Variability and feedback

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But first a review about matching

From MUD cards

- Could not understand the problem.
- What is variance?
- What is the relation between difference and sigma?
- Is it $\sigma(\Delta VT)$ or $\sigma_{\Delta VT}$?
- Why does 3σ mean ΔVT/3?
- The way we have multiplication of Fourier transform
- Good: example, but maybe unnecessary? Cleared up some problem with standard deviation
- Forgot to ask to clarify the difference between the formulas for absolute and relative variations.

The problem

- It is a problem that components are not exactly the same.
- Example: Resistor chain

Why is it a problem?

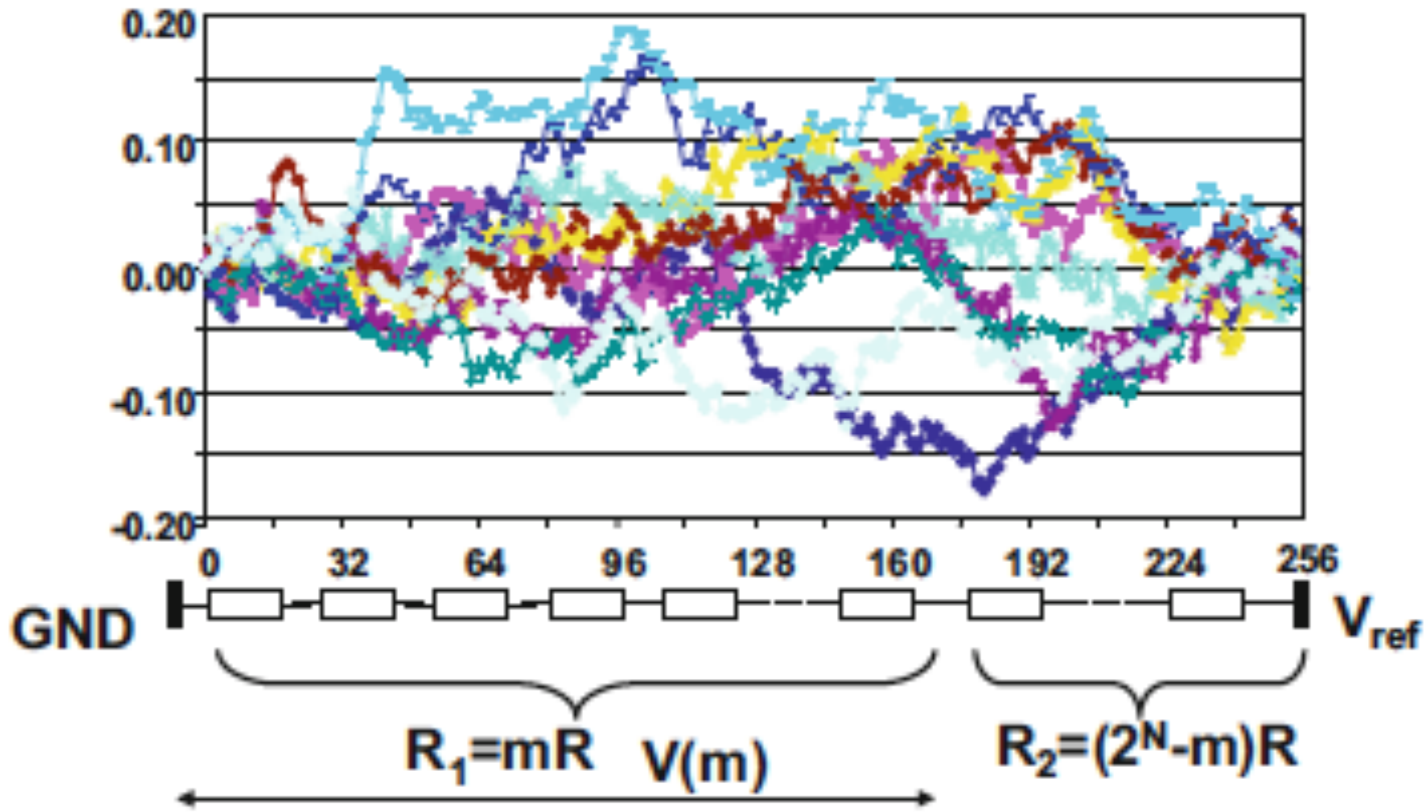
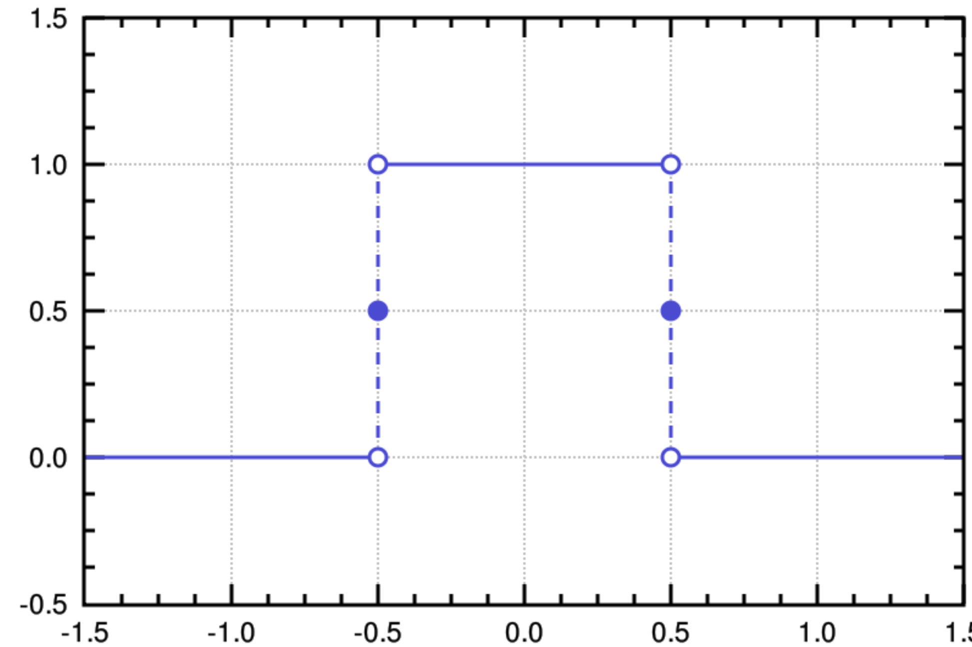


Fig. 7.16 A resistor string is connected between a reference voltage and ground. The simulation shows ten instances of the effect of $\sigma_R/R = 1\,\%$ mismatch. In this 256 devices ladder the maximum deviation is $\pm 20\,\%$

The problem

- We can make everything as similar as possible to make components equal (size, orientation, surrounding, etc)
- The remaining random variations we can decrease by making components physically larger.
- But, how large should we make them to make the error "small enough"? $\sigma^2_{\Delta P} = \frac{A_P^2}{WL}$
- What is "small enough" depends on the application and other errors in the system.
- Don't overengineer! It costs money and often power and speed too.

Rectangular function



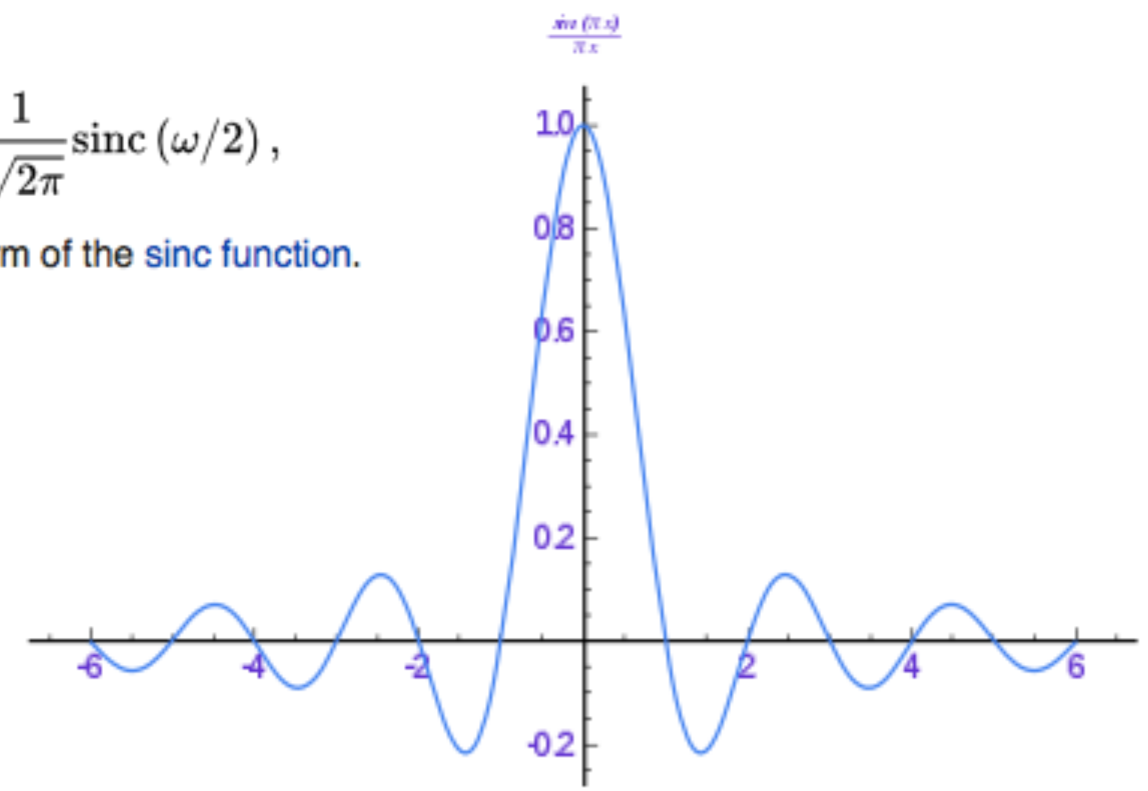
The unitary Fourier transforms of the rectangular function are[1]

$$\int_{-\infty}^{\infty} \mathrm{rect}(t)\cdot e^{-i2\pi ft}\,dt = \frac{\sin(\pi f)}{\pi f} = \mathrm{sinc}(f),$$

using ordinary frequency *f*, and

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \mathrm{rect}(t)\cdot e^{-i\omega t}\,dt = \frac{1}{\sqrt{2\pi}}\cdot\frac{\sin(\omega/2)}{\omega/2} = \frac{1}{\sqrt{2\pi}}\mathrm{sinc}(\omega/2),$$

using angular frequency ω, where sinc is the unnormalized form of the sinc function.



Convolution theorem

Fourier transform

$$\mathcal{F}\{g * f\} = \mathcal{F}\{g\} \cdot \mathcal{F}\{f\}$$

Convolution   Pointwise multiplication

Works also the other way around

$$\mathcal{F}\{g \cdot f\} = \mathcal{F}\{g\} * \mathcal{F}\{f\}$$

Pointwise multiplication   Convolution

We EE people
are most used to
convolutions of functions of time,
but it works for functions of space too

Very short

$$\Delta P(x_{12}, y_{12}) = P(x_1, y_1) - P(x_2, x_2)$$

same as

$$\Delta P(x_{12}, y_{12}) = \frac{1}{\text{area}} \left[\iint\limits_{\text{area}(x_1, y_1)} P(x', y') dx' dy' - \iint\limits_{\text{area}(x_2, y_2)} P(x', y') dx' dy' \right]$$

Take Fourier transform and use convolution theorem

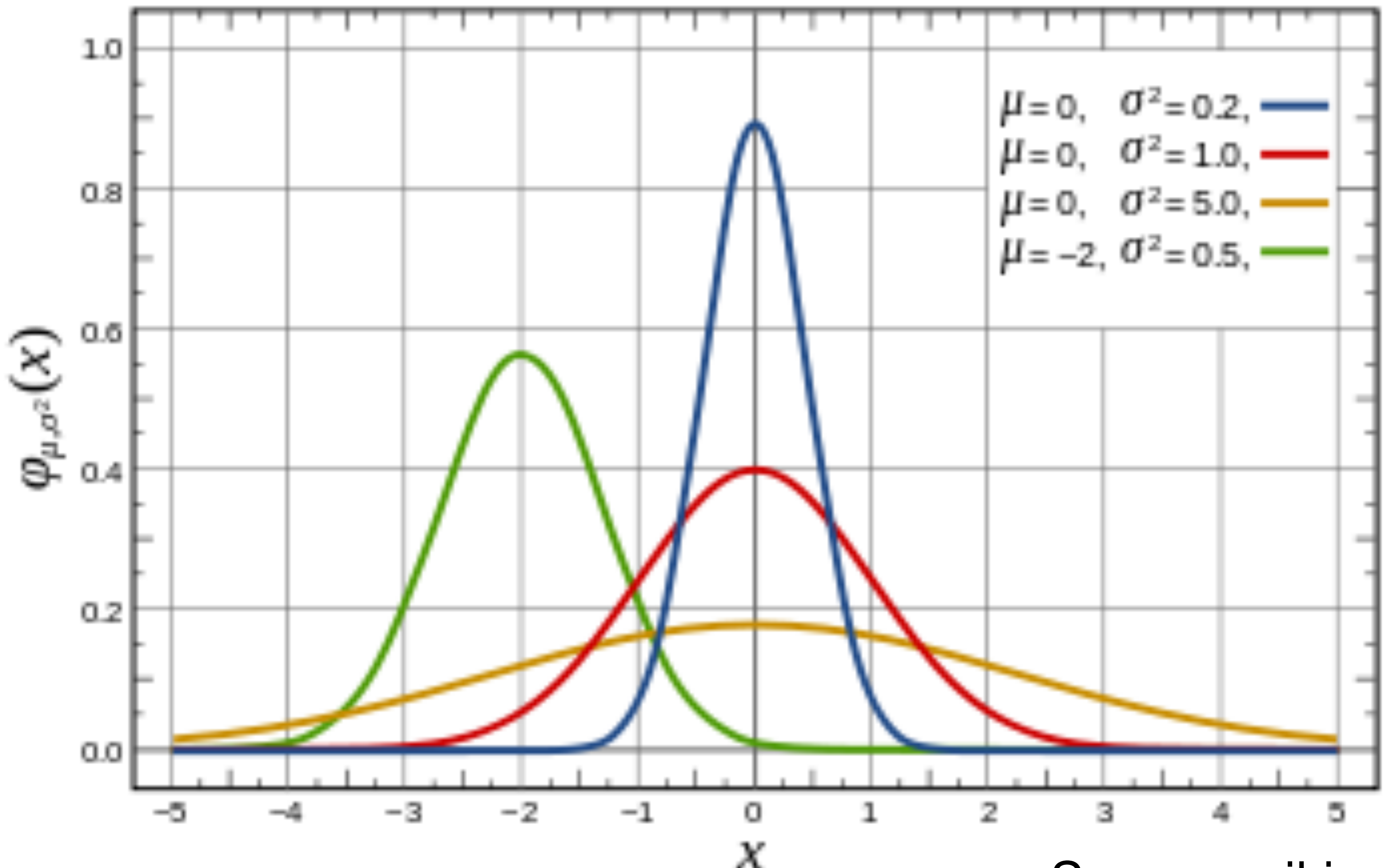
$$\Delta \mathcal{P}(\omega_x, \omega_y) = \mathcal{G}(\omega_x, \omega_y) \mathcal{P}(\omega_x, \omega_y)$$

sinc-y function constant

Sum the power contributions =
Integrate the squares over all spatial frequencies:

$$\sigma^2(\Delta P) = \frac{1}{4\pi^2} \int\limits_{\omega_y=-\infty}^{\infty} \int\limits_{\omega_x=-\infty}^{\infty} \left|\mathcal{G}(\omega_x, \omega_y)\right|^2 \left|\mathcal{P}(\omega_x, \omega_y)\right|^2 d\omega_x d\omega_y$$

Normal distributions



Source: wikipedia

Example

- Samples: 1, 4, -2, 0, -3
- Mean?
- Variance?
- Standard deviation?

In conclusion

- Do all to ensure similarity among matched components.
- Choose components large enough to ensure good enough matching!
- But don’t overengineer because it is expensive!
    - Can even make other requirements unachievable.

Over to feedback!

How bad is it?

OP27—SPECIFICATIONS

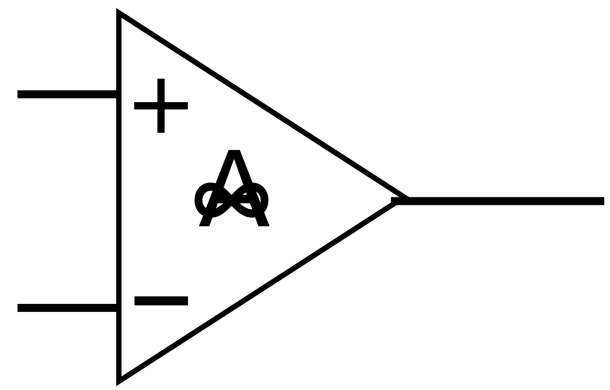
ELECTRICAL CHARACTERISTICS (@ $V_S = \pm 15$ V, $T_A = 25°C$, unless otherwise noted.)

Parameter	Symbol	Conditions	OP27A/E Min	OP27A/E Typ	OP27A/E Max	OP27F Min	OP27F Typ	OP27F Max	OP27C/G Min	OP27C/G Typ	OP27C/G Max	Unit
INPUT OFFSET VOLTAGE[1]	V_{OS}			10	25		20	60		30	100	μV
LONG-TERM V_{OS} STABILITY[2, 3]	V_{OS}/Time			0.2	1.0		0.3	1.5		0.4	2.0	μV/M_O
INPUT OFFSET CURRENT	I_{OS}			7	35		9	50		12	75	nA
INPUT BIAS CURRENT	I_B			±10	±40		±12	±55		±15	±80	nA
INPUT NOISE VOLTAGE[3, 4]	$e_{n\ p\text{-}p}$	0.1 Hz to 10 Hz		0.08	0.18		0.08	0.18		0.09	0.25	μV p-p
INPUT NOISE Voltage Density[3]	e_n	f_O = 10 Hz		3.5	5.5		3.5	5.5		3.8	8.0	nV/√Hz
		f_O = 30 Hz		3.1	4.5		3.1	4.5		3.3	5.6	nV/√Hz
		f_O = 1000 Hz		3.0	3.8		3.0	3.8		3.2	4.5	nV/√Hz
INPUT NOISE Current Density[3, 5]	i_n	f_O = 10 Hz		1.7	4.0		1.7	4.0		1.7		pA/√Hz
		f_O = 30 Hz		1.0	2.3		1.0	2.3		1.0		pA/√Hz
		f_O = 1000 Hz		0.4	0.6		0.4	0.6		0.4	0.6	pA/√Hz
INPUT RESISTANCE Differential Mode[6]	R		1.3	6		0.94	5		0.7	4		MΩ

Accuracy by feedback!

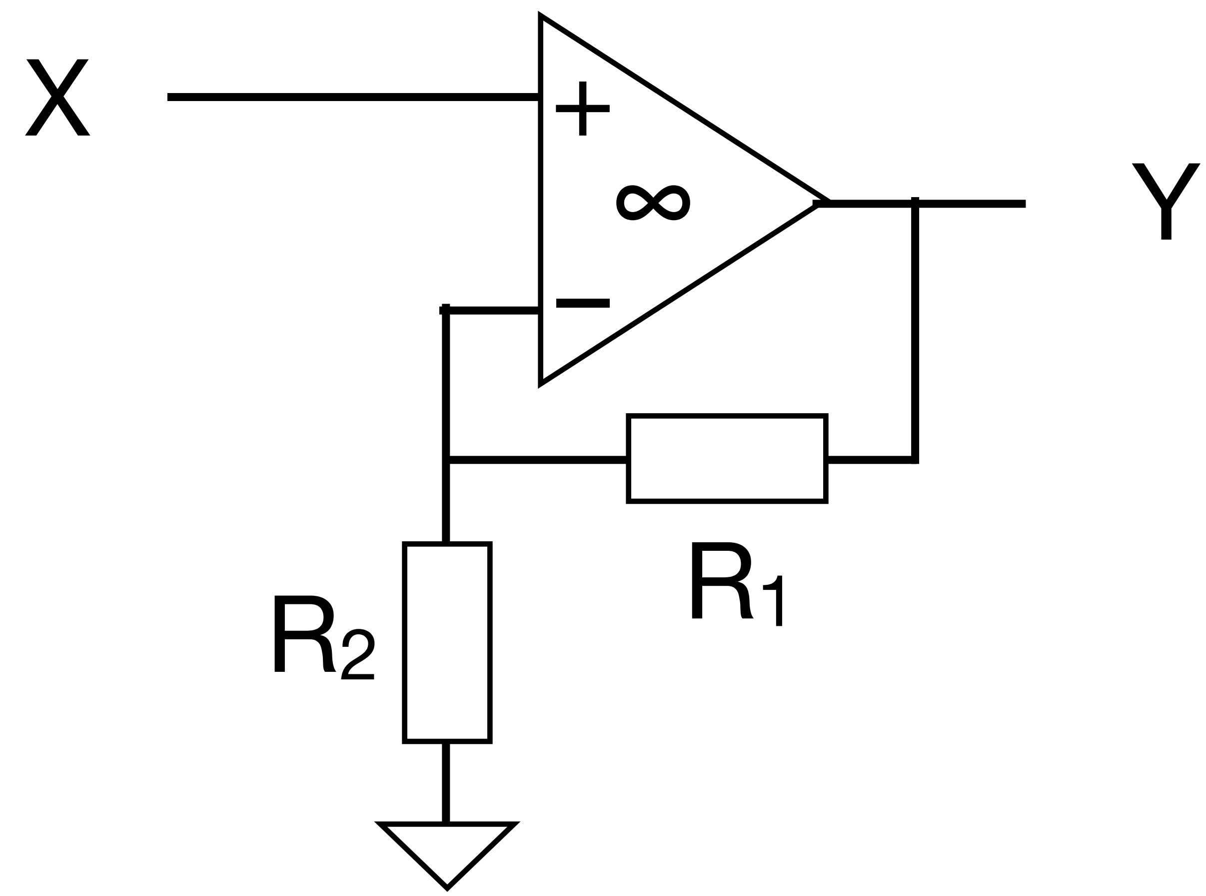
- Derive gain accuracy from **passive** components (R or C)
  - Much more precise than **active** components (transistors)
- Pay with gain magnitude for improved accuracy
  - That is: accurate but lower gain!
- Another issue: stability in feedback systems; not really dealt with in this course

Ideal amplifier model



- Output takes any voltage, sinks/sources any current
- No current into inputs, no voltage across
- Gain A towards ∞

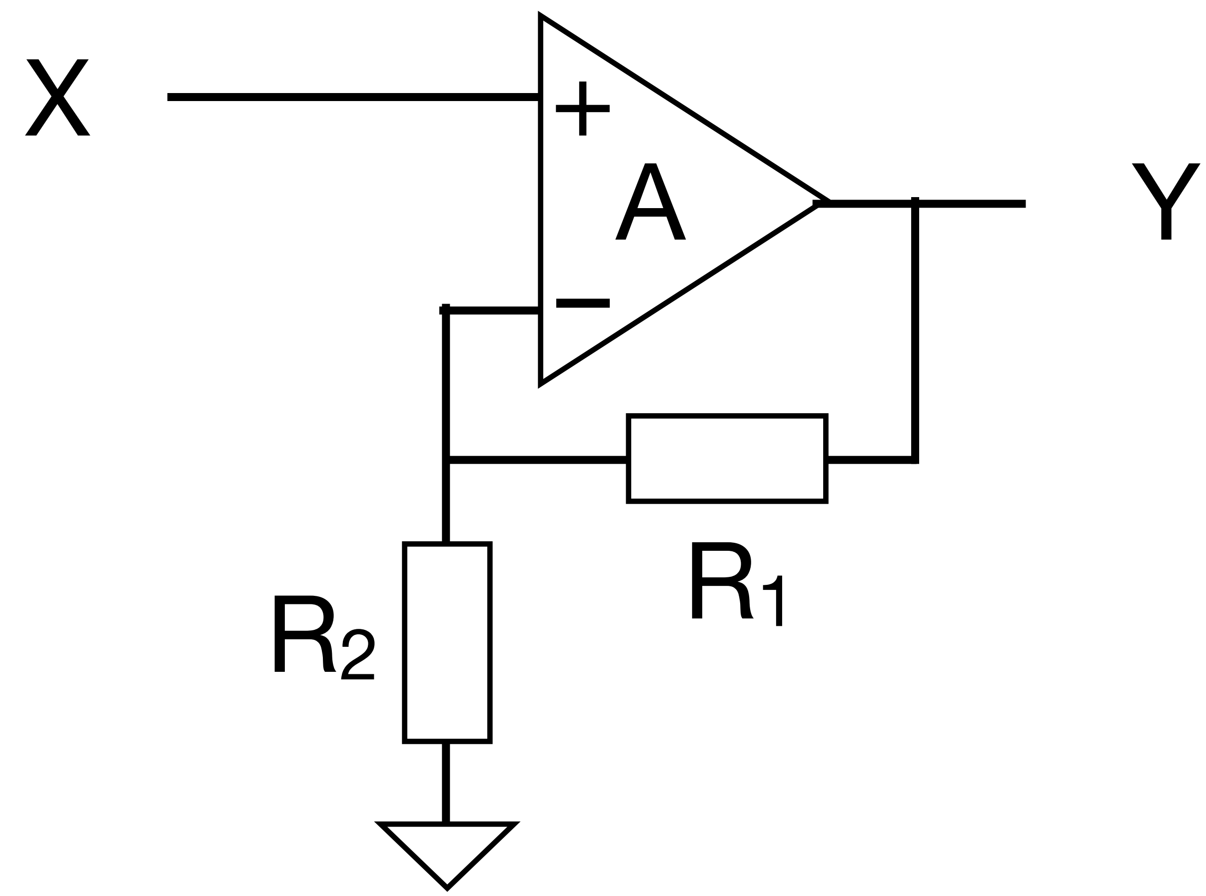
Example: voltage amplifier



- Voltage divider + amplifier!

$$X = Y \frac{R_2}{R_1 + R_2} \quad \text{=>} \quad \frac{Y}{X} = \frac{R_1 + R_2}{R_2}$$

Example: voltage amplifier



$$Y = A(X - Y\frac{R_2}{R_1 + R_2})$$

$$Y + AY\frac{R_2}{R_1 + R_2} = AX$$

=>

$$Y = \frac{AX}{1 + \frac{AR_2}{R_1+R_2}}$$

Let A grow large

- Let $R_2 / (R_1 + R_2) = \beta$
- $Y = X \cdot A / (1 + AR_2 / (R_1 + R_2)) =$

  $X \cdot A / (1 + A\beta) \approx$

  $\approx X \cdot A / (A\beta) =$

  $= X / \beta$
- When is "≈" allowed?
- Error magnitude?

Discrepancy

Deviation from “ideal” (A = ∞) performance

$$\frac{Y}{X} = \frac{A}{1 + A\beta} \approx \frac{1}{\beta}$$

Clearly, for approximation to hold, Aß >> 1

Discrepancy:

$$D = \frac{A}{1 + A\beta} / \frac{1}{\beta} = \frac{A\beta}{1 + A\beta}$$

D < 1; but close to 1 when Aß >> 1

Example

We wish $Y / X = 100$

Use $ß = 1 / 100 = 0.01$

We wish gain error to be $< 1\%$

Requirement is $D > 0.99$

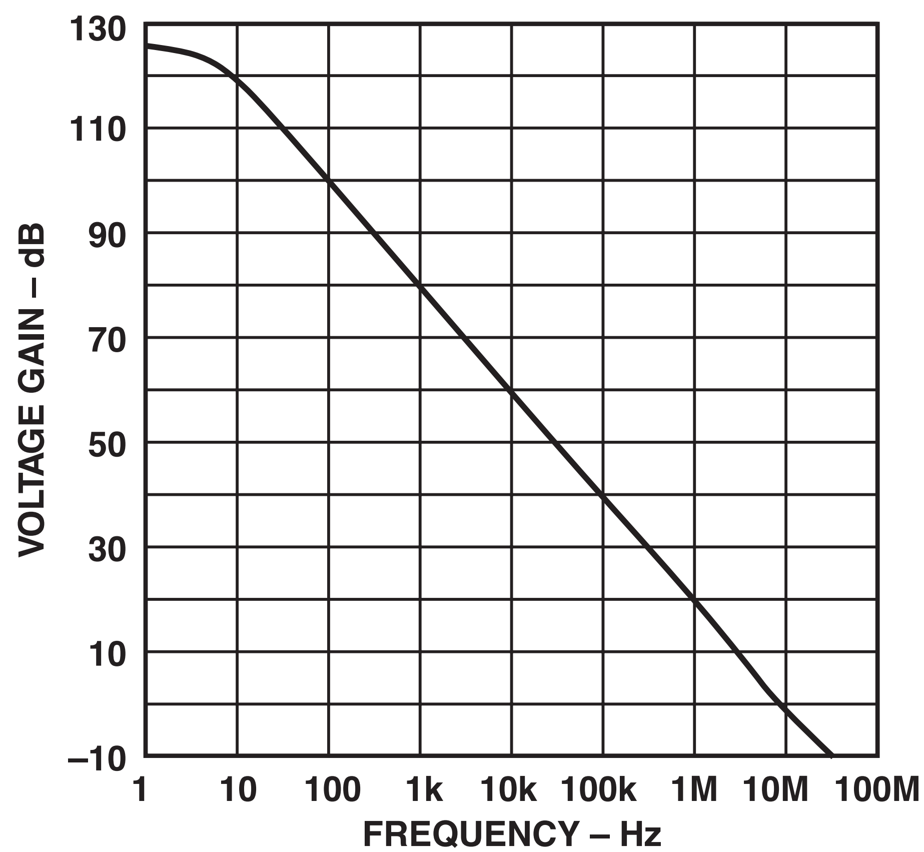
$D = Aß / (1 + Aß)$

What is required A?

$A = D / (ß(1 - D)) \approx 10^2 \cdot 10^2 = 10^4$

… or 80 dB

Dominant pole

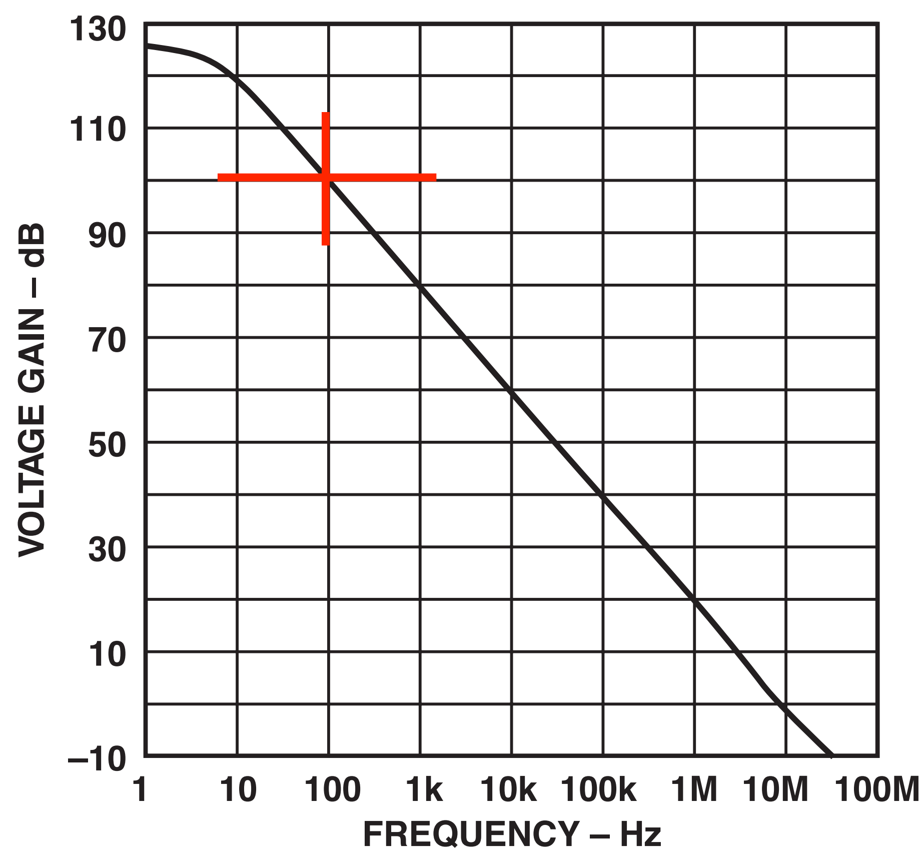


TPC 16. Open-Loop Gain vs. Frequency

- "Standard" op-amp transfer function Bode plot

[OP27]

GBW = gain-bandwidth product

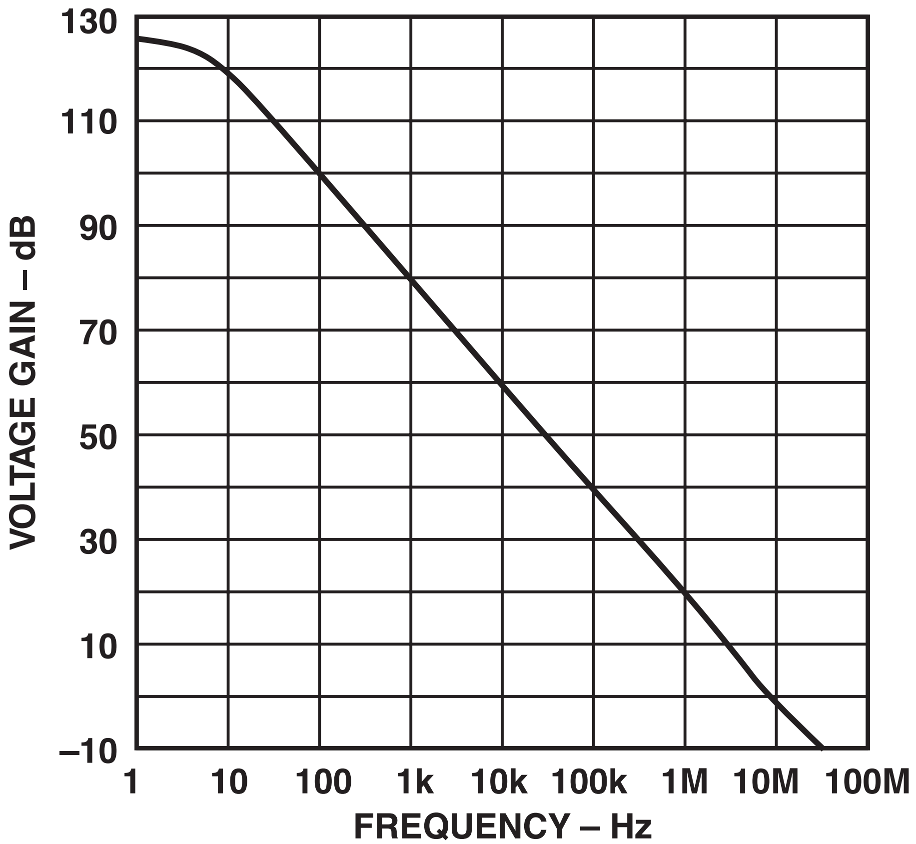


TPC 16. Open-Loop Gain vs. Frequency

- Standard parameter
  - Constant regardless of ß

Limitations

- Corrective power as function of frequency?
- Best at low frequencies, since higher gain!
- 80 dB only up to 1kHz!
    - Can't get Y / X = 100±1% at higher frequencies with OP27!



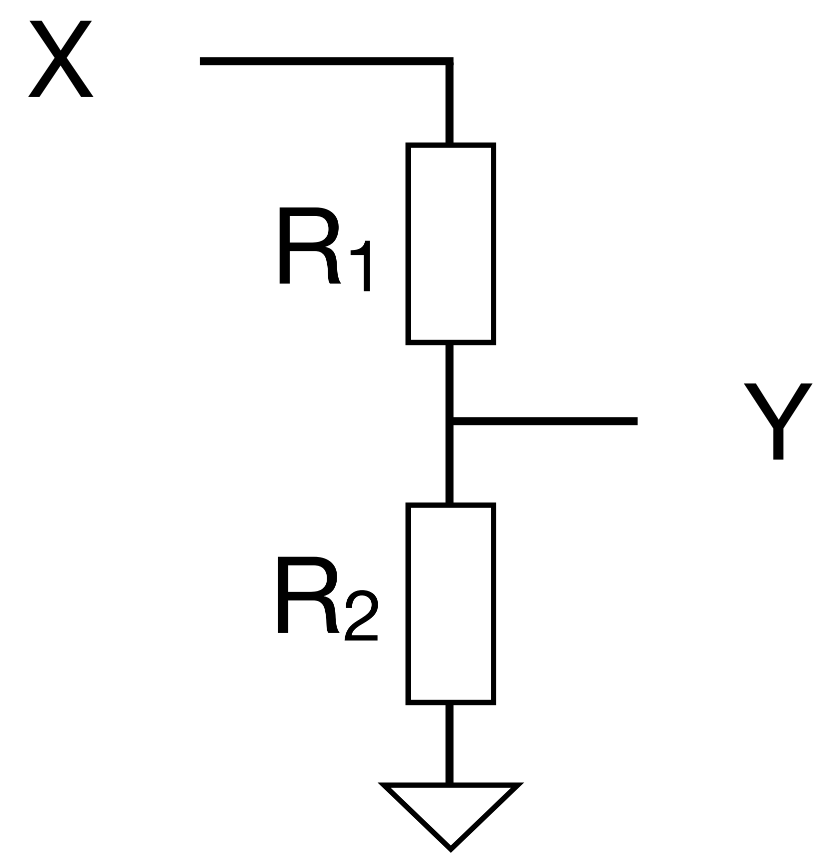
TPC 16. Open-Loop Gain vs. Frequency

What about the Rs?

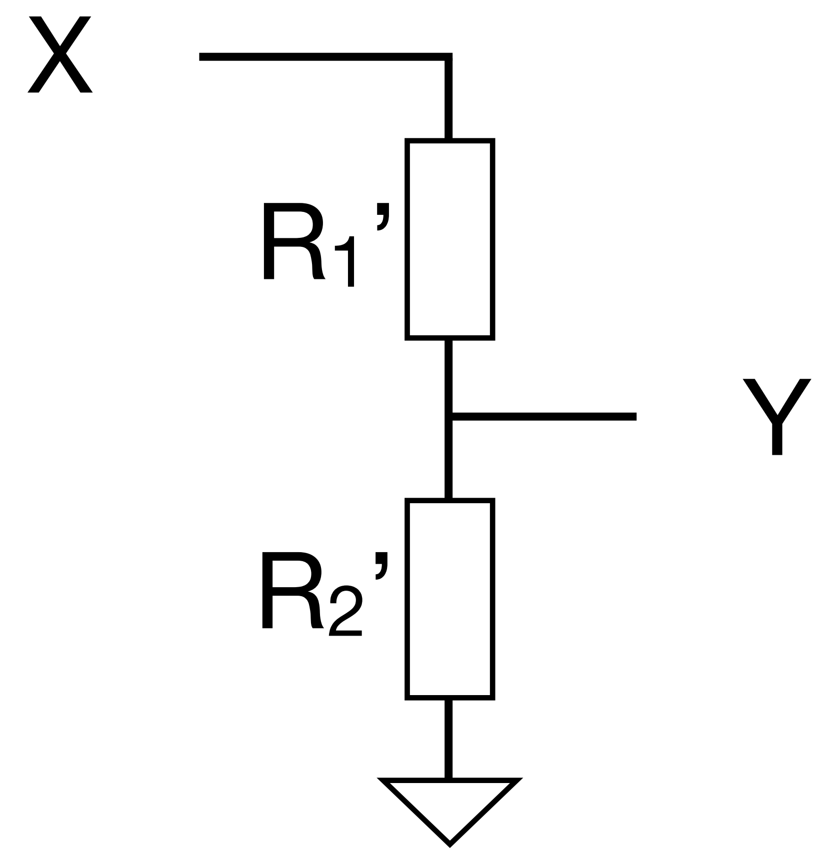
- Voltage divider

$$\frac{Y}{X} = \frac{R_2}{R_1 + R_2}$$

- Transfer set by <u>ratio</u> of resistors
    - Ratios more accurate than single values!



Mismatch influence?



- We wish $\frac{Y}{X} = \frac{R_2}{R_1 + R_2} = \frac{1}{r}$
  - so $R_1 = (r - 1) R_2$
- Assume worst-case deviation d:
  - $R_1' = (1 - d) R_1$
  - $R_2' = (1 + d) R_2$
- Then: $$\frac{Y}{X} = \frac{R_2'}{R_1' + R_2'} = \frac{R_2}{\frac{1-d}{1+d} R_1 + R_2}$$ $$= \frac{1}{\frac{(1-d)(r-1)}{1+d} + 1} \approx \frac{1}{\frac{r-1}{1-2d} + 1}$$
- For large r, a ±1% tolerance can affect ratio by ±2%

Example, again

- We wish Y / X = 100
  - ß = 1 / 100 = 0.01
- We wish gain error to be < 1%
  - …
  - 80 dB gain up to max frequency
- But also errors in ß due to passives!
  - Distribute error budget
  - 0.5% + 0.5%, or cheaper alternative

Reflection…

- How design to a given performance …
  - … with a given precision?
- Start with feedback network
  - Design/select amplifier that is “good enough”
- If not possible / affordable, split into several stages
  - Easier to provide 2 · 10x than 100x

Sources of variability

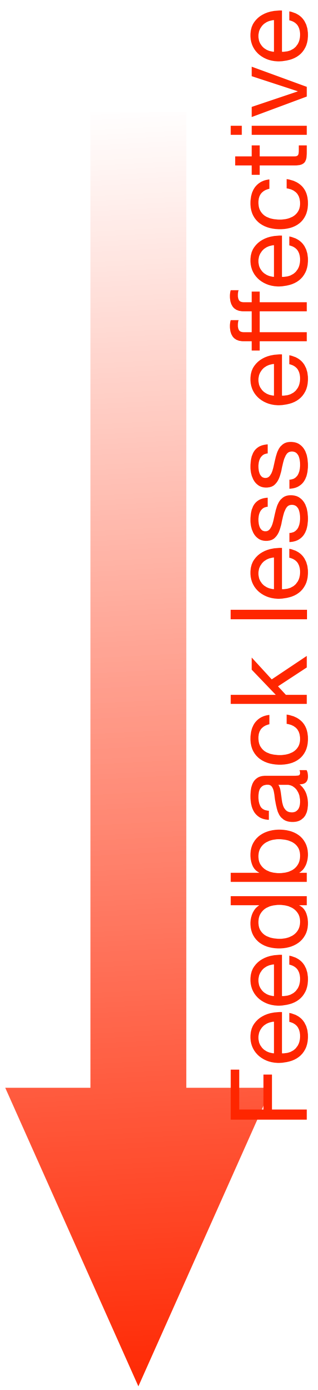
- "PVT":
  - Process (addressed through matching)
  - Voltage
  - Temperature
- "All" these affect active circuits more than passive ones
  - <u>Ratios</u> of passives especially stable

Other performance measures

- Voltage gain may be stabilized with passive feedback
- Current gain: other topologies, similar calculations
- Frequency-dependent gain: no simple ratios (how match R vs C?)
    - Week 4.
- Input/output impedances?
    - Also possible with two feedback loops

Time scales

- Static / years
  - Process corners, lithography, dopants
- Seconds
  - Supplies, temperature, 1/f noise
- Microseconds
  - IR drops, temperature gradients
- Picoseconds
  - IR drops, jitter, thermal noise



Summary

- Use feedback to control variability
  - Hide variations in active parts
  - Improvement set by gain given up
- Use ratios of passives to determine performance whenever possible
- Design feedback network first
  - Then select or design active parts:
    - How high open-loop gain and at what frequencies?
    - May have to spit in more stages if high requirements