Demonstration 1

Equations used from lecture 1

Fourier analysis

For an arbitrary function $f(t)$ with T as the period time, the Fourier series of $f(t)$ is

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos(n\omega t) + b_n \sin(n\omega t)\right]$$

$$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\cos(n\omega t)dt \qquad n = 0,1,2,3,...$$

$$b_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\sin(n\omega t)dt \qquad n = 1,2,3,...$$

If the function is given as function of angle instead of time the Fourier series can be written as

$$g(\theta) = f(t),\ \theta = \omega t$$

$$g(\theta) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos(n\theta) + b_n \sin(n\theta)\right]$$

$$a_n = \frac{1}{\pi}\int_{\theta_0}^{\theta_0+2\pi} g(\theta)\cos(n\theta)d\theta \qquad n = 0,1,2,3,...$$

$$b_n = \frac{1}{\pi}\int_{\theta_0}^{\theta_0+2\pi} g(\theta)\sin(n\theta)d\theta \qquad n = 1,2,3,...$$

RMS-value with Fourier-series

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos(n\omega t) + b_n \sin(n\omega t)\right] = F_0 + \sum_{n=1}^{\infty}\sqrt{2}F_h \sin(h\omega t + \varphi_h)$$

$$F_{rms} = \sqrt{F_0^2 + \sum_{n=1}^{\infty}F_n^2} = \sqrt{\left(\frac{a_0}{2}\right)^2 + \sum_{n=1}^{\infty}\left(\frac{a_n^2 + b_n^2}{2}\right)}$$

Definition of RMS-value

$$F_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} f^2(t)dt},\ G_{rms} = \sqrt{\frac{1}{2\pi}\int_{\theta_0}^{\theta_0+2\pi} g^2(\theta)d\theta}$$

Total harmonic distortion (THD)

$$\%THD = 100\frac{\sqrt{F_{rms}^2 - F_{1,rms}^2}}{F_{1,rms}} = 100\frac{F_{dis}}{F_{1,rms}}$$

Literature: Undeland book Chapter 1 and Chapter 3

Tutorial exercises

Problem 1 (P3-3 in Undeland book)

For the functions in figures below, calculate the average value and RMS-value of the fundamental and the harmonic frequency components.

Problem 2 (P3-4 in Undeland book)

In the waveforms of figures below, A=10.

(a) Calculate the RMS-value for the functions with Fourier series.
(b) Calculate the RMS-value for the functions with the RMS definition.

Problem 3 (P3-5 in Undeland book)

(a) Calculate the ratio of the fundamental frequency component to the total RMS-value.
(b) Calculate the ratio of the distortion component to the total RMS-value.
(c) Calculate the total harmonic distortion (THD) of the functions.
(c) Calculate the ratio of the average value to the total RMS-value.

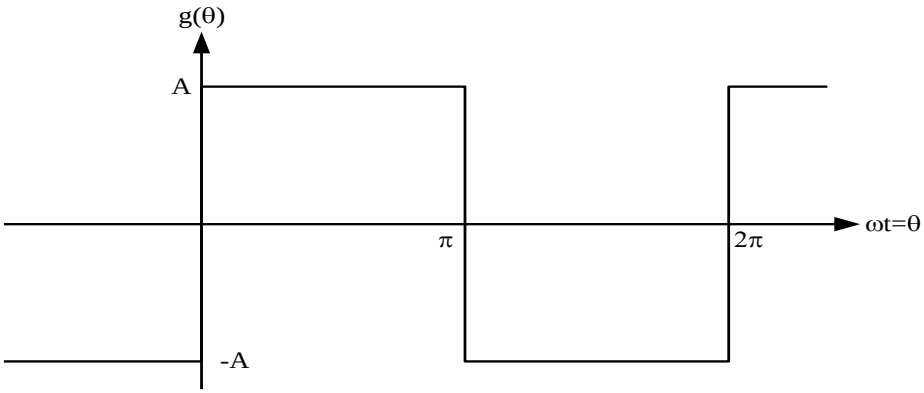

Figure 1 Waveform 1 (Figure P3-3 a in Undeland book)

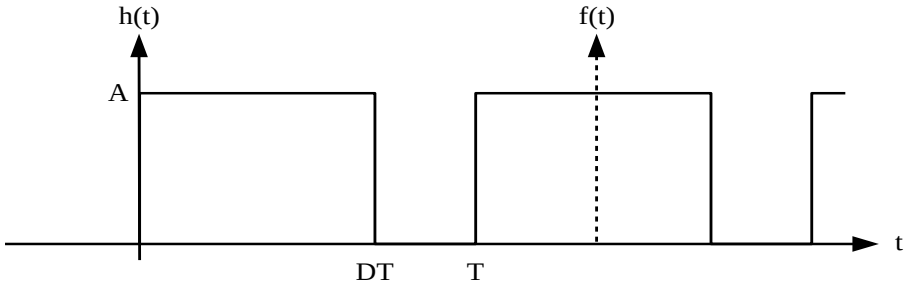

Figure 2 Waveform 2 (Figure P3-3 g in Undeland book)

Problem 4 (P3-7 in Undeland book)

A three-phase inductive load is supplied from a voltage source with $V_{phase} = 120V$. The load draws 10kW with a power factor of 0.85 (lagging).

(a) Calculate the RMS-value of the phase currents and the magnitude of the phase impedance.
(b) Draw a phasor diagram.

Problem 5 (Extra three-phase problem)

A three-phase load is consists of three identical impedances $\bar{Z}_L = 33.3 + j10\Omega/phase$ connected in Y. The load is connected to a symmetric three-phase 400V grid.

(a) Calculate the current in each phase.
(b) Calculate the active and reactive power.
(c) A capacitor, 31.8µF, is connected in parallel with the load, calculate the new active and reactive power.

Self-study exercises

From Undeland book:
P1-1, P1-3, P1-5
P3-2, P3-3 to P3-5 for Figure P3-3 b, c, f