Noise shaping

DAT116, 20181217

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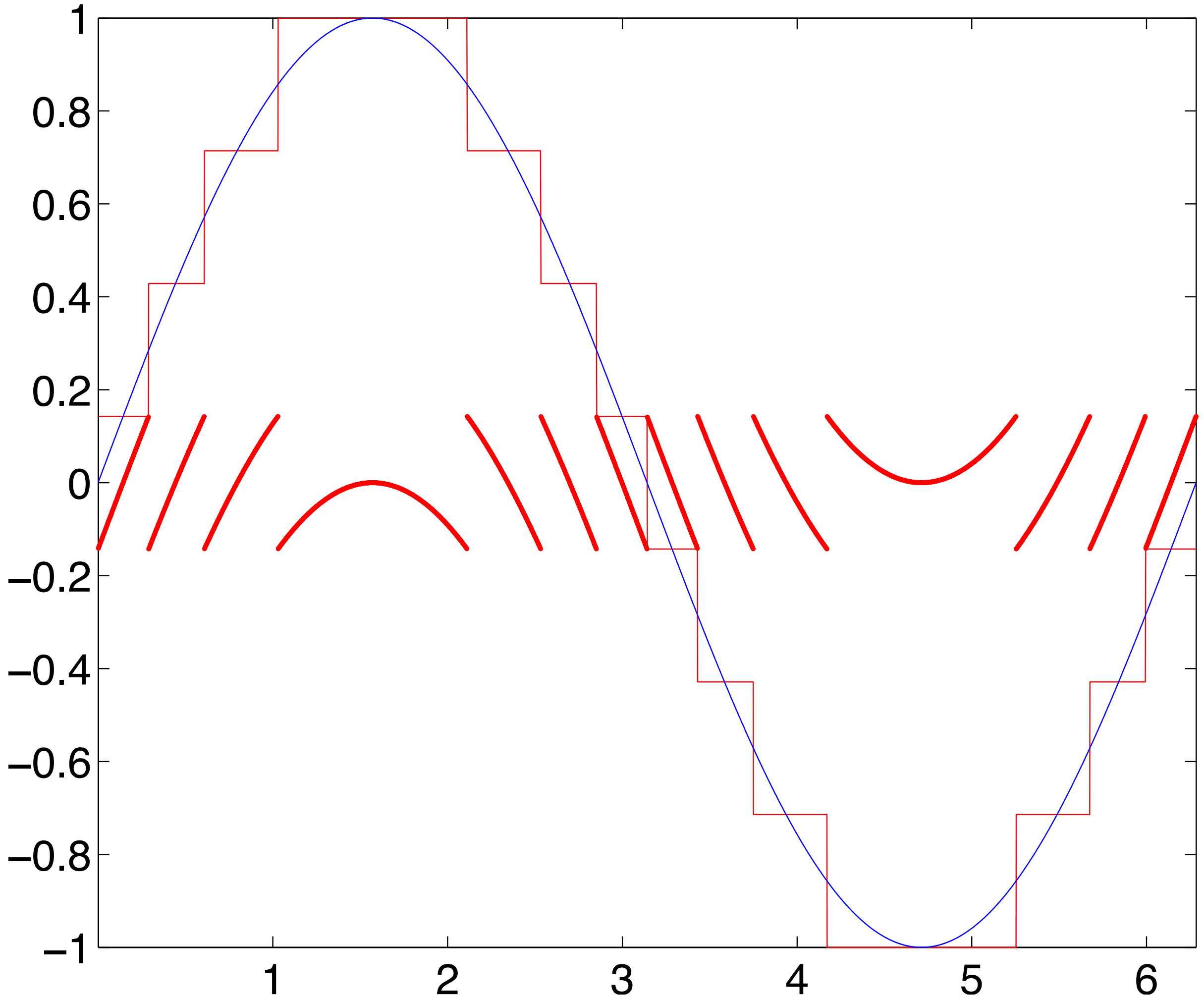
What?

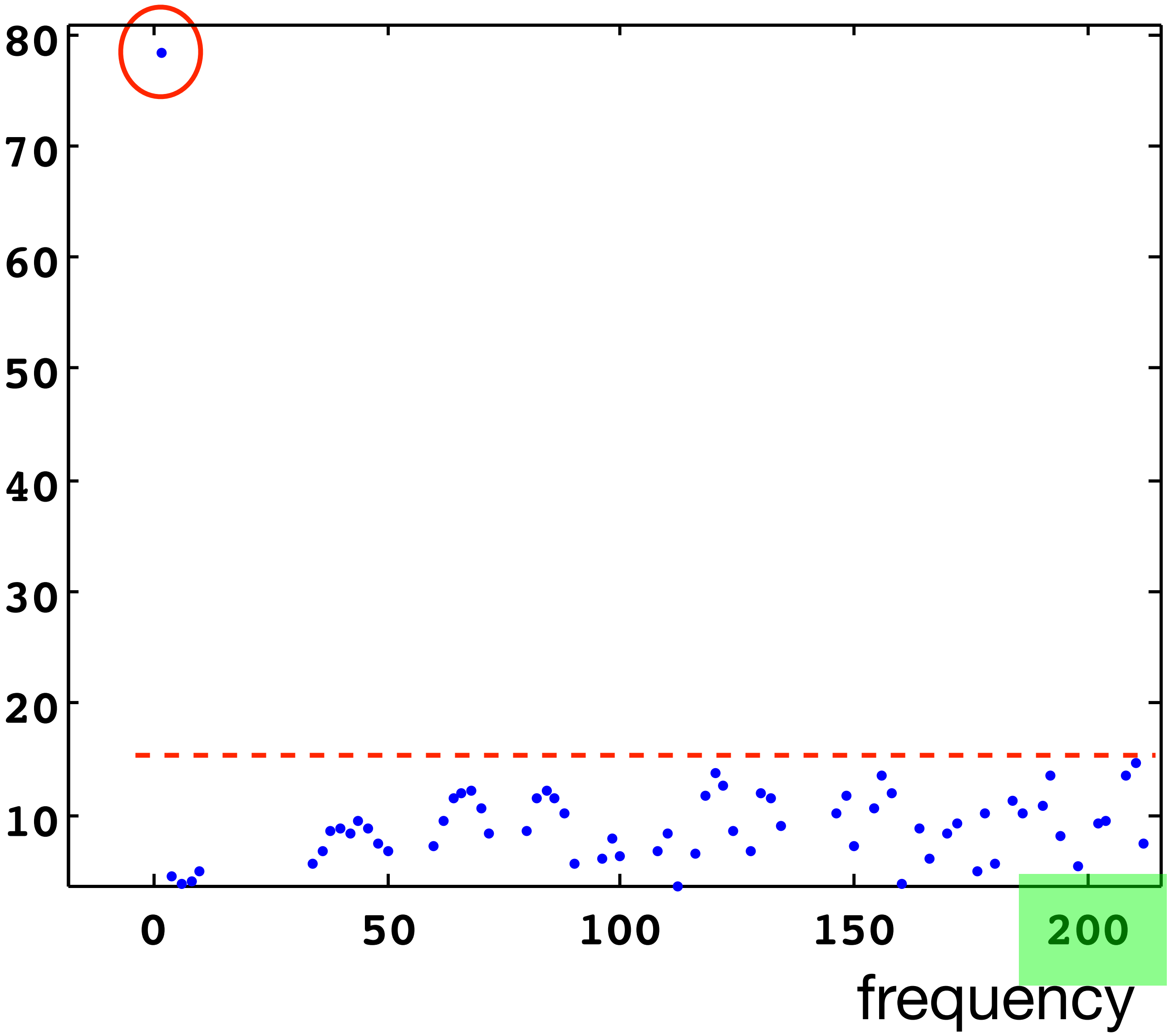
- Improve SNDR beyond 6.02 N + 1.76 dB

How?

- Force errors out of band-of-interest; filter

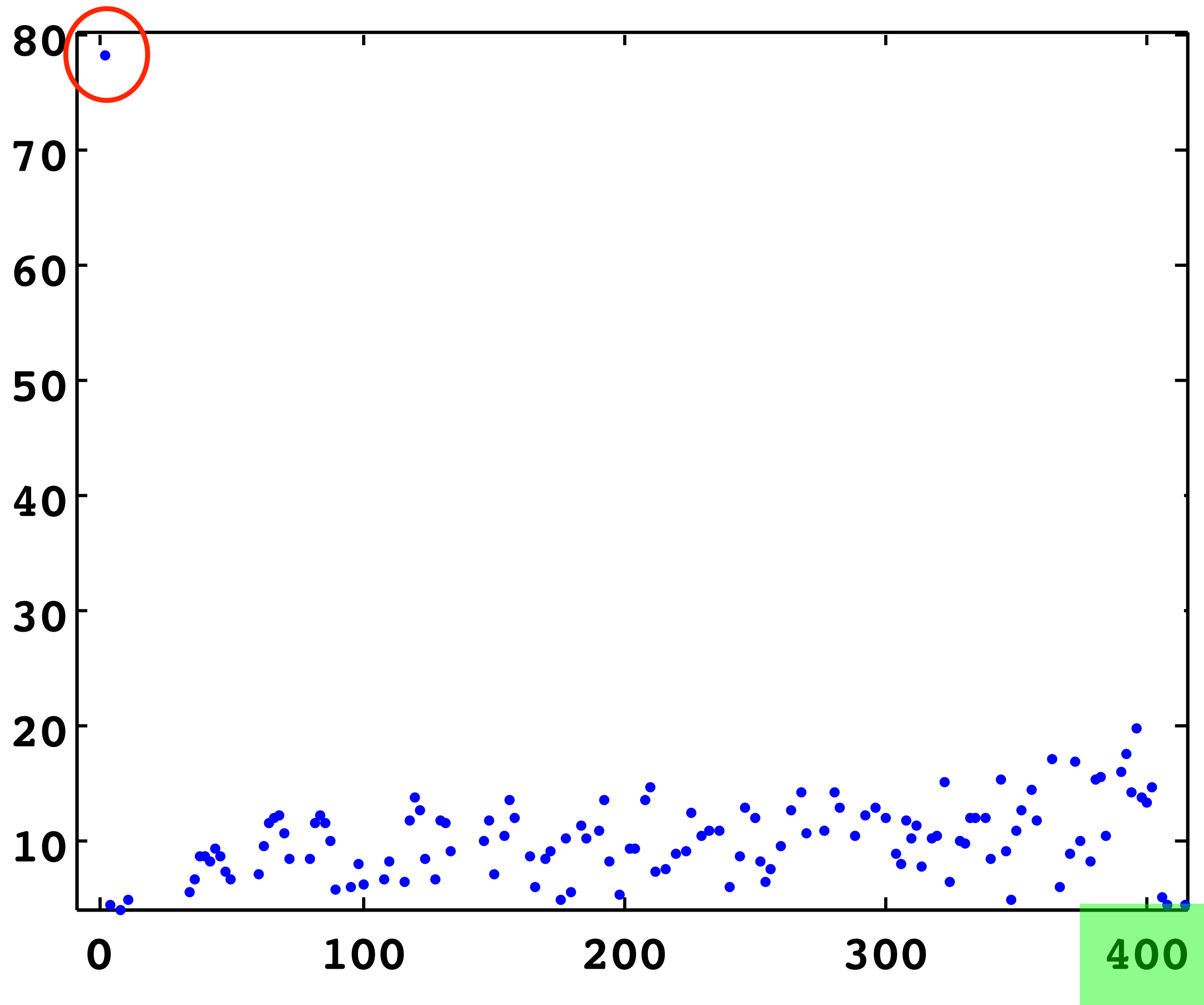
Quantization with Nyquist sampling





- SNR = 6.02 N + 1.76 dB (full-scale sinewave input)
- Noise power independent of sample rate

Oversample by factor 2

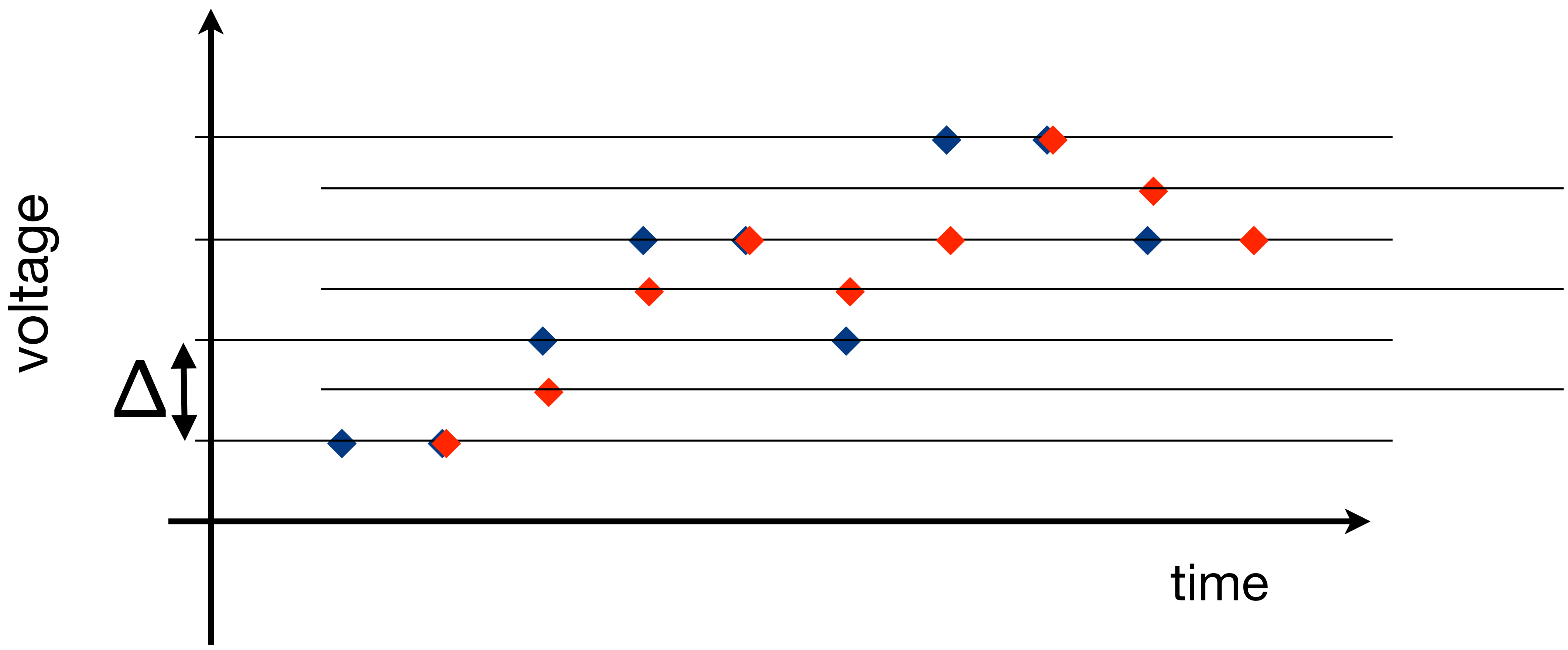


Here, OSR = 2

Design parameter!

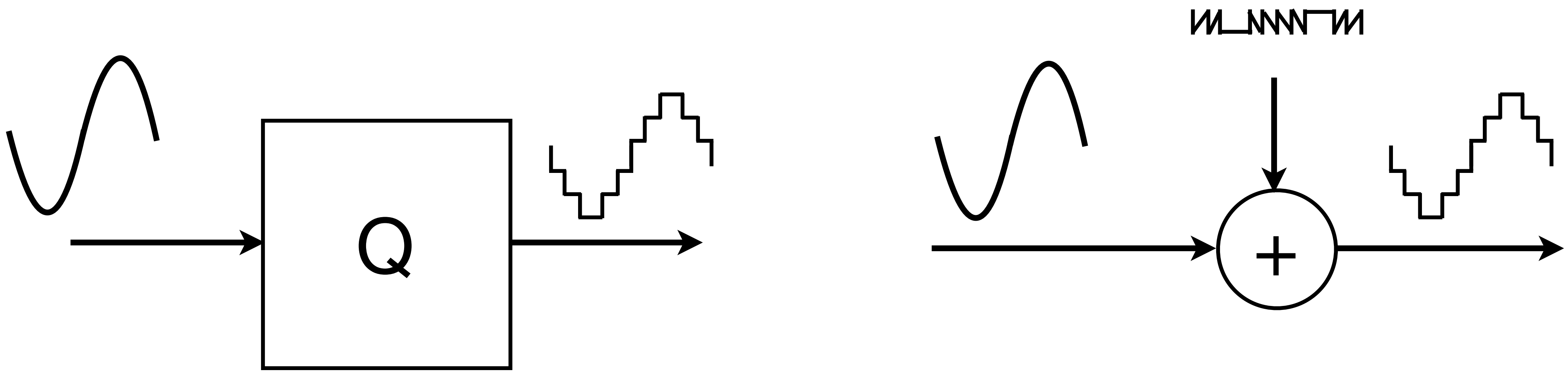
- Same total noise power ($\Delta^2/12$), but twice the frequency bins
- Post-sampling low-pass DT filter; removes half the noise power
- Then downsample by 2x by dropping every other sample
- Improve SNR by 3dB per factor 2 of oversampling! Yay!

Relation with resolution



- Intuitively:
  - Filter by $(1 + z^{-1}) / 2$ [simplest LP]
  - "New" "levels" introduced
  - Reduced Δ, so reduced noise!

Linear converter model

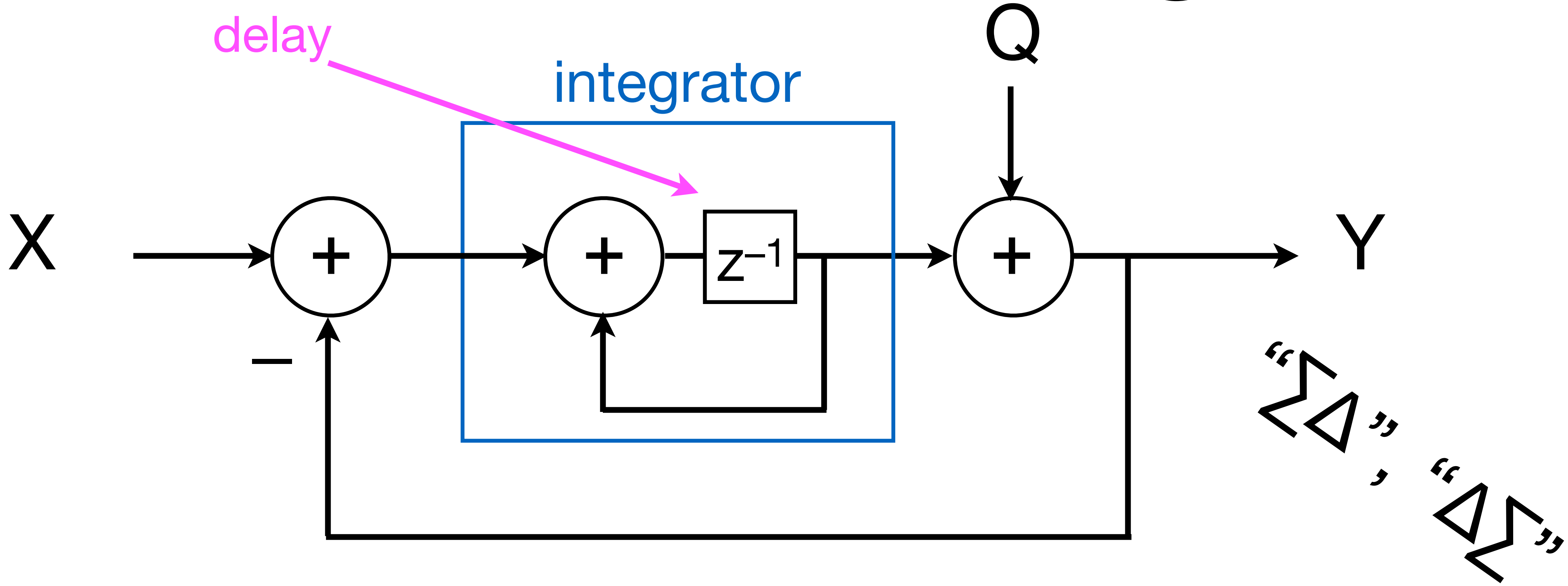


- Quantization is a non-linear process
- May be modelled as <u>noise addition</u>
  - Linear system!
- Assume added noise is white and uncorrelated with signal
  - OK if resolution is high

Noise shaping

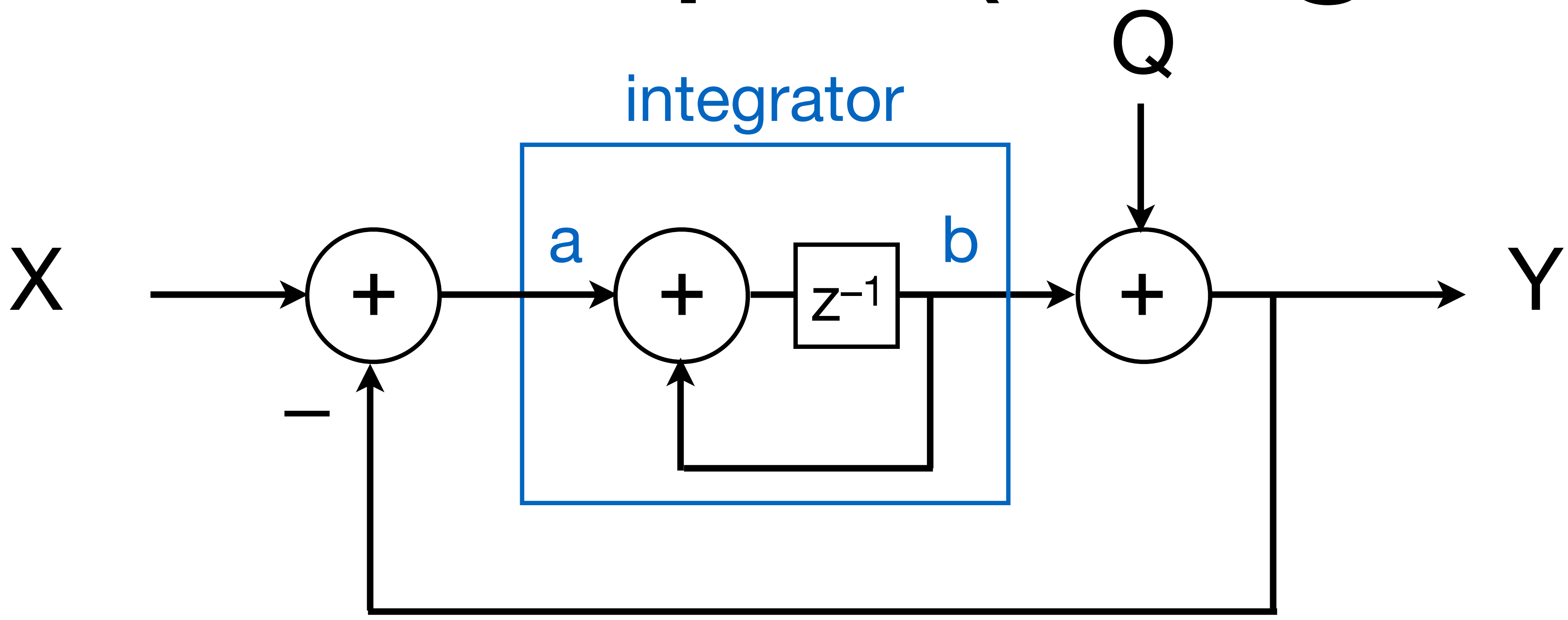
- Idea: use oversampling and push <u>more</u> of the noise out of signal band!
  - Feedback loop w/ linear filter
- Use digital post-filter (DT!) to suppress out-of-band noise
  - More noise removed than with “straight” oversampling
- SNR improves more than 3dB per x2 !
  - More effective for larger oversampling ratios

Simple example (integrator)



- Linear model: superposition, transfer functions
  - X to Y (Signal Transfer Function, STF)
  - Q to Y (Noise Transfer Function, NTF)
- Use z transform

Simple example (integrator)



integrator: $b = z^{-1}(a + b)$; $b(1 - z^{-1}) = a z^{-1}$

$H(z) = b / a = z^{-1} / (1 - z^{-1})$

$Y = Q + (X - Y) z^{-1} / (1 - z^{-1})$

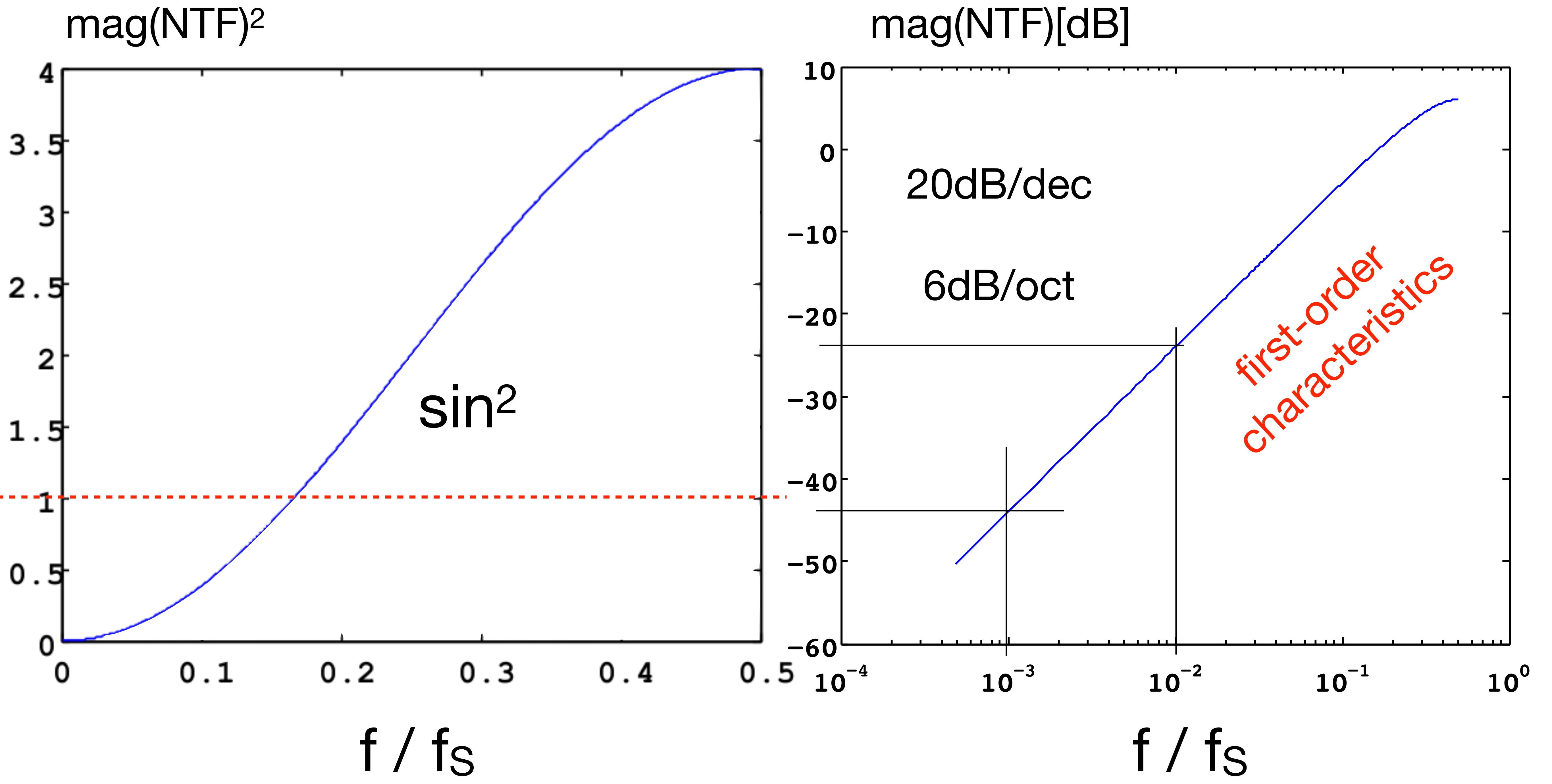
$Y = Q + X z^{-1} / (1 - z^{-1}) - Y z^{-1} / (1 - z^{-1})$

$Y = (1 - z^{-1}) Q + z^{-1} X$

STF: z^{-1} delay

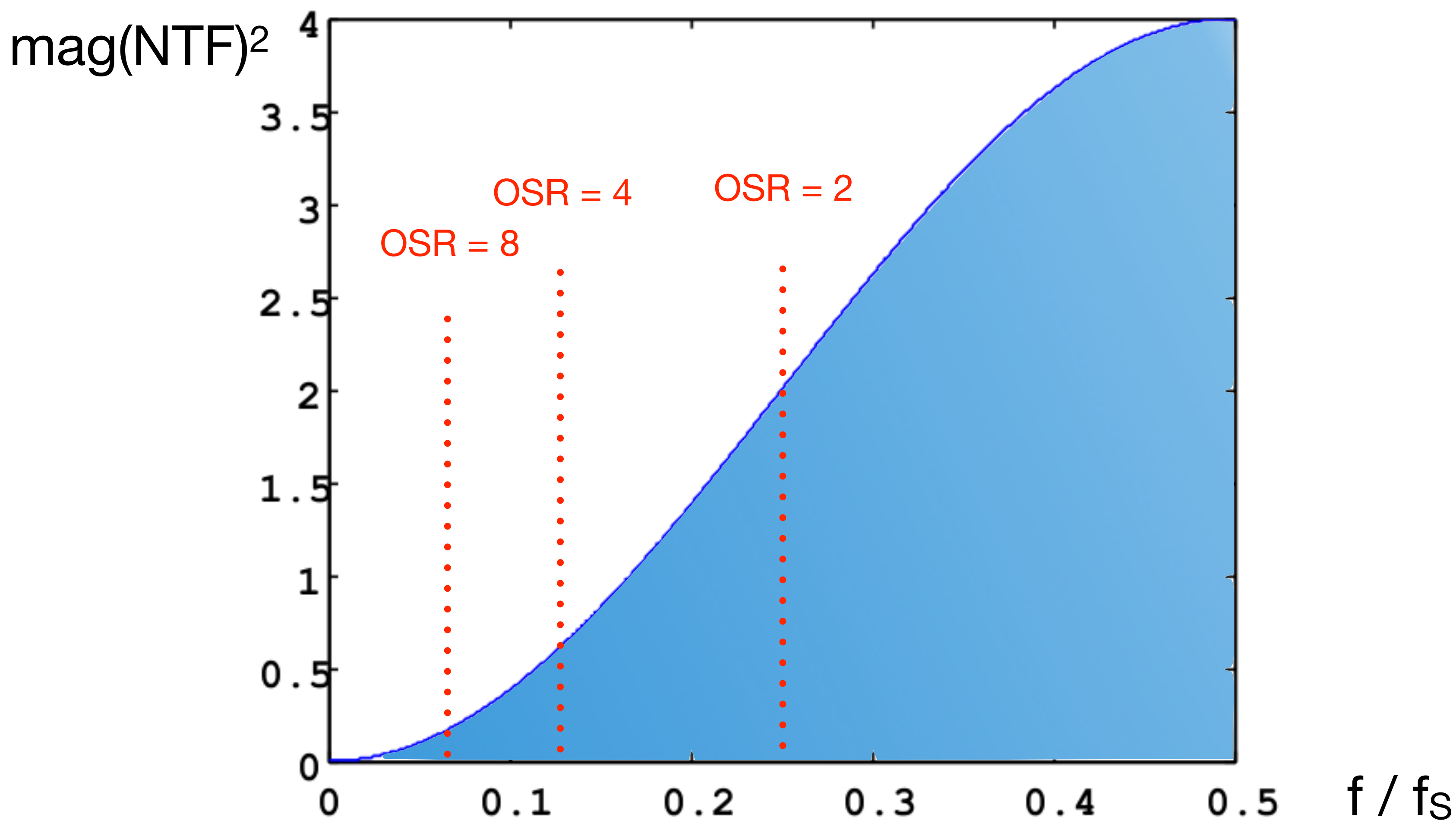
NTF: $(1 - z^{-1})$ highpass

NTF plots



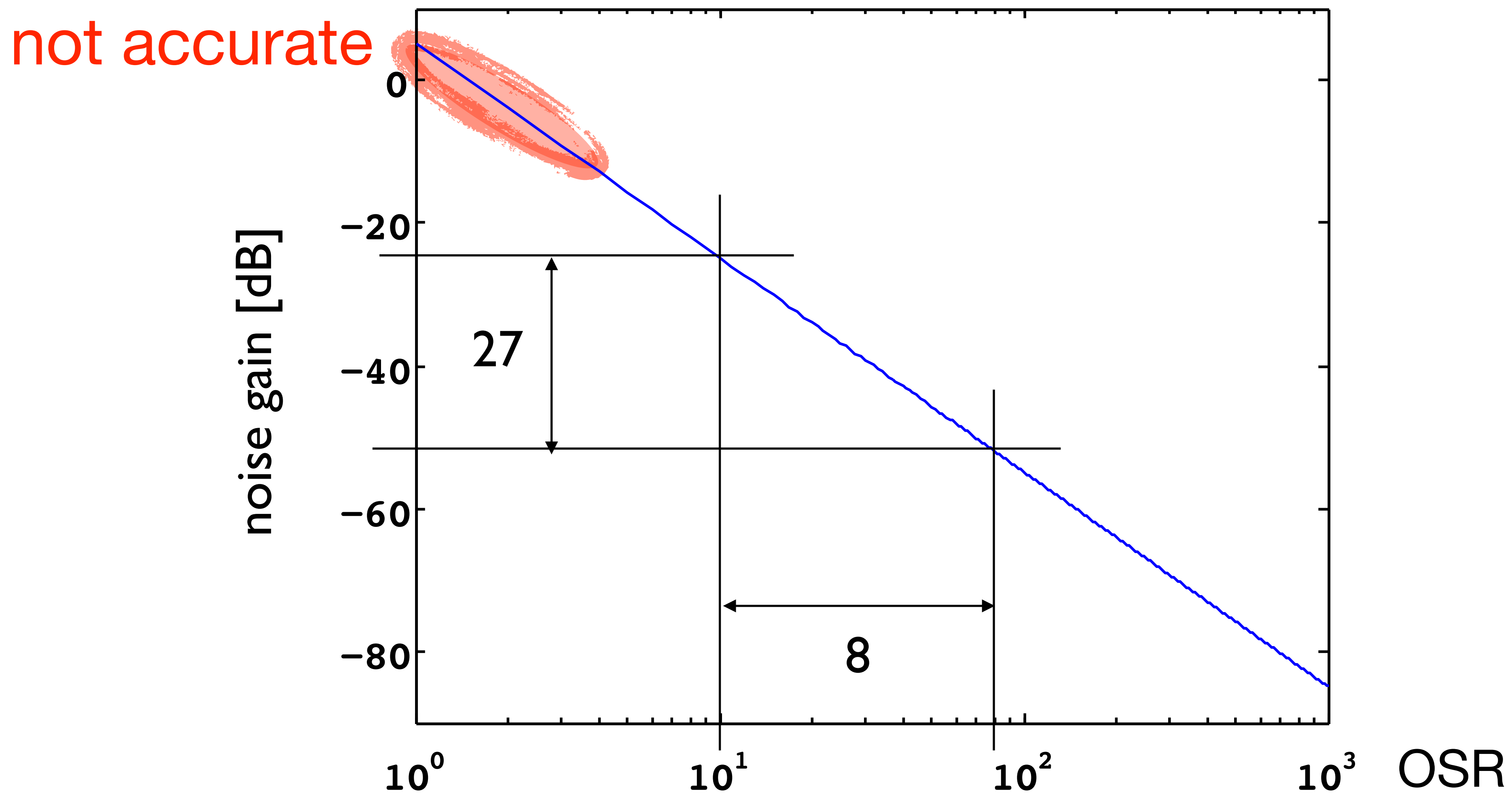
<u>Total</u> noise power is increased!

…but filtering saves us



- Small x: $\sin x \approx x$; $\sin^2 x \approx x^2$
- $\int x^2\,dx \sim x^3$; remaining noise ~ OSR^{-3}

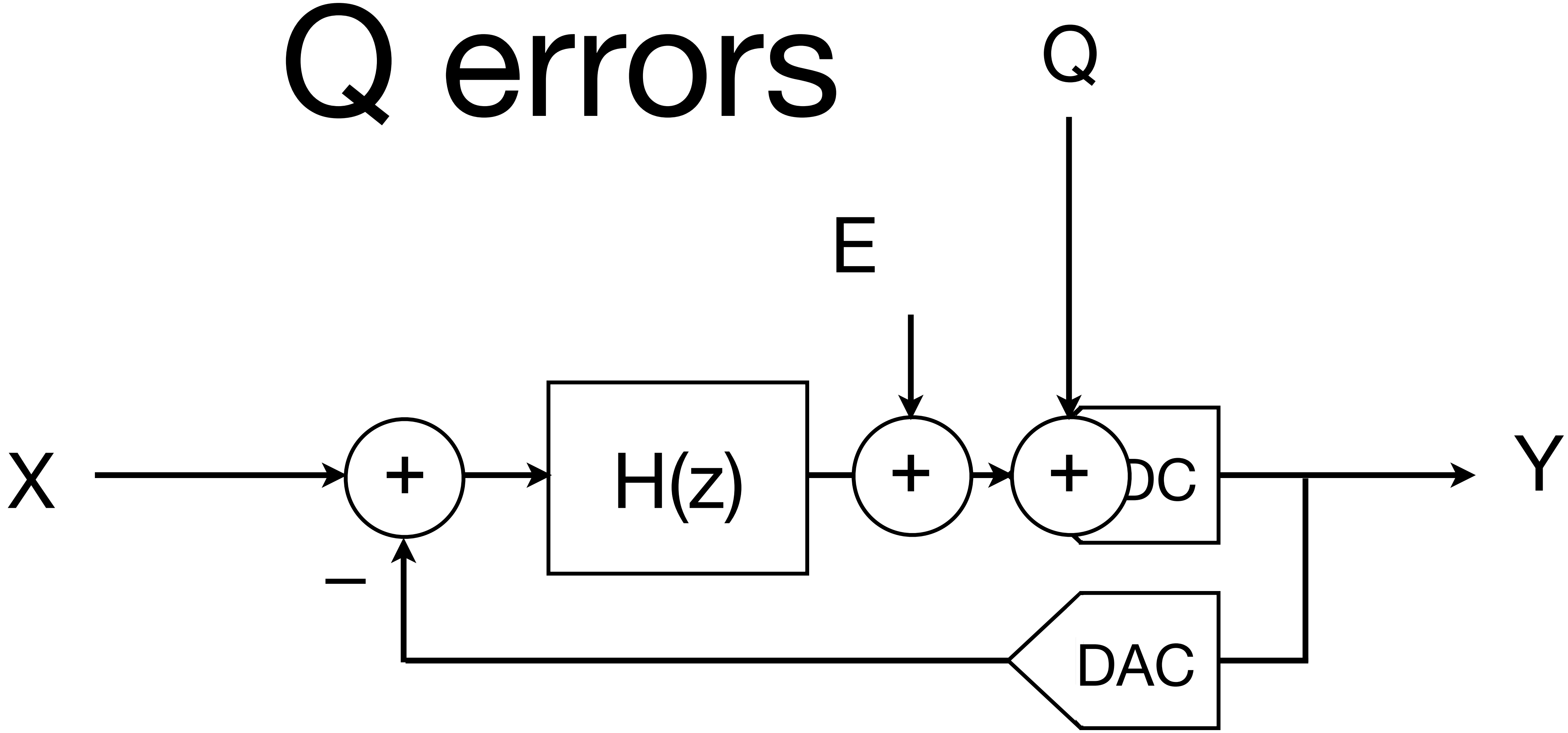
SNR vs oversampling ratio?



- Assume OSR is M (reasonably large)
- Then part of noise which falls in passband is $(\pi^2 / 3)\ M^{-3}$
- 9 dB SNR improvement for each doubling of OSR

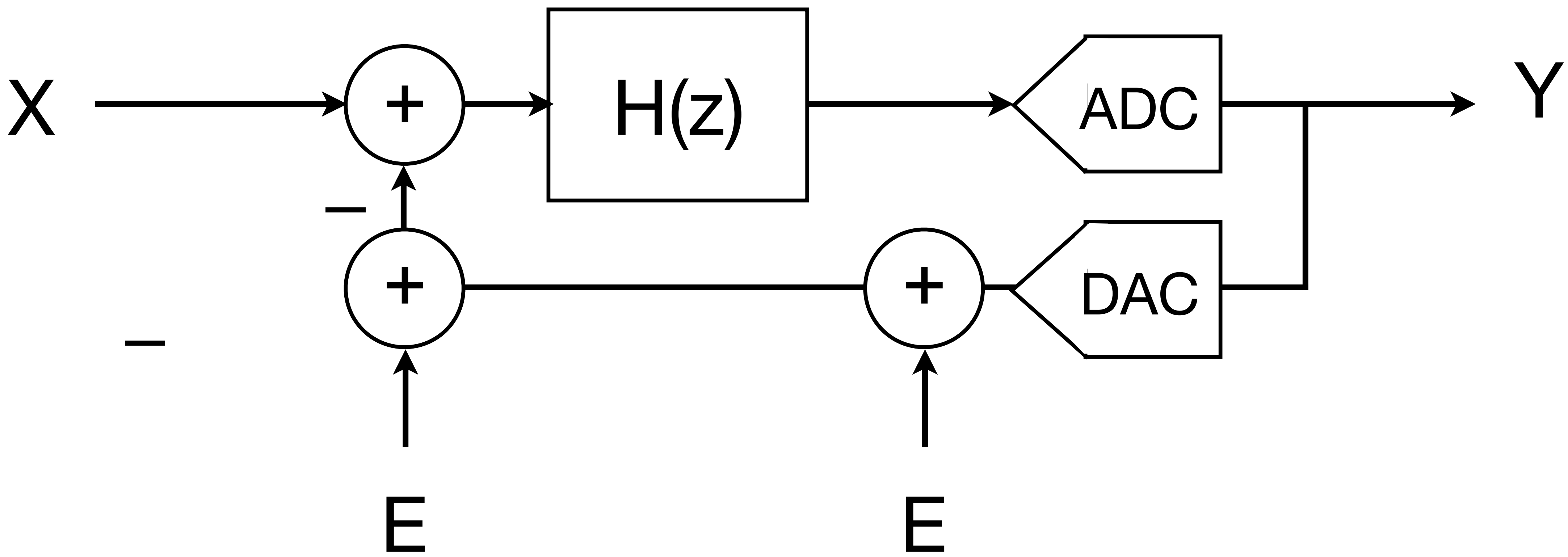
3dB w/o feedback

Q errors



- Additional quantizer errors (INL, DNL) indistinguishable from “ideal” quantization noise
  - Will be shaped out of band of interest
- Low-precision quantizer usable!

DAC errors



- DAC errors indistinguishable from input
  - DAC precision critical!

∑Δ features

+ Possible to improve SNR beyond quantizer capabilities
    - Get away with lower-precision quantizer for given SNR
    - Easy to implement in standard CMOS

– Low signal bandwidth

– High-speed circuits needed

– Higher latency due to feedback loop

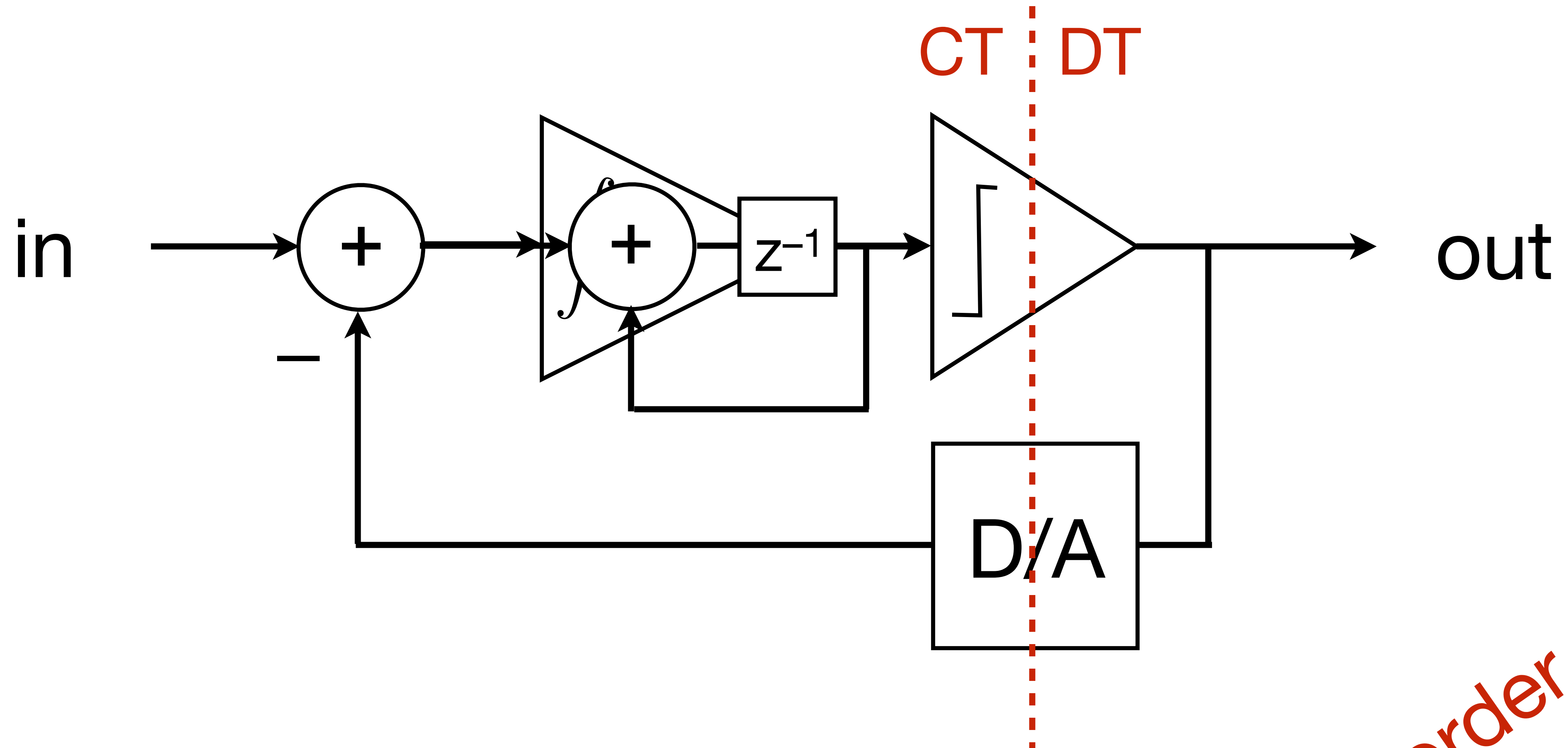
± No direct relation between input and output

Extreme ∑Δ: 1-bit quantizer

- Logical extreme 😀
- No DAC, ADC nonlinearity to worry about
- Needs large OSR for useful performance
  - “Base” SNR is less than 8dB!

- Some assumptions violated!
- Linear model not dependable

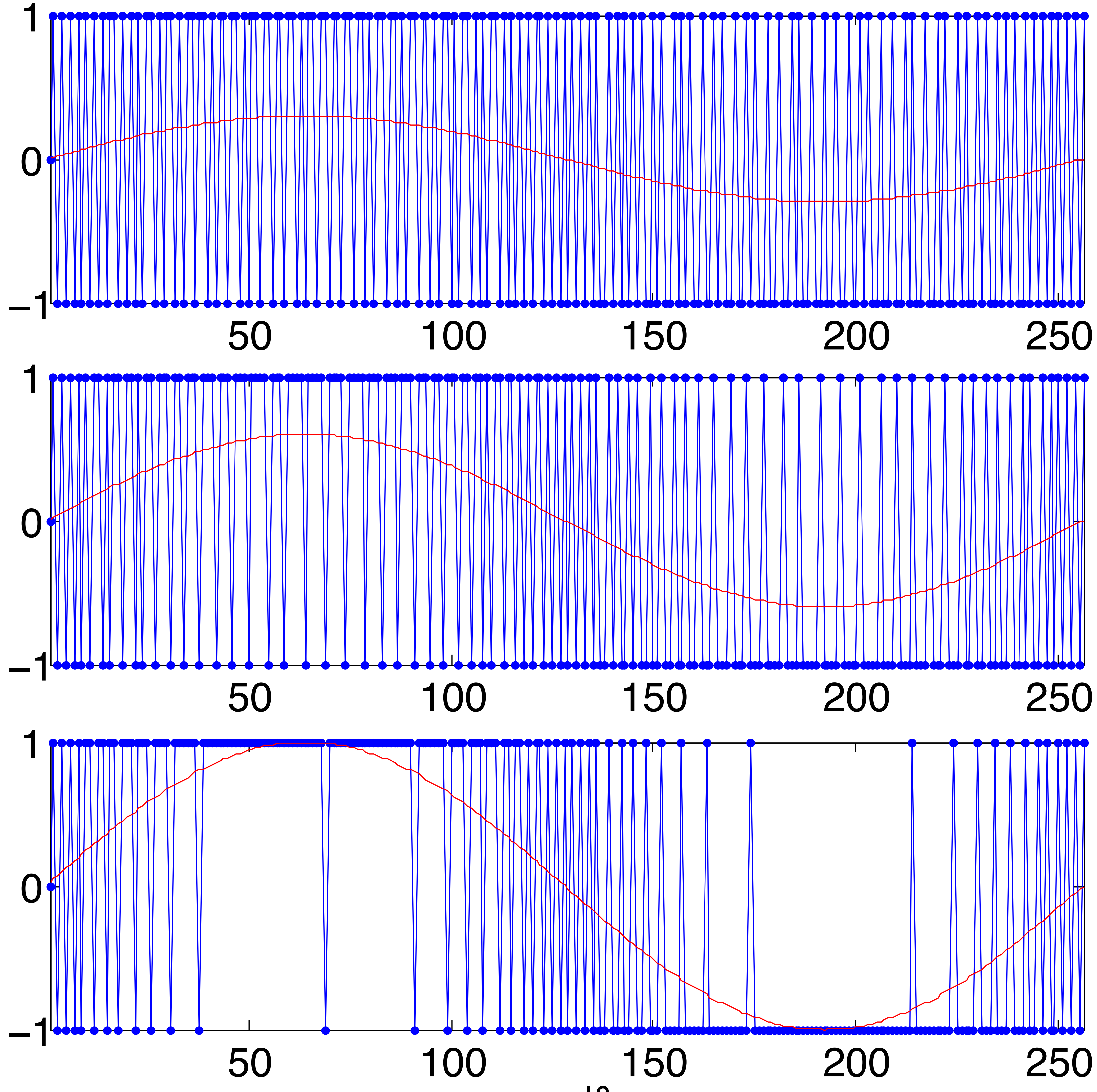
One-bit block diagram



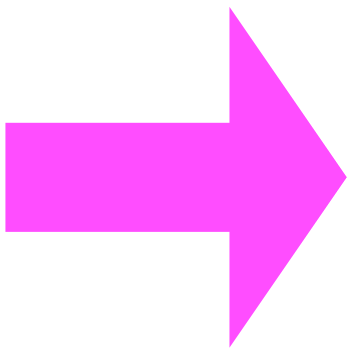
CT/DT border chosen per application

- Integrator, comparator, 1-bit D/A
- Error is integrated, compensated for
- Very simple hardware!

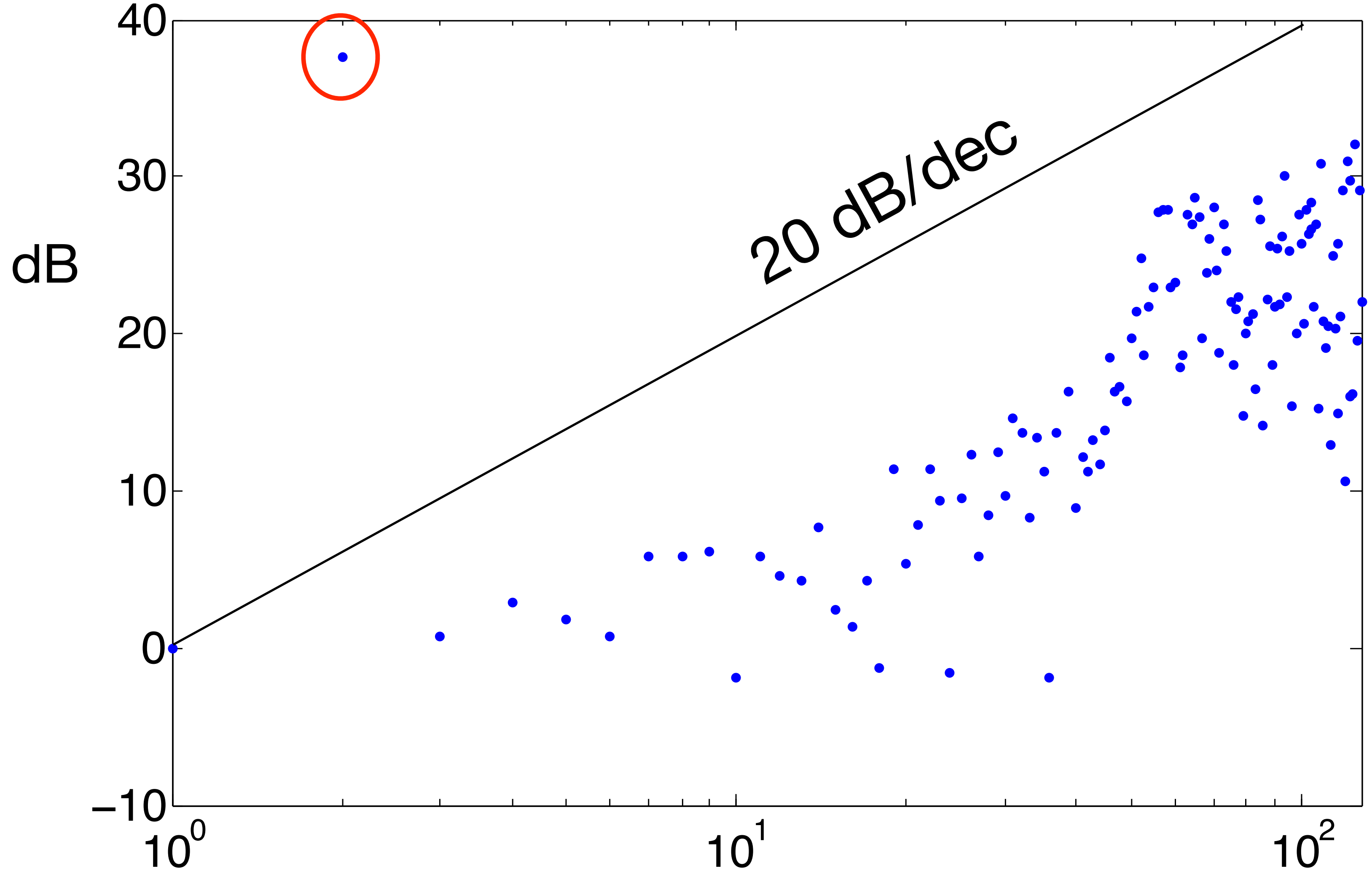
Time-domain behavior



Simulated behavior

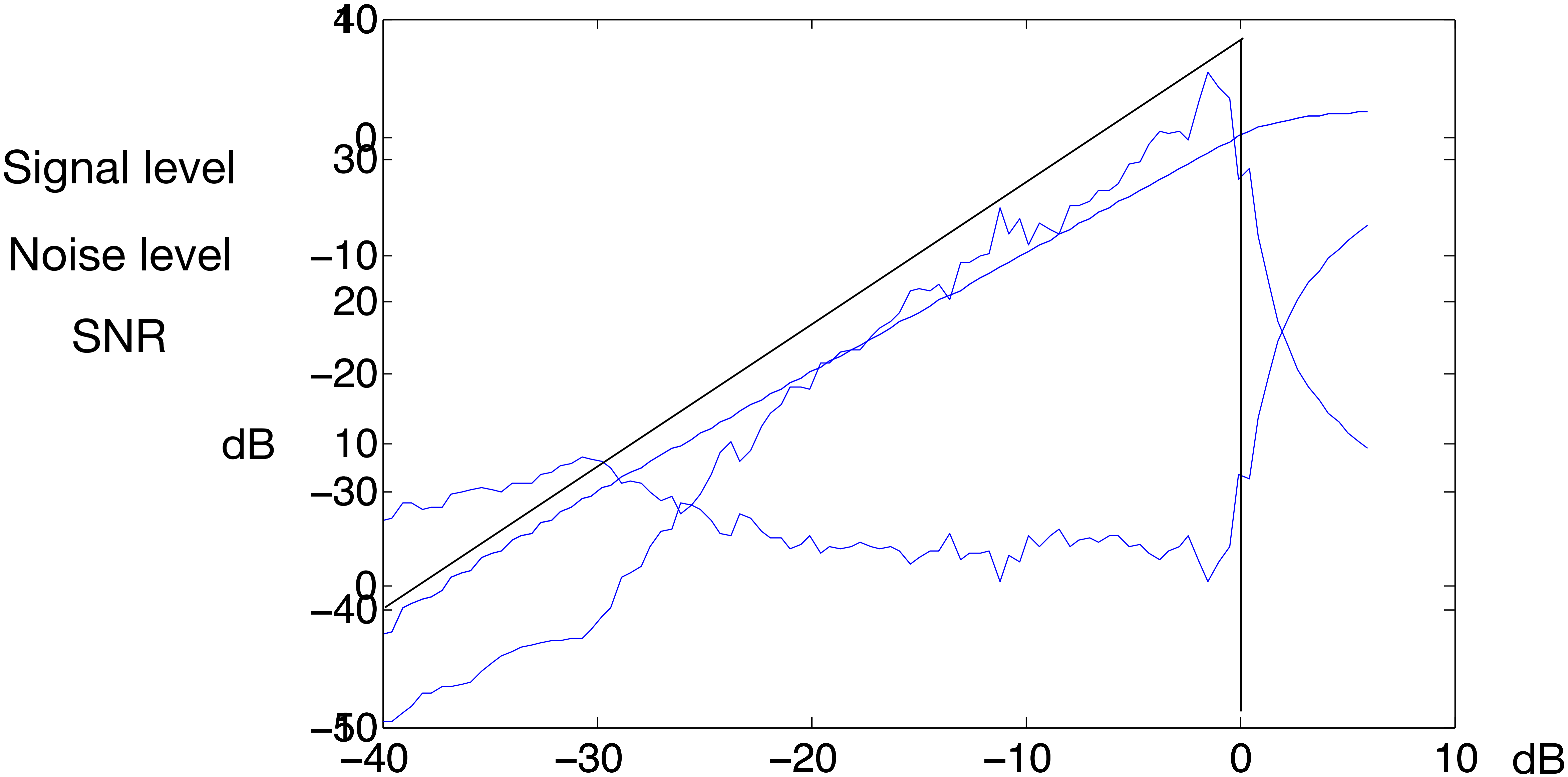


Frequency-domain behavior



- Obvious high-pass noise character!
  - Approximates first-order characteristics

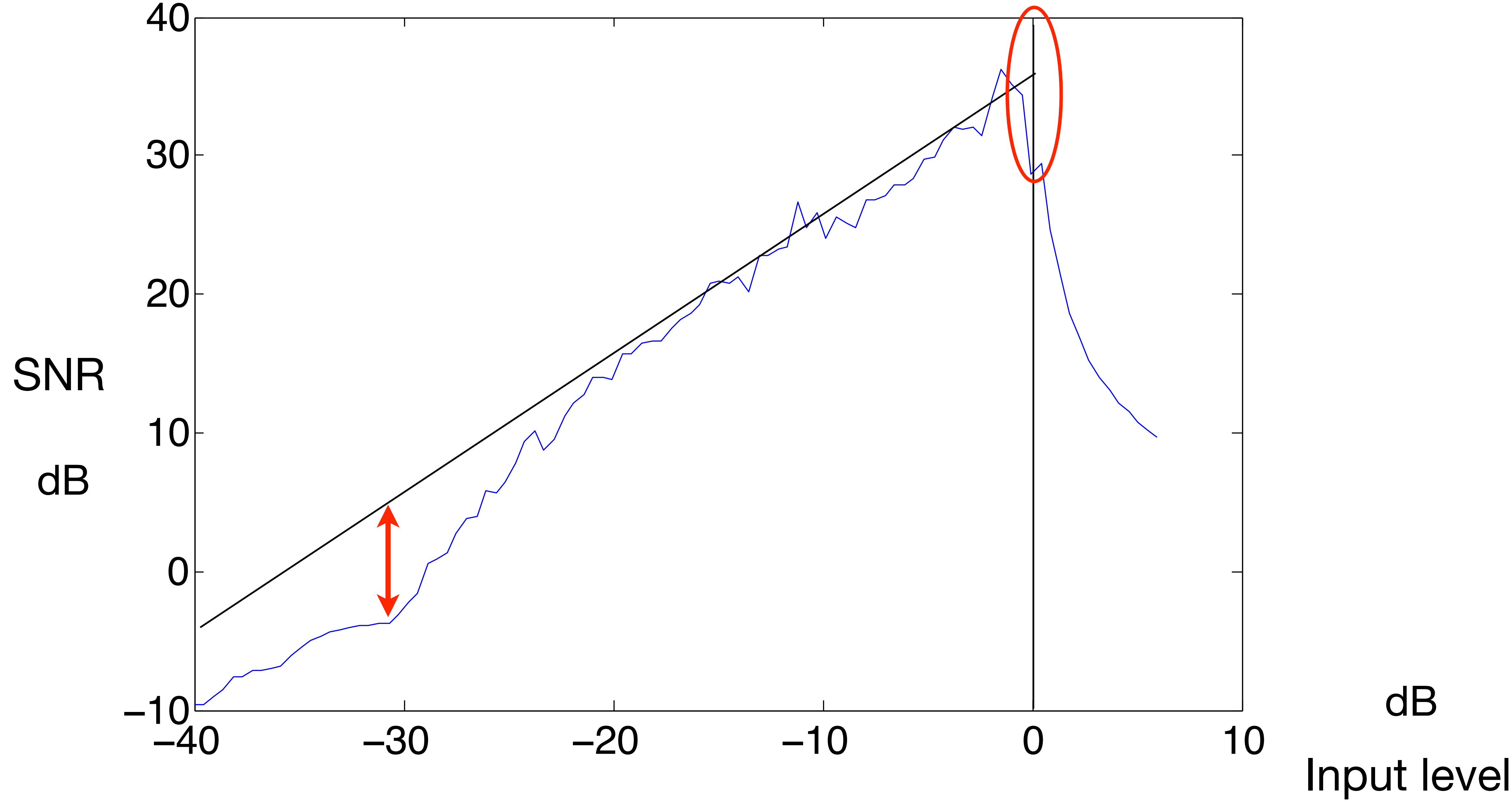
S, N, SNR vs input level



Input level

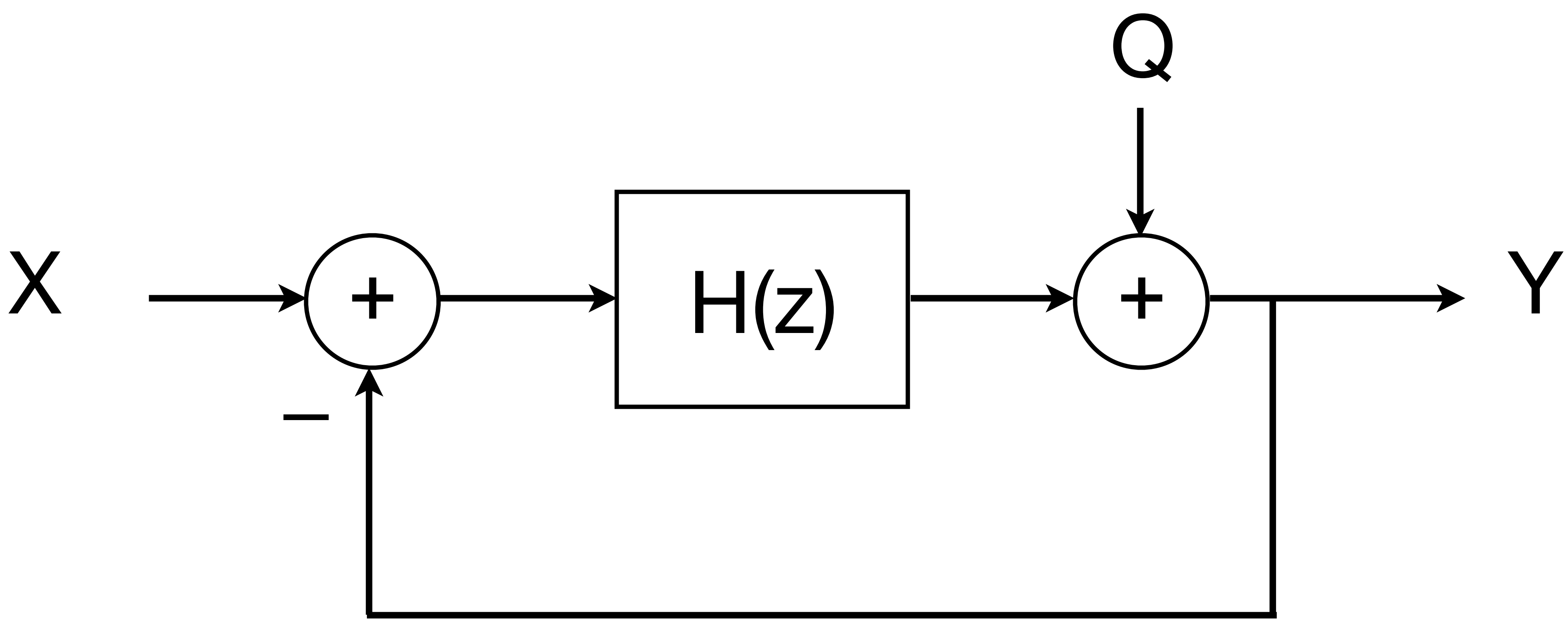
- Here, OSR = 16 = 2^4
  - <u>Theoretically</u> 6 + 2 – 5 + 4 · 9 dB = 39 dB
  - Worse in practice

Artifacts



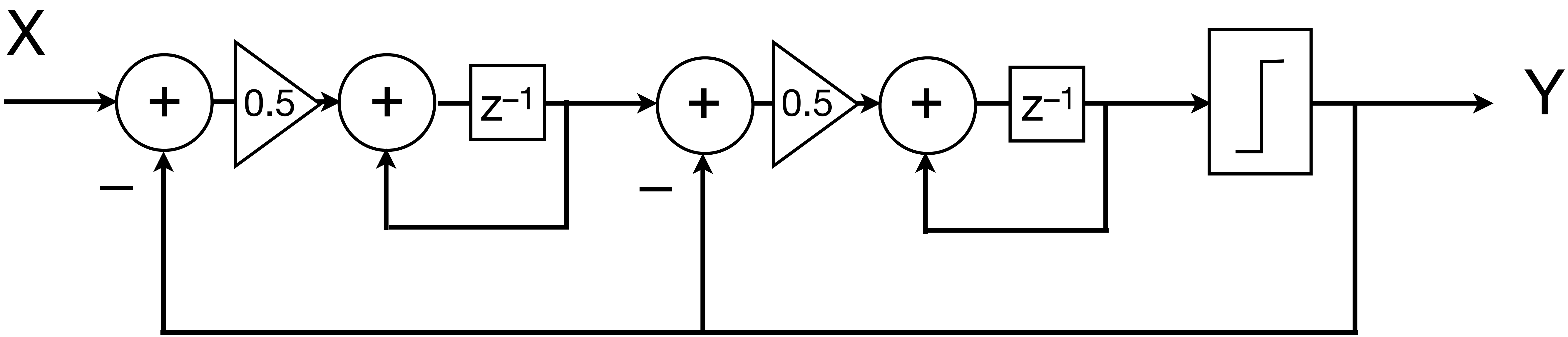
- SNR starts falling before input reaches full scale
- Extra noise at low input levels
  - Limit-cycle "tones" (as will be seen in lab)

Higher loop orders



- Integrator = first order H(z)
- Possible to use higher-order H(z)
  - Push out more of the noise!

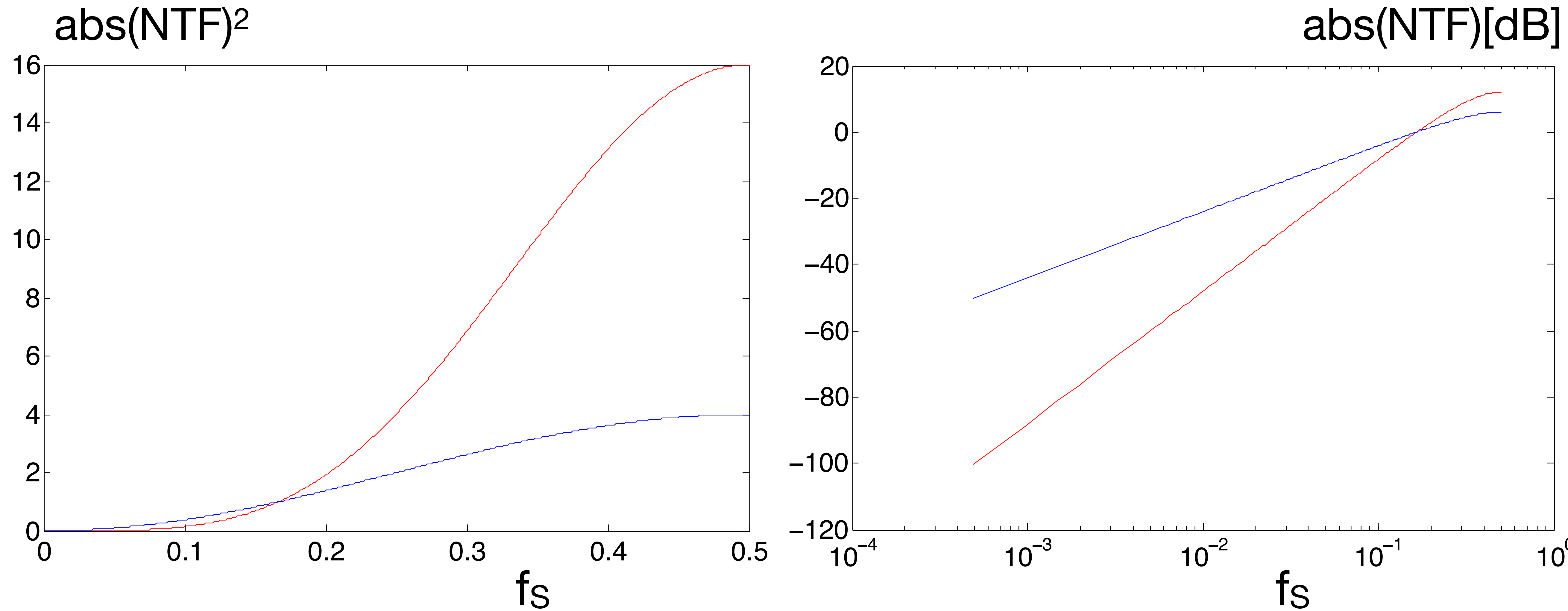
2nd-order ∑Δ loop



- $Y = X \cdot z^{-2} + Q\,(1 - z^{-1})^2$
  - Note squares!

- Simple loop structure; many alternatives exist

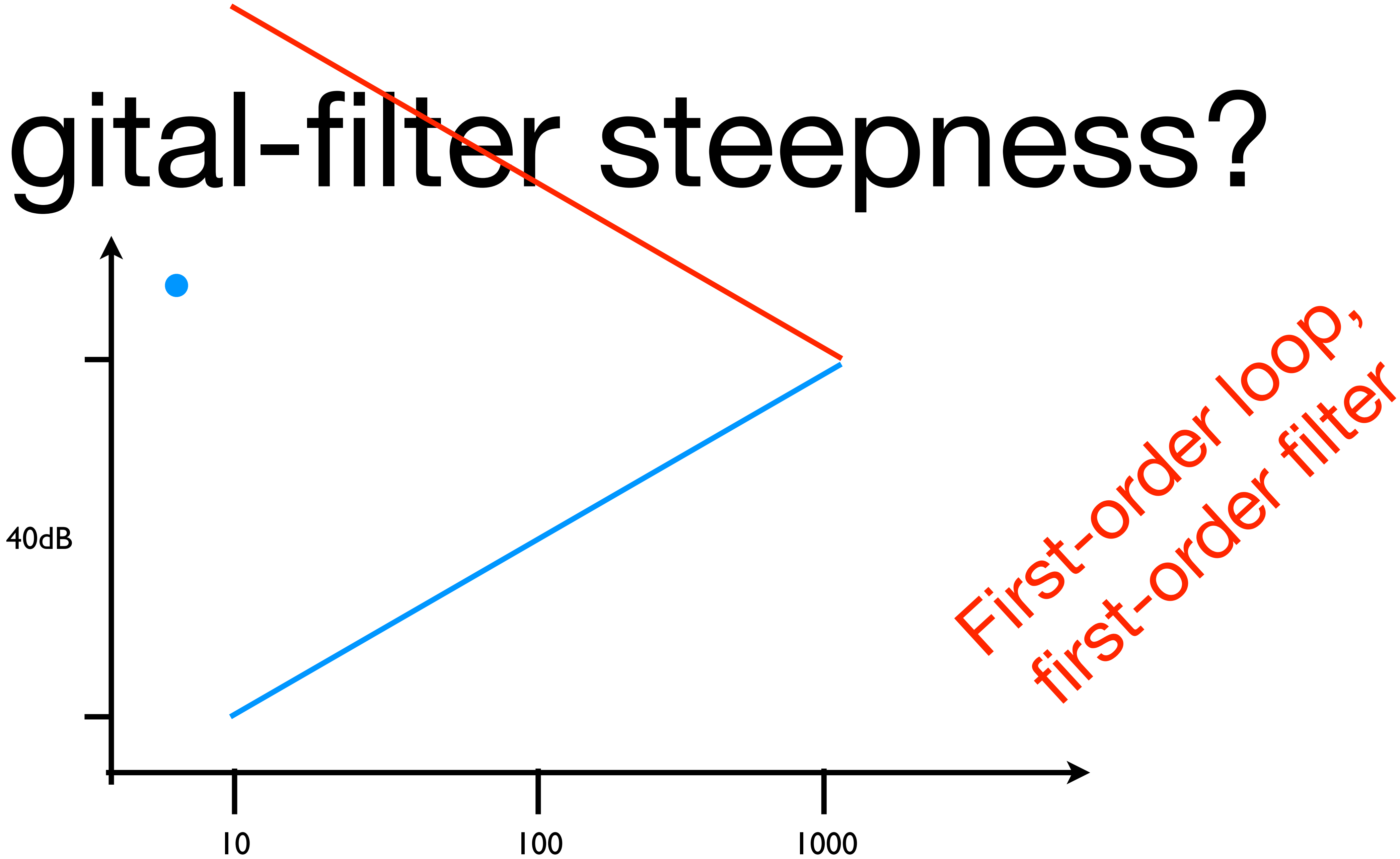
Ideal noise shaping



- Better suppression of in-band noise, steeper rise
- Higher <u>total</u> noise gain than for 1st order
- 15dB SNR improvement per doubling of OSR

Higher orders possible!

Digital-filter steepness?



- Noise rises by 20 dB / decade per loop order
- Filter must suppress noise <u>at least</u> as steeply!
    - ≥1st order filter for 1st order loop, etc

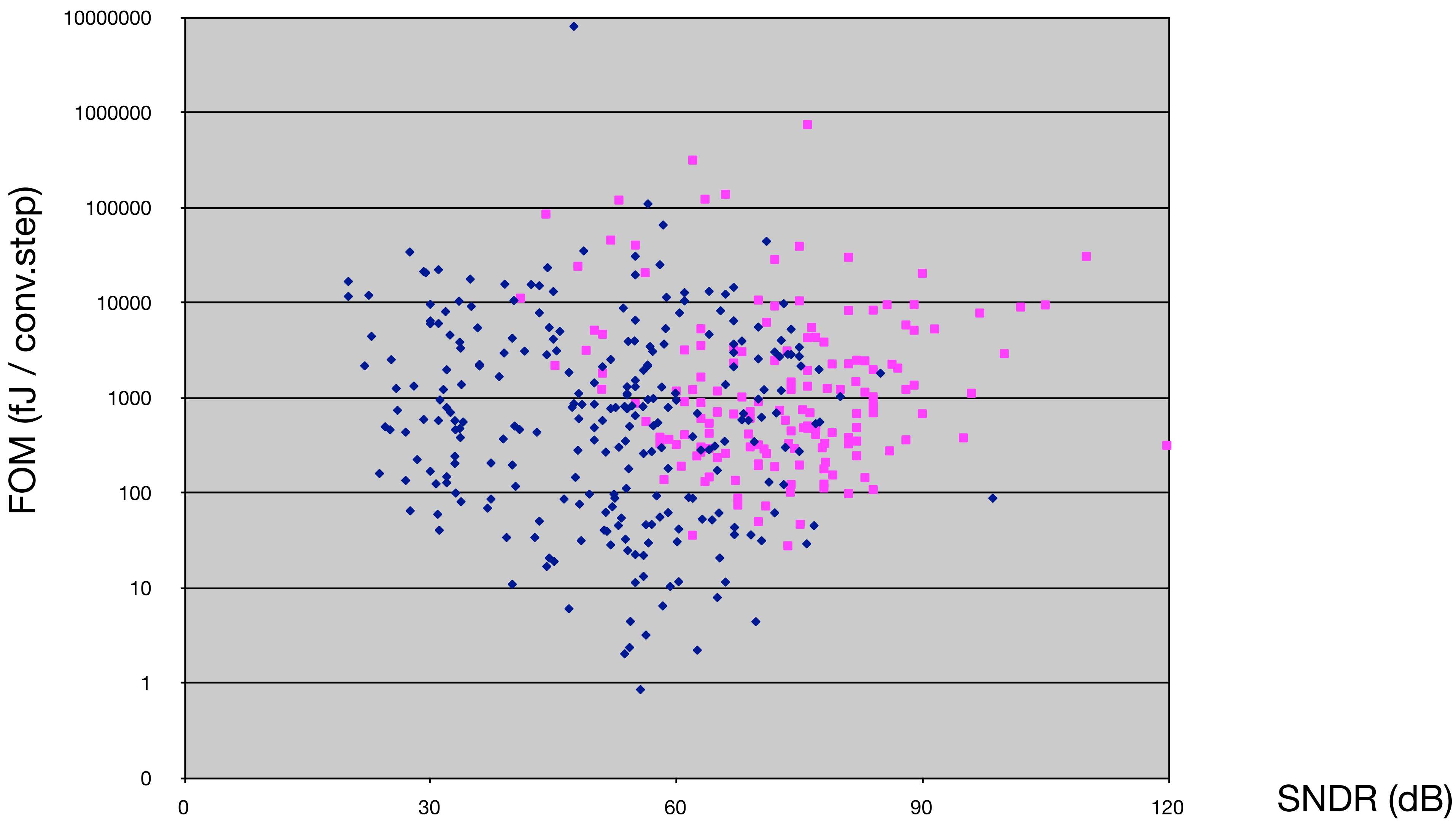
Sample rate reduction

- After low-pass filtering, typically reduce sample rate to Nyquist rate
  - For OSR = N, drop N–1 out of N samples
  - … or rather: don't even compute them in filter …
- Word length increase in filter
  - Averaging!
  - Longer words needed to support SNR

∑Δ design space

- Three ways to increase SNR:
  - Higher “core quantizer" resolution
  - Higher OSR
  - Better loop filter (higher order, better pole/zero placement)
- Stability issues for filter orders > 2
  - Possible to manage

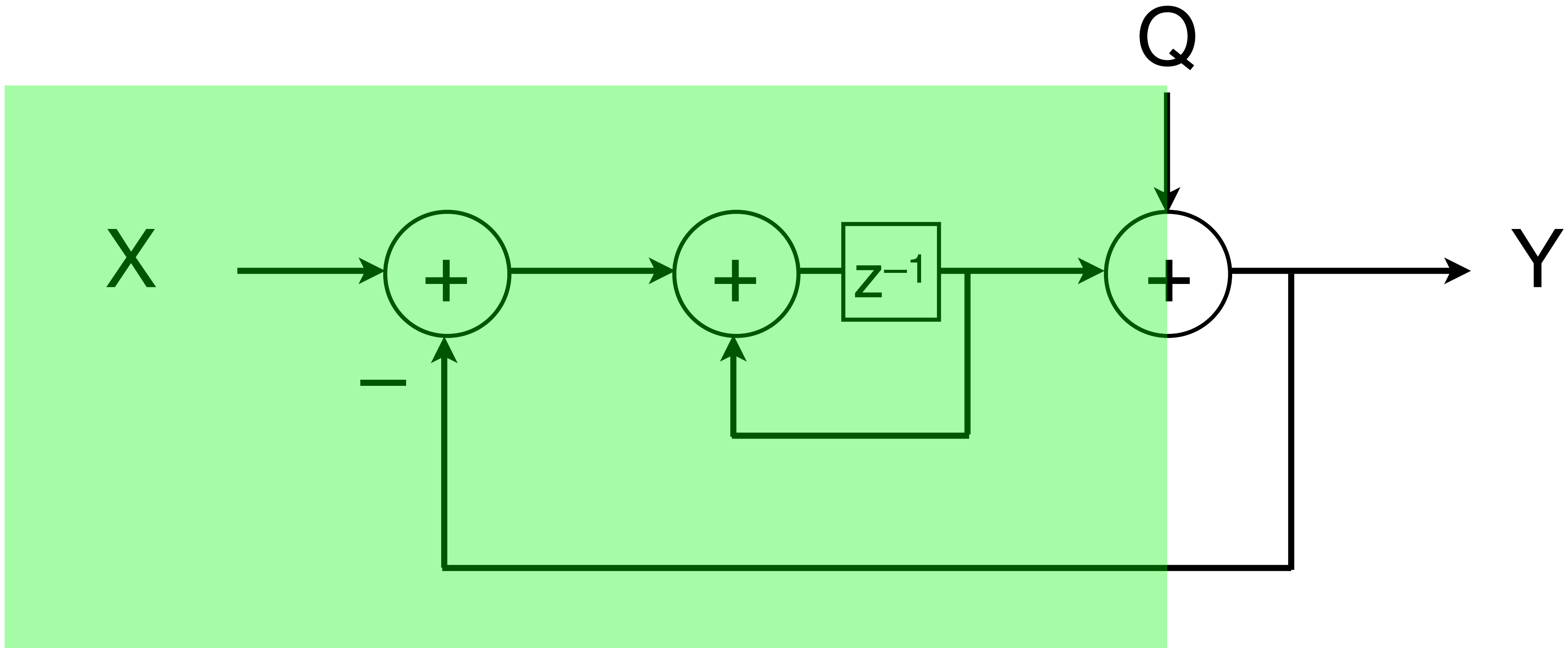
When useful?



- Magenta dots for oversampled ADCs
  - High SNDRs, not the very best FoMs
  - … and obviously not the highest signal bandwidths…

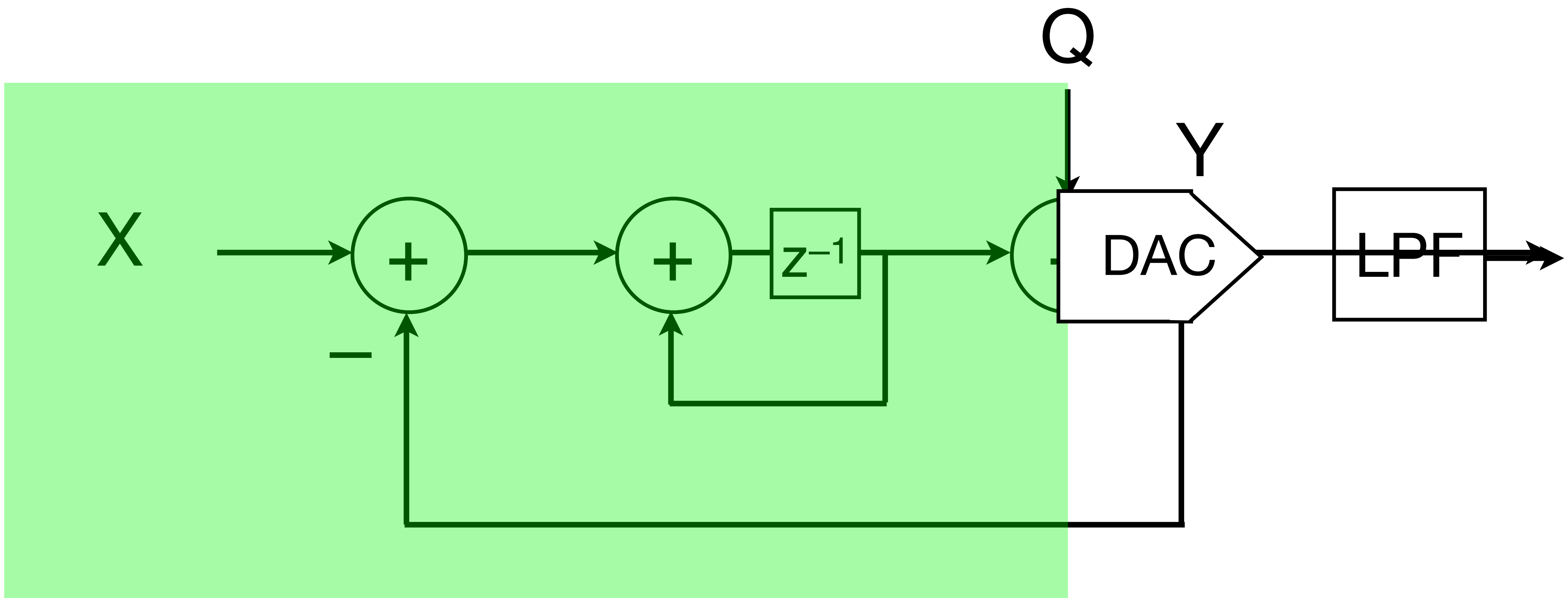
[Murmann data]

∑Δ DAC



- In ADC, green parts analog
- What if digital (high-resolution) instead?
  - Loop still works!
- Full theory still applicable!

∑Δ DAC



- Pre-processing for lower-precision DAC
  - X higher resolution than Y
  - Y at higher sample rate
- Quantization now corresponds to rounding / truncation
- Y may be a 1-bit signal (much upsampling needed)

Summary

- Noise shaping + filtering enables higher SNR than quantizer alone could give
  - High-pass noise filter not the only option!
- Simpler analog at cost of higher sample rate
- Feedback loop neutralizes (most…) errors
- DT lowpass filter (digital) for downsampling / reconstruction
- Also useful as pre-processing for low-resolution DACs