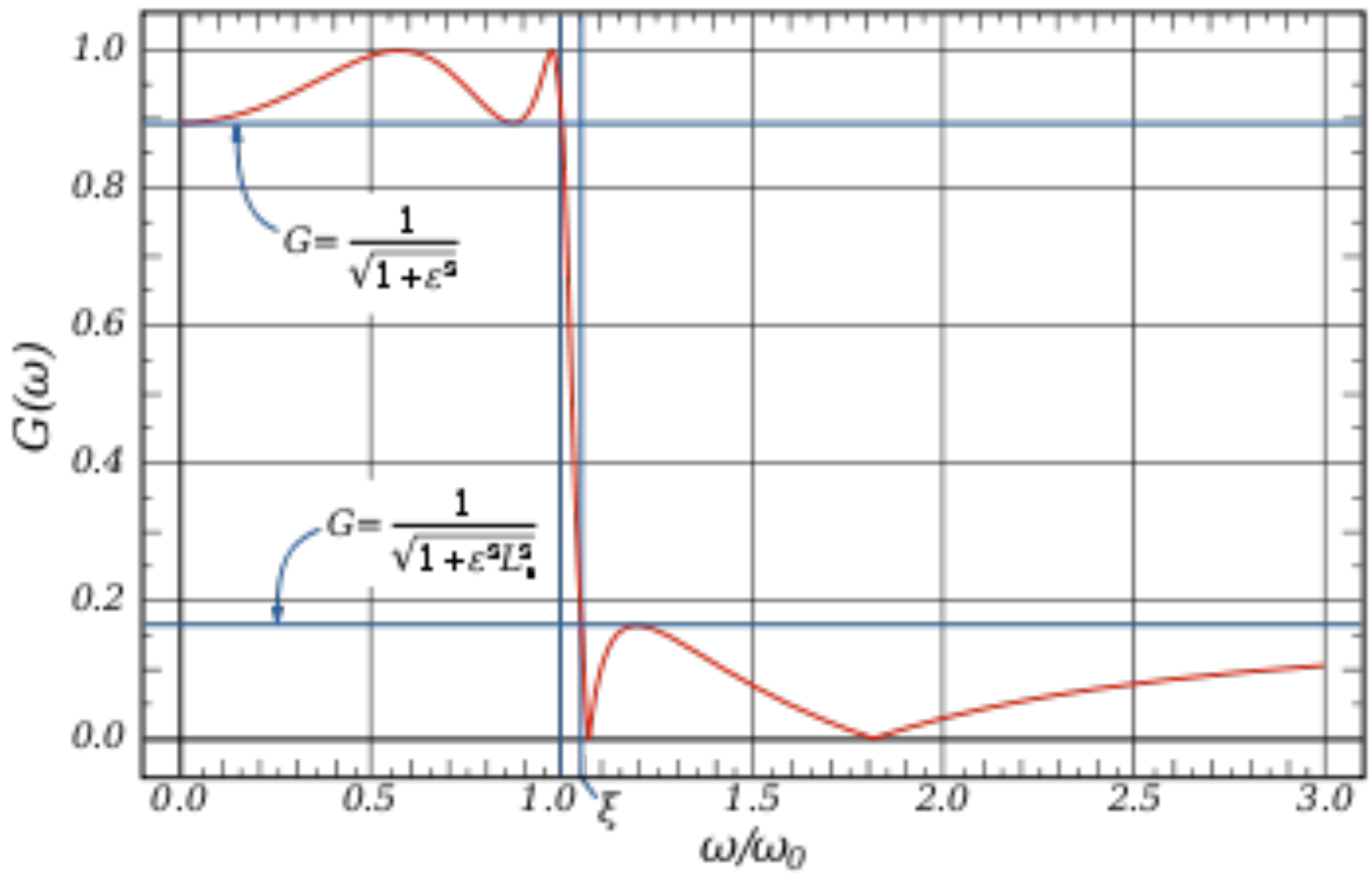
8th-order filter

Inverse Chebyshev



Steeper close to stopband for same order, but has ripple in stop band
Constraints: certain ripple in stopband; monotonic in passband
Parameter: ripple factor

Elliptic (Cauer)



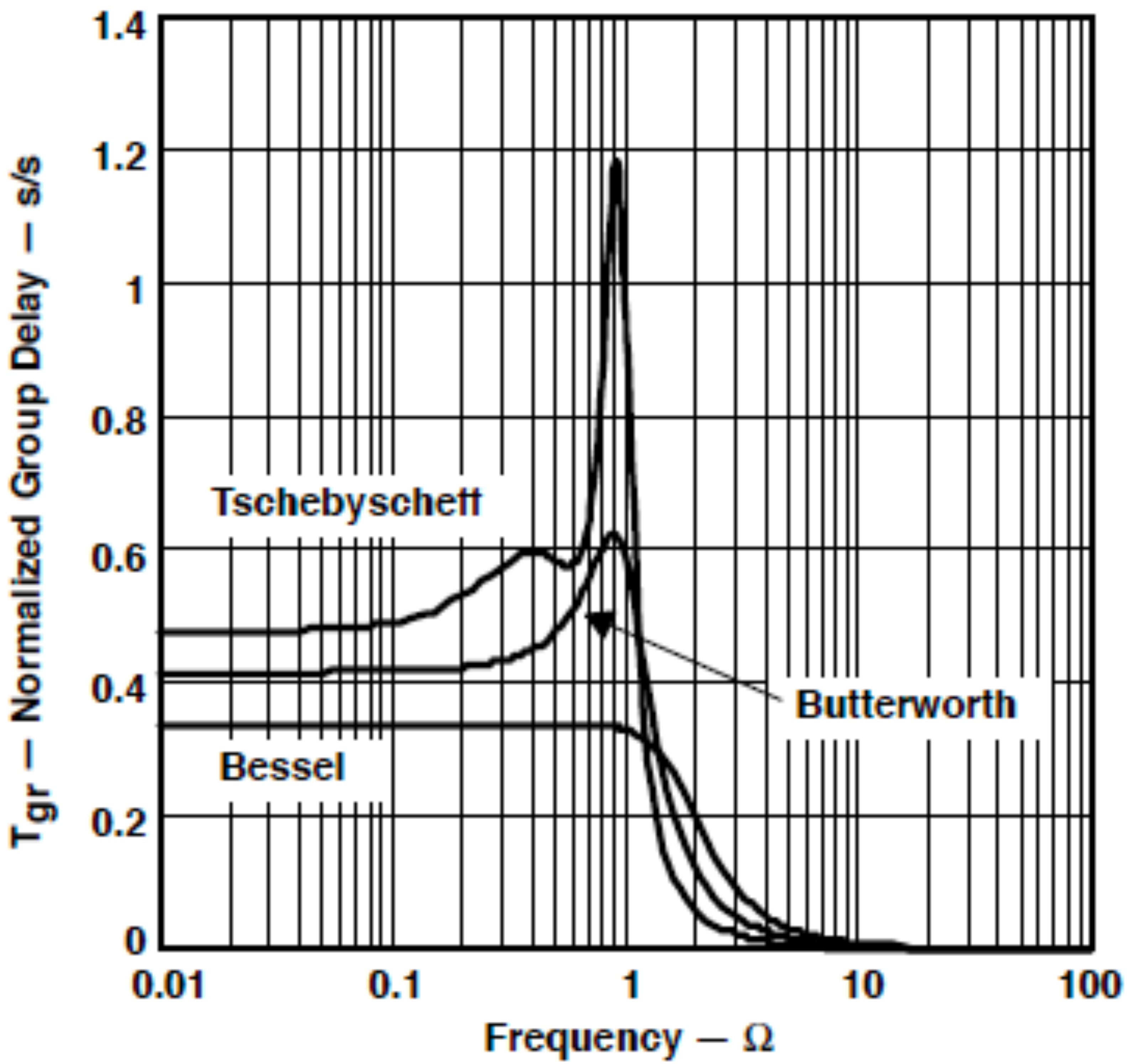
Steepest for certain order, but has ripple in **both** bands

Constraints: certain ripple in passband;

certain (possibly other) ripple in stopband

Parameters: ripple factor(s)

Bessel = linear phase



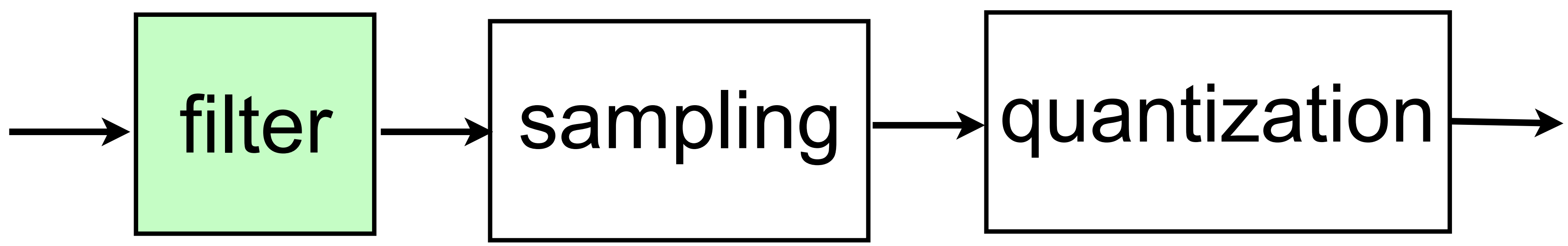
Conclusion: Finding a xfer function that fits spec

- At certain order:
  - Tradeoff between steepness and flatness in pass & stopband
  - Consequences for phase too!
- Note: Many other constraints on gain and phase (delay) can be used when specifying a filter!
- These are the classics.

How find pole and zero placement?

- Matlab
- Table
- Calculate yourself
    - Take SSY130 Applied signal processing to learn how.

Example: A/D path



- How does the design of the sampling box influence the **specification** and **transfer function** of the filter?

From poles and zeros to ckts Implementation styles

- Filters with cascade of 1st and 2nd order sections
  - Often active filters
- Coupled-form filter
  - Often passive filters

Nomenclature

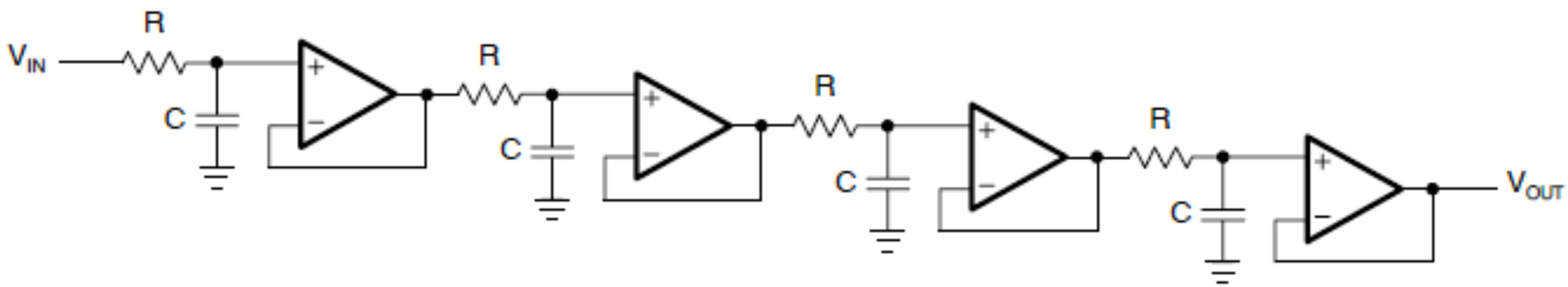
- **Passive** filters - only use passive components: R, L, C
- **Active** filters - use passive (usually no L:s) and active components = components with gain (usually OPamps or transconductors)

- **Sampled** filters - signal is sampled (discrete time) (in week 6)
- **Continuous-time** (CT) filters - signal is not sampled

- Analog active filters - use components with gain + passives, sampled or CT (analog = signal is not discretized)

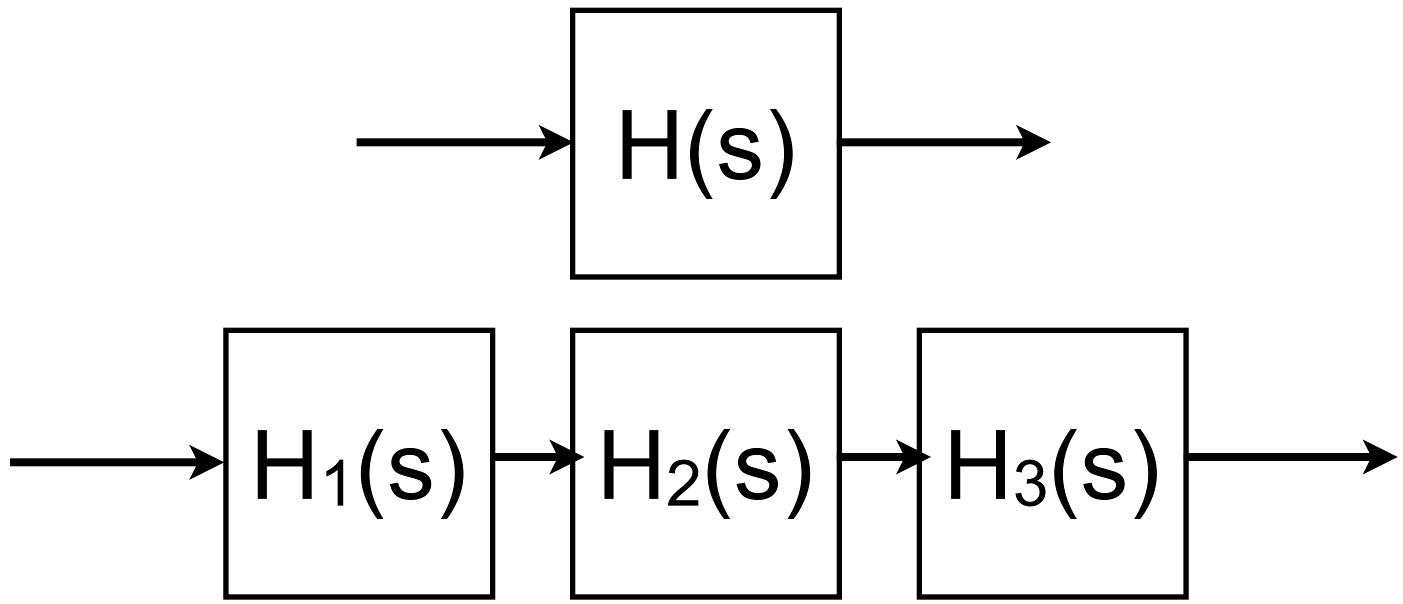
(Digital filters are sampled and discretized)

How to implement



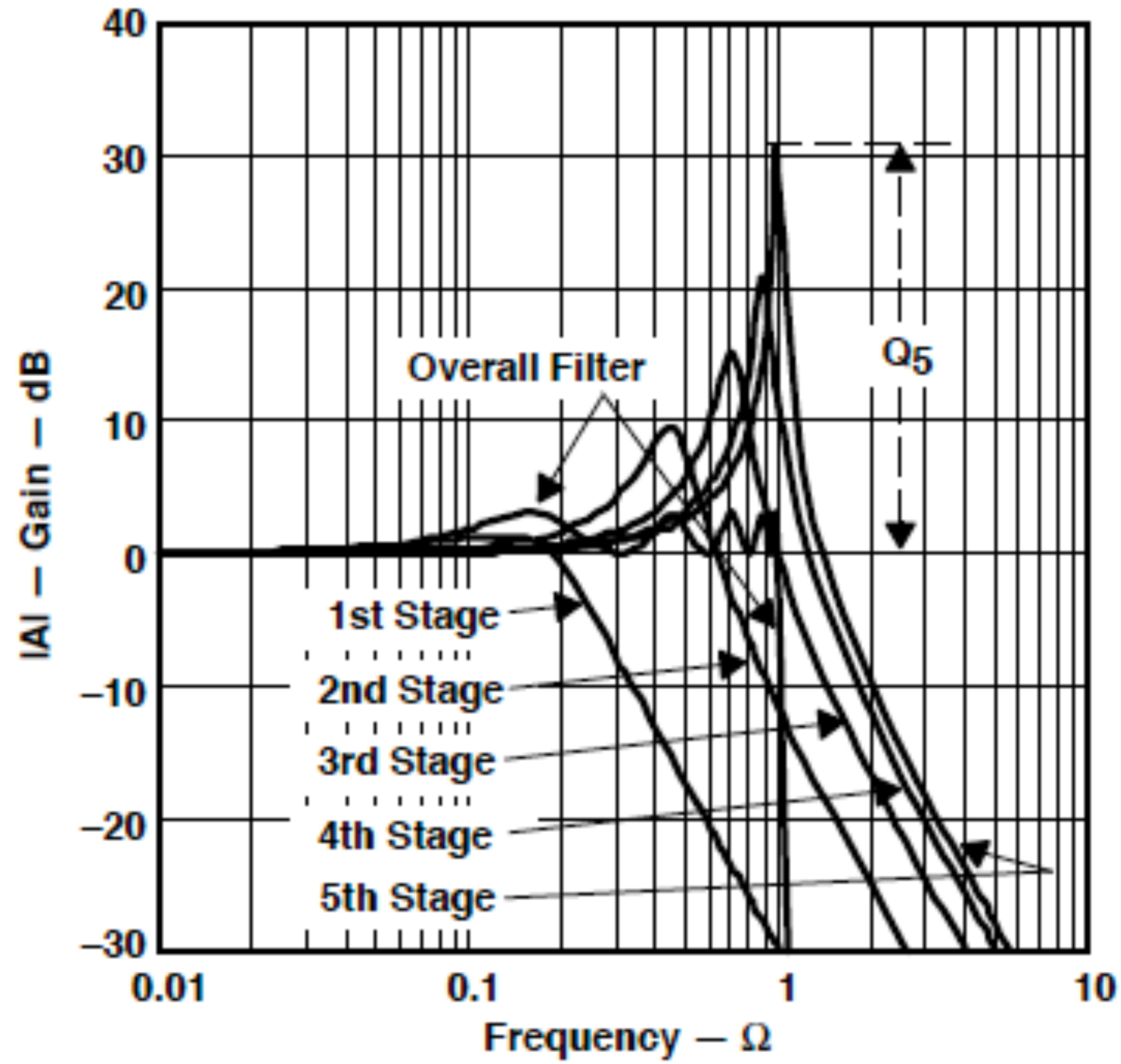
Is this a good idea?

Cascade main idea



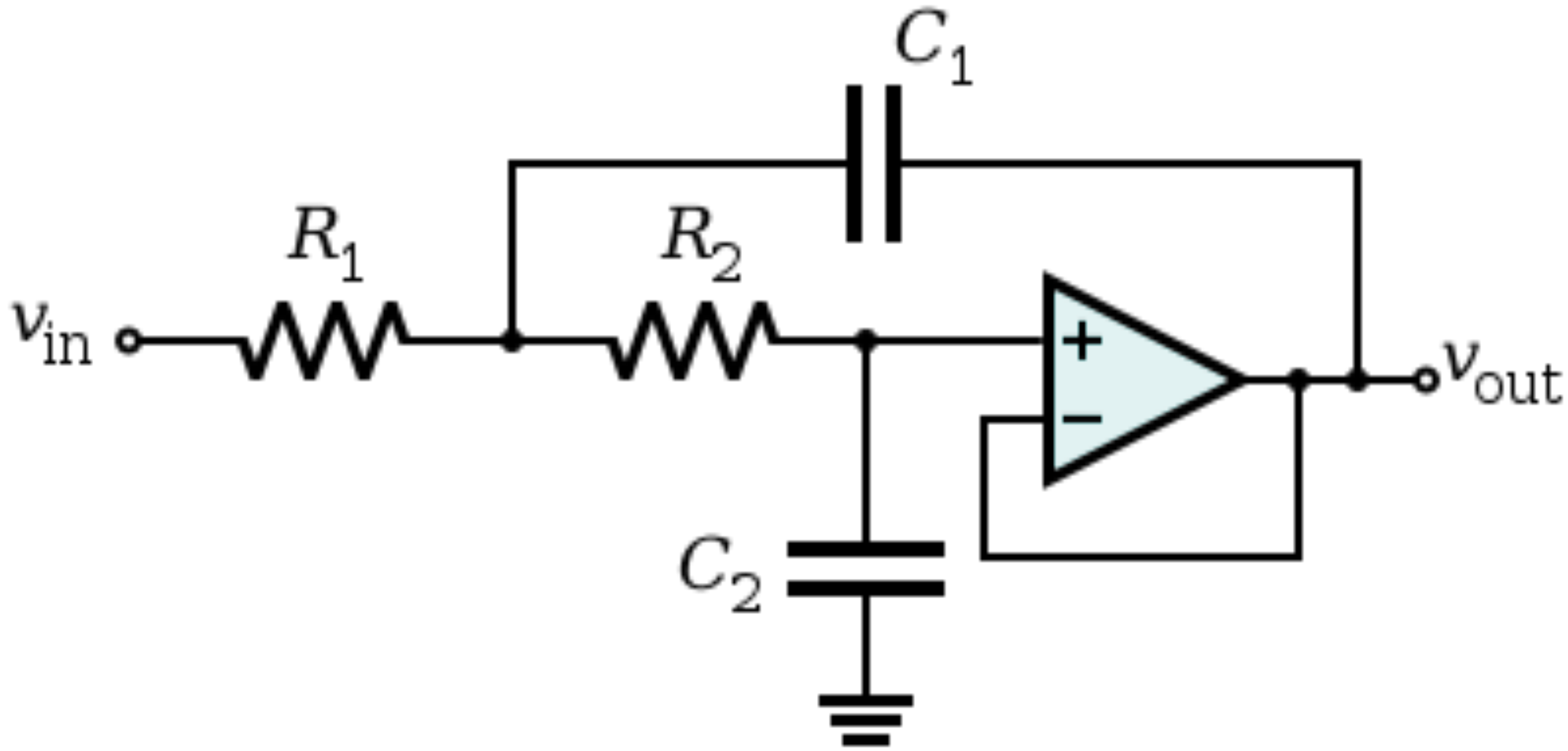
- Implement filter as cascade of blocks
- Split xfer function into factors
  - $H(s) = \prod H_i(s)$
- Implement at most 2 poles and at most 2 zeros per block

Higher Q for higher order filter



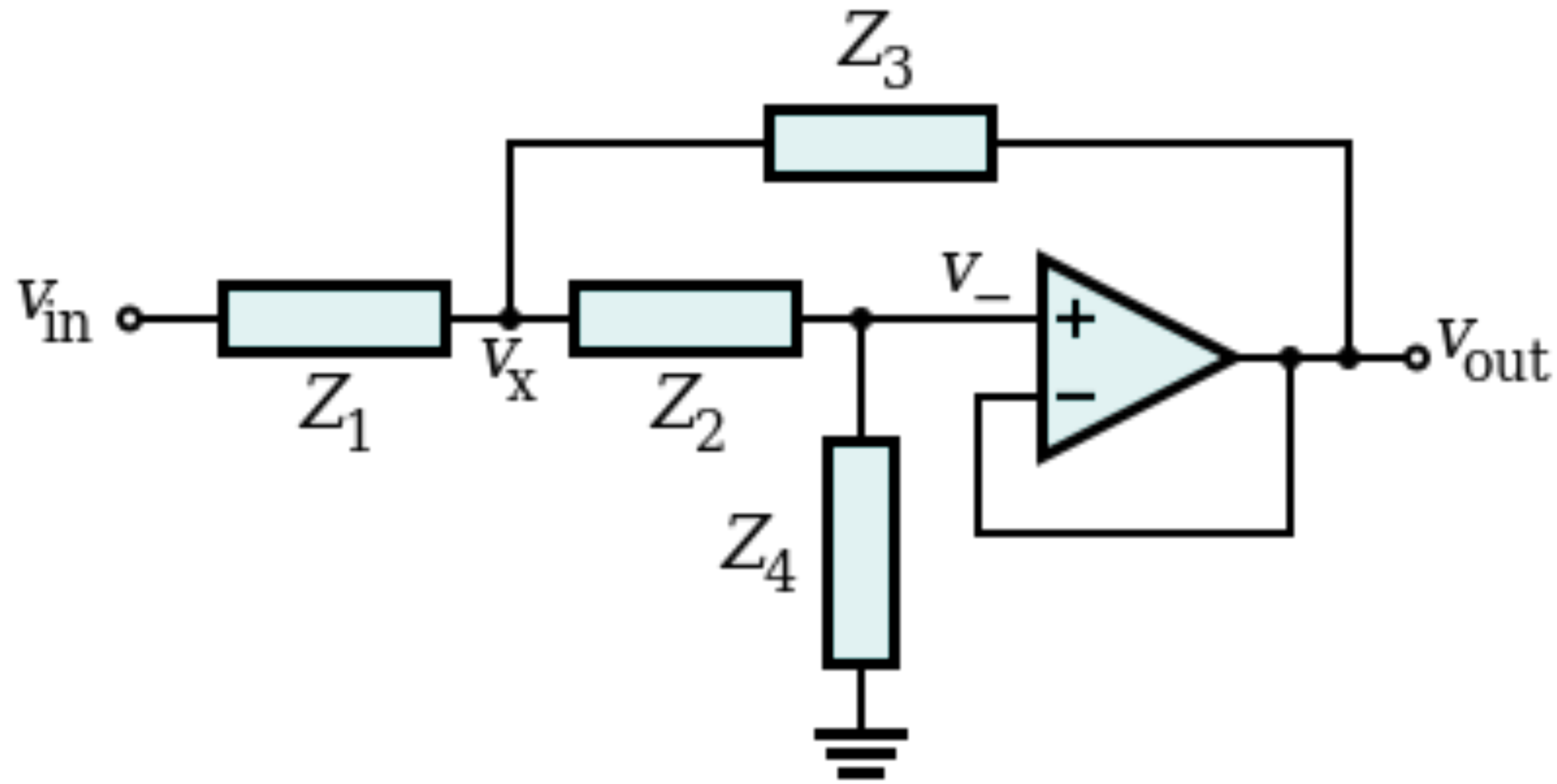
Example: 10th order Chebysjev filter

2nd-order-block example



- Sallen-Key LP link
- Feedback helps set poles
    - Here beta is 1 = unity-gain configuration

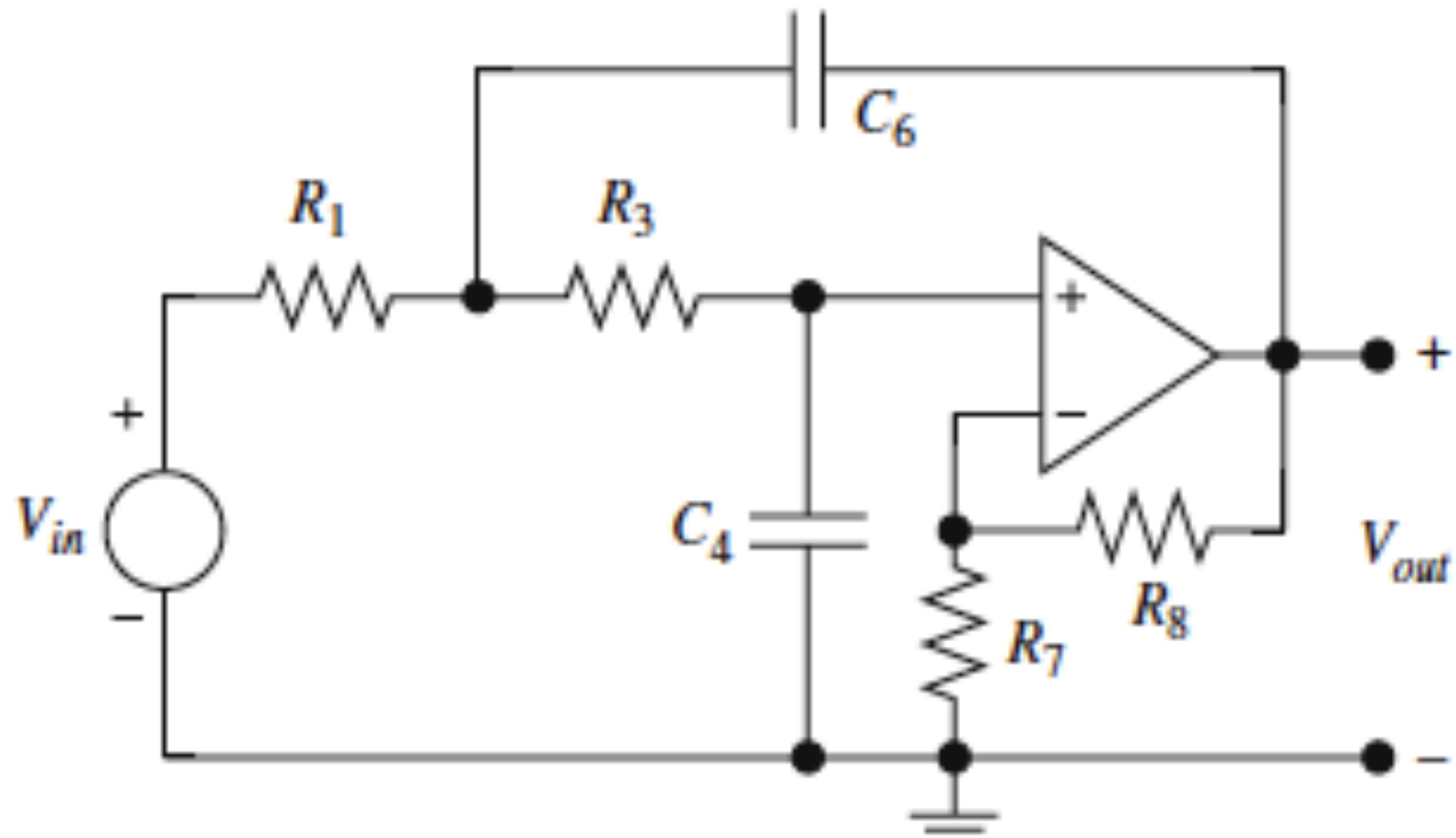
Sallen-Key block
the general case



$$\frac{v_{out}}{v_{in}} = \frac{Z_3 Z_4}{Z_1 Z_2 + Z_3(Z_1 + Z_2) + Z_3 Z_4},$$

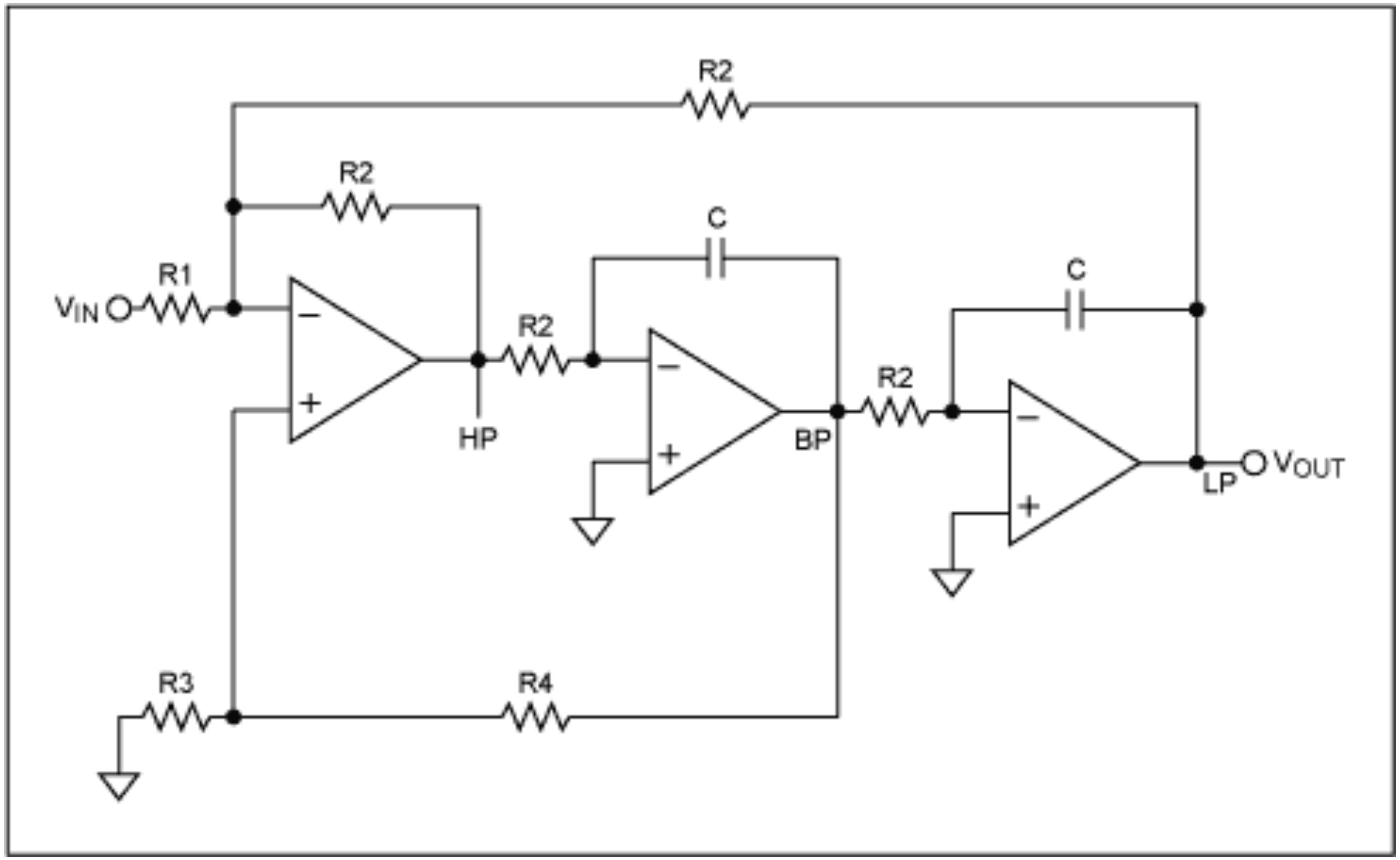
How derive xfer function?

Sallen-Key topology for lab 3

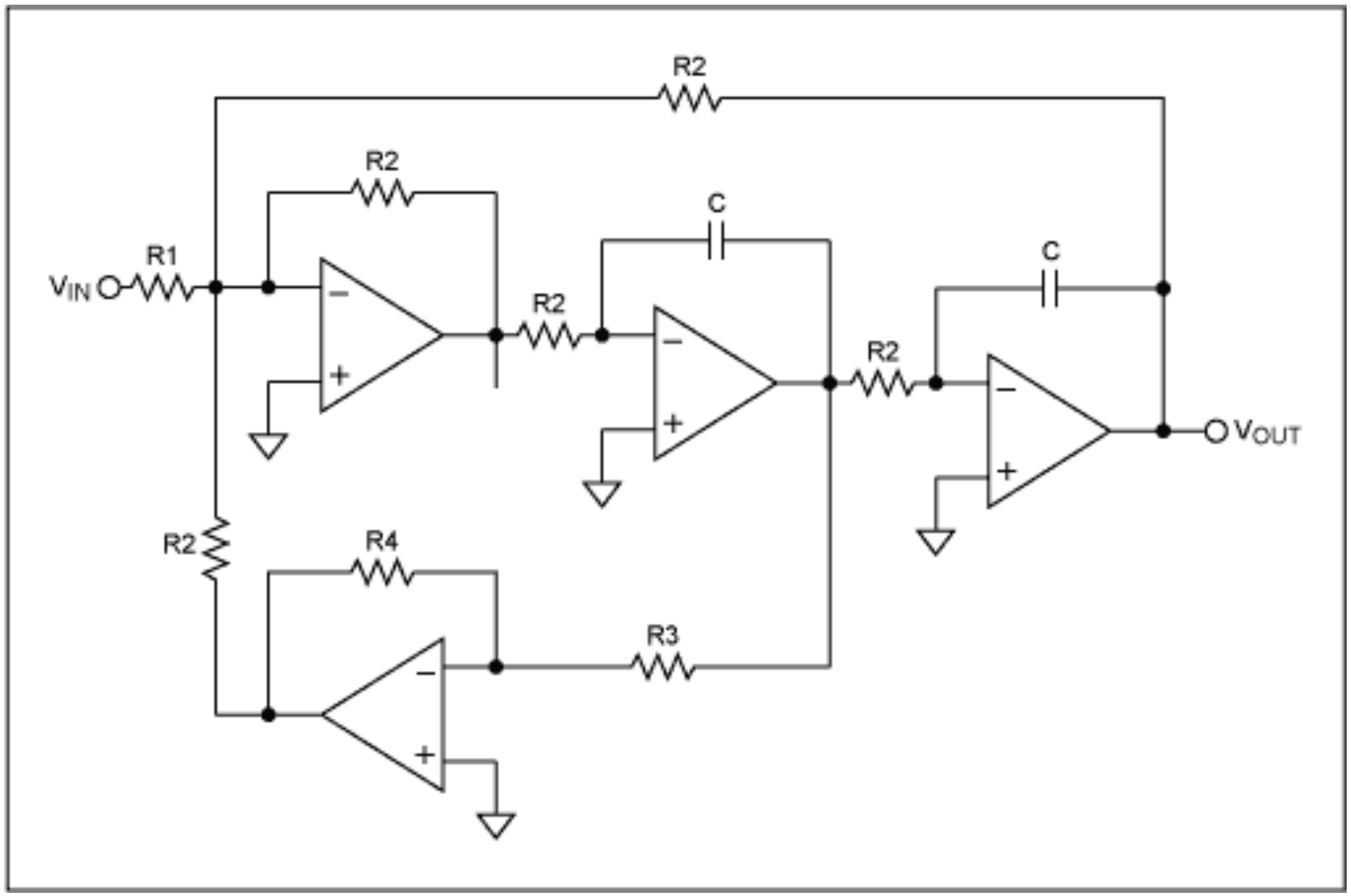


Gain ≠ 1 otherwise the same

General biquad



With independent Q setting



http://www.maximintegrated.com/en/app-notes/index.mvp/id/1762

How assign poles/zeros?

- Application dependent…
  - …but some advice can be given
- Dynamic range and/or sensitivity
- Poles close to zeros mean high sensitivities
- Higher order gives higher sensitivities
- Distortion and noise generated in one stage filtered by subsequent stages
  - => Low-Q first, higher later

Summary

- Practically important and theoretically large field
- Active filters easier to integrate
- Higher-order filters cost more
  - Both power and hardware

- Next time: Coupled forms
  - More on implementations

Mörk koppar	Koppar	Löv	Granit	Himmel
Kaffe	Tegel	Energi	Väst-kust	Skym-ning