Dept. Of Electric Power Engineering
Chalmers University of Technology

Solution of demonstration 1

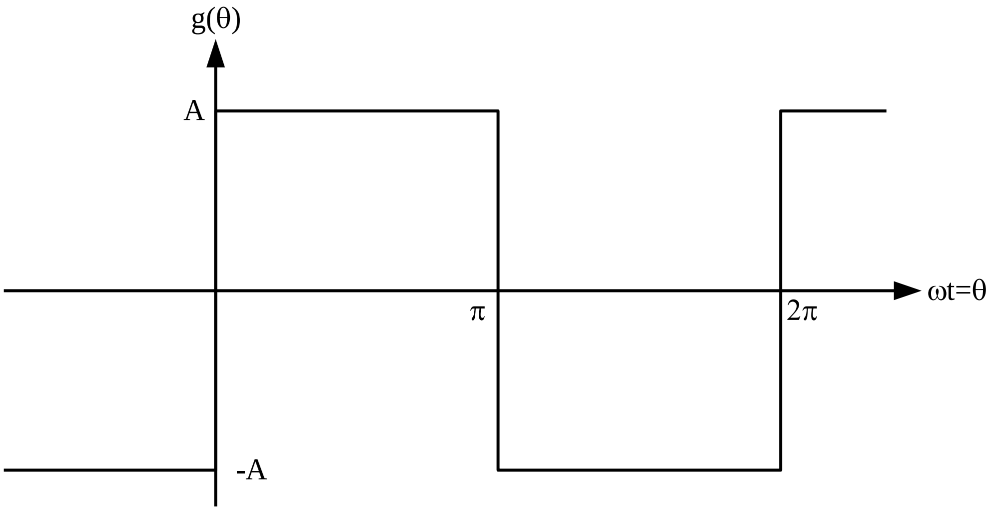


Figure 1 Waveform 1 (Figure P3-3 a in Undeland book)

Problem 1 (P3-3 in Undeland book)

For the functions in figures below, calculate the average value and RMS-value of the fundamental and the harmonic frequency components.

Solution

$$g(\theta) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos(n\theta) + b_n \sin(n\theta)\right]$$

$$a_n = \frac{1}{\pi}\int_{\theta_0}^{\theta_0+2\pi} g(\theta)\cos(n\theta)d\theta \qquad n = 0,1,2,3,...$$

$$b_n = \frac{1}{\pi}\int_{\theta_0}^{\theta_0+2\pi} g(\theta)\sin(n\theta)d\theta \qquad n = 1,2,3,...$$

From the figure we see that $g(\theta)$ is odd, $-g(\theta) = g(-\theta)$ which gives that $a_n = 0$ for all n. The function is also half-wave, $g(\theta) = -g(\theta+\pi)$, which gives that $b_n = 0$ for all even n.

$$b_n = \frac{4}{\pi}\int_{\theta_0}^{\theta_0+\pi/2} g(\theta)\sin(n\theta)d\theta \qquad \text{for } n=1, 3, 5\ldots \text{ (odd)}$$

$$b_n = 0 \qquad \text{for } n=2, 4, 6\ldots \text{ (even)}$$

For odd *n*:

$$b_n = \frac{4}{\pi}\int_0^{\pi/2} A\sin(n\theta)d\theta = -\frac{4}{\pi}\frac{A}{n}\left[\cos(n\theta)\right]_0^{\pi/2} = \frac{4}{\pi}\frac{A}{n}(1-\underbrace{\cos(n\frac{\pi}{2})}_{=0\ for\ odd\ n}) = \frac{4}{\pi}\frac{A}{n}$$

The average of the function is the half DC-component:

$$G_{av} = \frac{a_0}{2} = 0$$

The RMS-value of the harmonics

$$G_{n,rms} = \sqrt{\frac{a_n^2 + b_n^2}{2}} = \frac{b_n}{\sqrt{2}} = \frac{2\sqrt{2}}{\pi}\frac{A}{n}$$ for *n*=1, 3, 5… (odd)

Problem 2 (P3-4 in Undeland book)

In the waveforms of figures below, A=10.

(a) Calculate the RMS-value for the functions with Fourier series.
(b) Calculate the RMS-value for the functions with the RMS definition.

Solution

(a) $$G_{rms} = \sqrt{G_0^2 + \sum_{n=1}^{\infty} G_n^2} = \sqrt{\left(\frac{a_0}{2}\right)^2 + \sum_{n=1}^{\infty}\left(\frac{a_n^2 + b_n^2}{2}\right)}$$

Applied to our case, where $a_0 = a_n = 0$ and the expression for $b_n$only is valid for $n = odd$:

$$G_{rms} = \sqrt{\sum_{n=1}^{\infty}\frac{1}{2}\left(\frac{4A}{n\pi}\right)^2} = \frac{4A}{\sqrt{2}\pi}\sqrt{\sum_{n=1}^{\infty}\left(\frac{1}{n}\right)^2} = \frac{4A}{\sqrt{2}\pi}\sqrt{(1+\frac{1}{3^2}+\frac{1}{5^2}+...)}$$

$$\sum_{n=1}^{\infty}\left(\frac{1}{n}\right)^2 = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + ... = \frac{\pi^2}{6}$$

$$\frac{1}{2^2}\sum_{n=1}^{\infty}\left(\frac{1}{n}\right)^2 = \frac{1}{2^2} + \frac{1}{4^2} + ... = \frac{\pi^2}{24}$$

$$\sum_{n=1}^{\infty}\left(\frac{1}{n}\right)^2 - \frac{1}{2^2}\sum_{n=1}^{\infty}\left(\frac{1}{n}\right)^2 = 1 + \frac{1}{3^2} + \frac{1}{5^2} + ... = \frac{\pi^2}{8}$$

$$G_{rms} = \frac{4A}{\sqrt{2}\pi}\sqrt{\frac{\pi^2}{8}} = A = 10$$

(b) $$G_{rms} = \sqrt{\frac{1}{2\pi}\int_{\theta_0}^{\theta_0+2\pi} g^2(\theta)d\theta} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} A^2 d\theta} = A = 10$$

Problem 3 (P3-5 in Undeland book)

(a) Calculate the ratio of the fundamental frequency component to the total RMS-value.
(b) Calculate the ratio of the distortion component to the total RMS-value.
(c) Calculate the total harmonic distortion (THD) of the functions.
(c) Calculate the ratio of the average value to the total RMS-value.

Solution

(a) $$\frac{G_{1,rms}}{G_{rms}} = \frac{\frac{2\sqrt{2}A}{\pi}}{A} = \frac{2\sqrt{2}}{\pi} = 0.9$$

(b) $$\frac{G_{dis}}{G_{rms}} = \frac{\sqrt{G_{rms}^2 - G_{1,rms}^2}}{G_{rms}} = \frac{\sqrt{10^2 - (\frac{2\sqrt{2}}{\pi}.10)^2}}{10} = 0.435$$

(c) $$\%THD = 100\frac{G_{dis}}{G_{1,rms}} = 100\frac{4.35}{9} = 48.31\%$$

(d) $$\frac{G_{av}}{G_{rms}} = 0$$

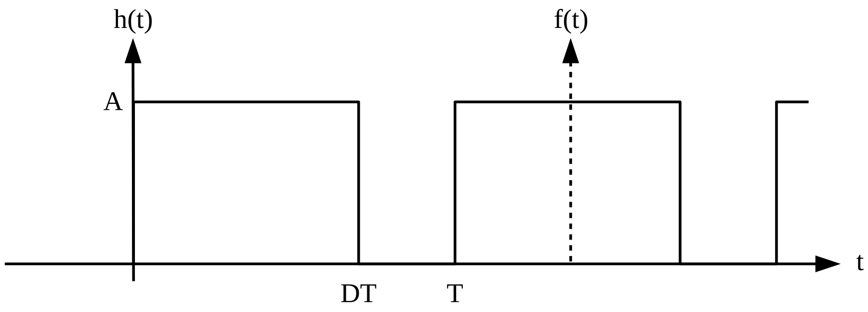


Figure 2 Waveform 2 (Figure P3-3 g in Undeland book)

Problem 1 (P3-3 in Undeland book)

For the functions in figures below, calculate the average value and RMS-value of the fundamental and the harmonic frequency components.

Solution

According to the task we shall calculate the average and the RMS-value of the fundamental and the harmonics of the function $h(t)$. But since the angle of the fundamental and the harmonics is not asked for, we start by defining a new starting point of the function and thereby obtaining the new even function $f(t)$ for which the Fourier series is calculated. The error that is introduced by redefining the starting point is only related to the phase of the components.

The function becomes even, hence can the a_n and b_n component be calculated as:

$b_n = 0$ for all n

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos(n\omega t)dt = \frac{4A}{T}\int_0^{DT/2}\cos(n\omega t)dt = \frac{4A}{T}\frac{1}{n\omega}\left[\sin(n\omega t)\right]_0^{DT/2} = \{\ \omega = 2\pi f\} =$$

$$= \frac{4A}{T}\frac{1}{n\cdot 2\pi f}\sin(n\cdot 2\pi f\frac{DT}{2}) = \left\{T = \frac{1}{f}\right\} = \frac{2A}{n\pi}\sin(\pi Dn)$$

for all $n = 1,2,3,...$

$$a_0 = \frac{4}{T}\int_0^{T/2} f(t)dt = \frac{4A}{T}\int_0^{DT/2} dt = 2AD$$

The average of the function is

$$H_{av} = \frac{a_0}{2} = AD$$

The rms of the harmonics

$$H_{n,rms} = \frac{a_n}{\sqrt{2}} = \frac{\sqrt{2}A}{n\pi}\sin(\pi Dn)$$ for n=1,2,3,…

Problem 2 (P3-4 in Undeland book)

In the waveforms of figures below, A=10.

(a) Calculate the RMS-value for the functions with Fourier series.
(b) Calculate the RMS-value for the functions with the RMS definition.

Solution

(a) $$H_{rms} = \sqrt{H_0^2 + \sum_{n=1}^{\infty} H_n^2} = \sqrt{\left(\frac{a_0}{2}\right)^2 + \sum_{n=1}^{\infty}\left(\frac{a_n^2 + b_n^2}{2}\right)}$$

Applied to our case, where $b_n = 0$

$$H_{rms} = \sqrt{\left(\frac{a_0}{2}\right)^2 + \sum_{n=1}^{\infty}\left(\frac{a_n^2}{2}\right)} = \sqrt{(AD)^2 + \sum_{n=1}^{\infty}\frac{1}{2}\left(\frac{2A}{n\pi}\sin(\pi Dn)\right)^2}$$

(b) $$H_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} f^2(t)dt} = \sqrt{\frac{1}{T}\int_{0}^{DT} A^2 dt} = A\sqrt{D}$$

Problem 3 (P3-5 in Undeland book)

(a) Calculate the ratio of the fundamental frequency component to the total RMS-value.
(b) Calculate the ratio of the distortion component to the total RMS-value.
(c) Calculate the total harmonic distortion (THD) of the functions.
(c) Calculate the ratio of the average value to the total RMS-value.

Solution

(a) $$\frac{H_{1,rms}}{H_{rms}} = \frac{\frac{\sqrt{2}A}{\pi}\sin(\pi D)}{A\sqrt{D}} = \frac{\sqrt{2}\sin(\pi D)}{\pi\sqrt{D}}$$

(b) $$\frac{H_{dis}}{H_{rms}} = \frac{\sqrt{H_{rms}^2 - H_{1,rms}^2}}{H_{rms}} = \frac{\sqrt{A^2 D - (\frac{\sqrt{2}A}{\pi}\sin(\pi D))^2}}{A\sqrt{D}} = \sqrt{1 - \frac{2}{D\pi^2}\sin^2(\pi D)}$$

(c) $$\%THD = 100\frac{H_{dis}}{H_{1,rms}} = 100\frac{\sqrt{A^2 D - (\frac{\sqrt{2}A}{\pi}\sin(\pi D))^2}}{\frac{\sqrt{2}A}{\pi}\sin(\pi D)} = 100\sqrt{\frac{D\pi^2}{2\sin^2(\pi D)} - 1}$$

(d) $$\frac{H_{av}}{H_{rms}} = \frac{AD}{A\sqrt{D}} = \sqrt{D}$$

Problem 4 (P3-7 in Undeland book)

A three-phase inductive load is supplied from a voltage source with $V_{phase} = 120V$. The load draws 10kW with a power factor of 0.85 (lagging).

(a) Calculate the RMS-value of the phase currents and the magnitude of the phase impedance.
(b) Draw a phasor diagram.

Solution

(a) 3-phase system $V_{phase} = 120V$ (RMS), $P = 10W$ at 0.85 PF (lagging).

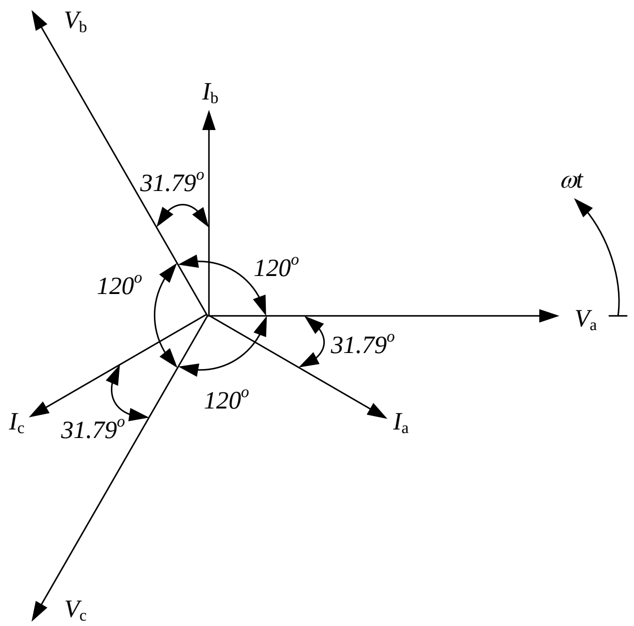
Power factor angle: $\varphi = \arccos(0.85) = 31.8°$.

$P = 3 \cdot I_{phase} \cdot V_{phase} \cdot \cos(\varphi)$

$$I_a = \frac{P}{3 \cdot V_{phase} \cdot \cos(\varphi)} = \frac{10kW}{3 \cdot 120V \cdot 0.85} = 32.7A$$

$$|Z| = \left|\frac{V_{phase}}{I_{phase}}\right| = \frac{120V}{32.7A} = 3.7\Omega$$

(b)



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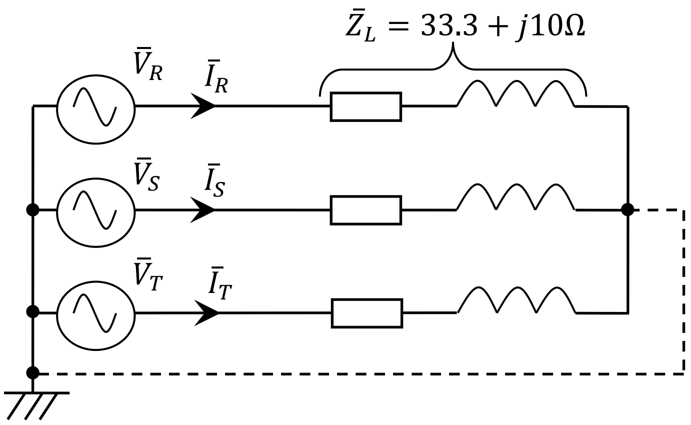
Problem 5 (Extra three-phase problem)

A three-phase load is consists of three identical impedances $\bar{Z}_L = 33.3 + j10\Omega/phase$ connected in Y. The load is connected to a symmetric three-phase 400V grid.

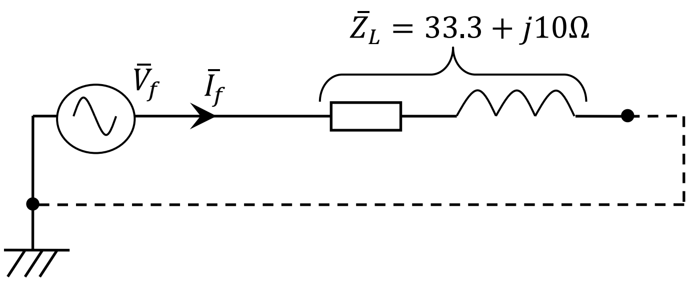
(a) Calculate the current in each phase.
(b) Calculate the active and reactive power.
(c) A capacitor, 31.8μF, is connected in parallel with the load, calculate the new active and reactive power.

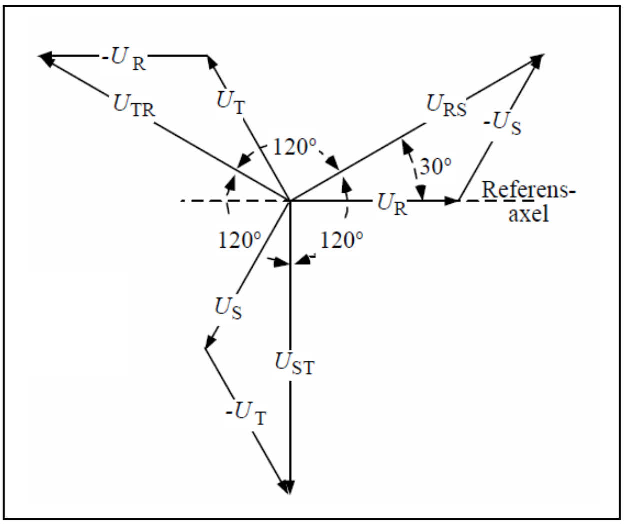
Solution

(a) The circuit can be drawn as:



The load is symmetrical and can hence be drawn as an equivalent y-phase:





The voltage source in the equivalent y-phase is is equal to the phase-voltage. In the graph to the right, the relation between phase voltages and main voltages is described. Note that there will be a 30° difference, but since the phase voltage is used as reference phase, this phase difference does not contribute to the results.

$$\bar{V}_f = \frac{400V}{\sqrt{3}}\angle 0^\circ = 231V\angle 0^\circ$$

The current in each phase can be calculated as:

$$\overline{I}_f = \frac{\overline{V}_f}{R + j\omega L} = \frac{231V\angle 0^\circ}{(33.3 + j10)\Omega} = 6.64A\angle{-16.7^\circ}$$

The effective value of the current in each phase will be the same due to the symmetrical load.

(b) The complex apparent power can be calculated with help of the phase votlage and phase current:

$$\overline{S}_{3-phase} = 3 \cdot \overline{V}_f \cdot \overline{I}_f^* = 3 \cdot 231V\angle 0^\circ \cdot 6.64A\angle 16.7^\circ = 4600VA\angle 16.7^\circ = \\ = 4400W + j1300VAr$$

Note that since both P and Q are positive, the load consumes both active (4400W) and reactive (1300VAr) power.

(c) The new total complex impedance can be calculated as:

$$\overline{Z}_{tot} = (33.3 + j10)\Omega \ // \ \frac{1}{j2 \cdot \pi \cdot 50 \cdot 31.8\mu F}\Omega = (36.2 - j2.29)\Omega = 36.3\Omega\angle{-3.62^\circ}$$

The same phase voltage is applied over the new complex load.

$$\overline{I}_f = \frac{\overline{V}_f}{\overline{Z}_{tot}} = \frac{231V\angle 0^\circ}{(36.2 - j2.29)\Omega} = 6.37A\angle 3.62^\circ$$

The new total apparent power now becomes:

$$\overline{S}_{3-phase} = 3 \cdot \overline{V}_f \cdot \overline{I}_f^* = 3 \cdot 231V\angle 0^\circ \cdot 6.37A\angle{-3.62^\circ} = 4415VA\angle{-3.6^\circ} = \\ = 4400W - j279VAr$$

Note that the active power is the same, but the reactive power has been lowered.