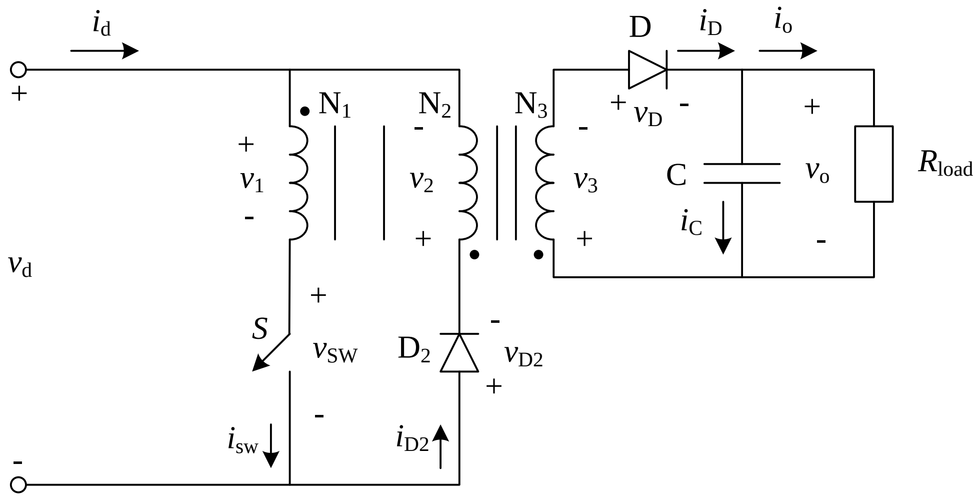
Operation of a 3-winding flyback converter

A flyback converter with a protective winding (N_2) and turns ratio given by $(N_1\colon N_2\colon N_3)$.



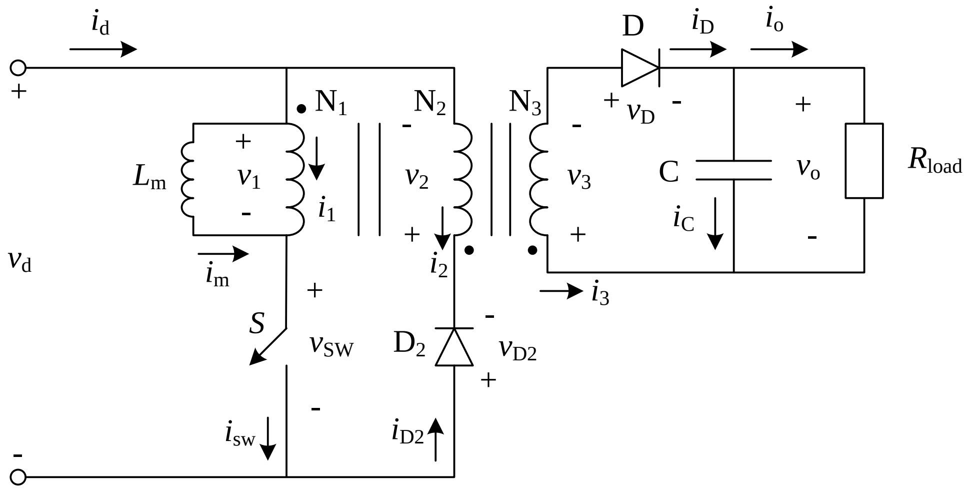
When and how does the winding N_2 and diode D_2 become operational to limit the output voltage?

For CCM/ DCM operation, derive the voltage transfer ratio V_o/V_d in terms of the load resistance R_{load}, switching frequency (f_{sw}), transformer mutual inductance (L_m) and duty ratio(D).

How does these factors $(R_{load}, f_{sw}, L_m, D)$ decide if the mode of operation of the converter is in CCM or DCM?

Self-exercise: plot the current and voltage waveforms for CCM/DCM/no load operations?

The transformer in a flyback converter is represented by three windings with ideal transformation ratio and a mutual inductance for energy storage as



Because of the ideal transformer assumption and the three windings share the same magnetic core, the voltage rations are derived as

$$v_1 = N_1 \frac{d\phi}{dt} = L_m \frac{di_m}{dt}$$
$$v_2 = N_2 \frac{d\phi}{dt} \qquad (1)$$
$$v_3 = N_3 \frac{d\phi}{dt}$$

$$\Rightarrow v_1 : v_2 : v_3 = N_1 : N_2 : N_3 \qquad (2)$$

The notation of the dots show that for a positive voltage at the dot on winding 1, then we get a positive voltage at the dot on the other windings. At the same time, if we have a current going in at the dot on the input winding 1, we must then have a current going out from the dot on the other windings. From conservation of energy for the three windings representing an ideal transformer, the input power at the input terminal is balanced by the other terminals as

$$\Rightarrow v_1 i_1 = v_2 i_2 + v_3 i_3$$
$$\Rightarrow v_1 (i_{sw} - i_m) = -v_2 i_{D2} - v_3 i_D \qquad (3)$$
$$\Rightarrow v_1 (i_m - i_{sw}) = v_2 i_{D2} + v_3 i_D$$

The voltage and current relationships of the windings in (1) - (3) should be fulfilled in any mode of operation for the converter. To obtain the output-input voltage ratio for the converter, we have to determine first if the converter is operating in CCM or DCM operation. For a given set of operating points, this can be obtained through comparison of the average magnetization current (I_m) to its peak-to-peak value (Δi_m). i.e.,

If $I_m \geq \Delta i_m/2$, the converter is CCM else the converter is in DCM.

This means that both I_m and Δi_m should be calculated from given operating points for checking the mode of operation. This is also dependent if the protective winding N_2 is operating or not which again depends on the given set of operating points. For the sake of discussion, let's discuss all the possible combinations of the converter operation.

Consider the states of the converter for the switch-on and switch-off period,

When the switch S is on ($v_{sw} = 0$), using (1) – (3), we have

$$v_1 = v_d = L_m \frac{di_m}{dt} \Rightarrow v_2 = \frac{N_2}{N_1} v_1 = \frac{N_2}{N_1} v_d \Rightarrow v_3 = \frac{N_3}{N_1} v_1 = \frac{N_3}{N_1} v_d$$
$$\Rightarrow v_D = -(v_3 + v_o) < 0 \Rightarrow i_D = -i_3 = 0$$
$$\Rightarrow v_{D2} = -(v_2 + v_d) < 0 \Rightarrow i_{D2} = -i_2 = 0$$

$$v_1 i_1 = v_2 i_2 + v_3 i_3 \Rightarrow i_1 = \frac{N_2}{N_1} i_2 + \frac{N_3}{N_1} i_3 = 0$$

$$\Rightarrow i_{sw} = i_1 + i_m = i_m \Rightarrow i_d = i_{sw} + i_2 = i_{sw} = i_m$$

This means that during this switch-on period, both the diodes ***D*** and ***D₂*** are off due to the negative voltage applied on them. Therefore, energy is stored in the mutual inductance and the current i_m is linearly increasing.

When the switch S is off, we have

$$i_{sw} = i_1 + i_m = 0 \Rightarrow i_1 = -i_m$$

$$v_D = -(v_3 + v_o)$$

$$v_{D2} = -(v_2 + v_d) = -\left(\frac{N_2}{N_3} v_3 + v_d\right)$$

Now, we have to first decide which diode (***D*** or $\boldsymbol{D_2}$) is conducting. The condition for this is given by

D conducts if $v_D = -(v_3+v_o) \geq 0$

$\boldsymbol{D_2}$ conducts if $v_{D2} = -(v_2+v_d) \geq 0$

To arrive at a conclusion, we consider that the converter is operating in steady-state. If we start with the assumption with the diode ***D*** is conducting, what does this mean to the magnitude of the output voltage v_o and the conduction state of diode $\boldsymbol{D_2}$?

D conducts means that $v_D = -(v_3+v_o) \geq 0 \Rightarrow v_3 \leq -v_o$. If we assume that the diode forward voltage is zero, this means that $\boldsymbol{v_3 = -v_o}$. This gives the voltage over the diode $\boldsymbol{D_2}$ as

$v_{D2} = -(v_2+v_d) = -([N_2/N_3]\ v_3+v_d) = -(-[N_2/N_3]\ v_0+v_d) =[N_2/N_3]\ v_0 - v_d$

Considering the above equation, we have the diode ***D*** conducting and the diode $\boldsymbol{D_2}$ off at the same time if $v_{D2} = [N_2/N_3]\ v_0 - v_d < 0 \Rightarrow \boldsymbol{v_0 < [N_3/N_2]v_d}$. Now we have arrived at the following first conclusion:

Conclusion 1: ***Diode D conducts and diode*** $\boldsymbol{D_2}$ ***is off if*** $\boldsymbol{v_0 \leq [N_3/N_2]v_d}$.

Now, we continue the second assumption that the diode $\boldsymbol{D_2}$ is conducting. What does this mean to the magnitude of the output voltage v_o and the conduction state of diode ***D***?

$\boldsymbol{D_2}$ conducts means that $v_{D2} = -(v_2+v_d) \geq 0 \Rightarrow v_2 \leq -v_d$. If we assume that the diode forward voltage is again zero, this means that $\boldsymbol{v_2 = -v_d}$. This gives the voltage over the diode ***D*** as

$v_D = -(v_3+v_o) = -([N_3/N_2]\ v_2+v_0) =-(-[N_3/N_2]\ v_d+v_0)= [N_3/N_2]\ v_d-v_0$

Considering the above equation, we have the diode $\boldsymbol{D_2}$ conducting and the diode $\boldsymbol{D}$ off at the same time if $v_D = [N_3/N_2]\, v_d - v_o < 0 \Rightarrow \boldsymbol{v_o > [N_3/N_2]v_d}$. Now we have arrived at the following second conclusion:

Conclusion 2: ***Diode D_2 conducts and diode D is off if $v_0 > [N_3/N_2]v_d$.***

The two conclusions above state that during the switch-off period, the conduction of the diodes $\boldsymbol{D}$ or $\boldsymbol{D_2}$ is decided by the magnitude of the output voltage. If a choice of an operating point results in an output voltage higher than $\boldsymbol{v_{0,max} = [N_3/N_2]v_d}$, the diode $\boldsymbol{D_2}$ will start to conduct and as a result disconnecting diode $\boldsymbol{D}$. This makes the output voltage not to increase beyond the maximum value. As such the output voltage is limited to the maximum value of $\boldsymbol{v_{0,max} = [N_3/N_2]v_d}$. Now, the output voltage to the input voltage ratio is derived for various cases as follow.

Case 1: CCM

Switch-on => from previous analysis, the diodes $\boldsymbol{D}$ and $\boldsymbol{D_2}$ are both off and we have

$$v_1 = v_d$$

Switch-off => from previous analysis, the diode $\boldsymbol{D}$ or $\boldsymbol{D_2}$ will be conducting. Assuming that the output voltage is less than the maximum value of $\boldsymbol{v_{0,max} = [N_3/N_2]v_d}$, the diode $\boldsymbol{D}$ will be conducting and we have

$$v_3 = -v_o \Rightarrow v_1 = (N_1/N_3)\, v_3 = -(N_1/N_3)\, v_o$$

From the fact that the average voltage of v_1 is zero in steady-state, we have

$$v_{1,avg} = 0 = [v_d.DT_{sw} - (N_1/N_3)\, v_o.(1-D)T_{sw}]/T_{sw}$$

$$\Rightarrow \quad \frac{V_o}{V_d} = \frac{N_3}{N_1}\frac{D}{1-D}$$

The restriction for the above equation is that

$$\frac{V_o}{V_d} = \frac{N_3}{N_1}\frac{D}{1-D} \le \frac{N_3}{N_2}$$

If the above restriction is not fulfilled due to the choice of the turns ratio or the duty cycle, the output voltage will be limited to $\boldsymbol{(N_2/N_3)v_d}$. In this case, once the output voltage is the maximum value, only the diode $\boldsymbol{D_2}$ will be conducting returning the energy stored during the switch-on period back to the source through the protective winding. From the above equation, we can express the maximum duty cycle that is allowed for the converter to operate in CCM as

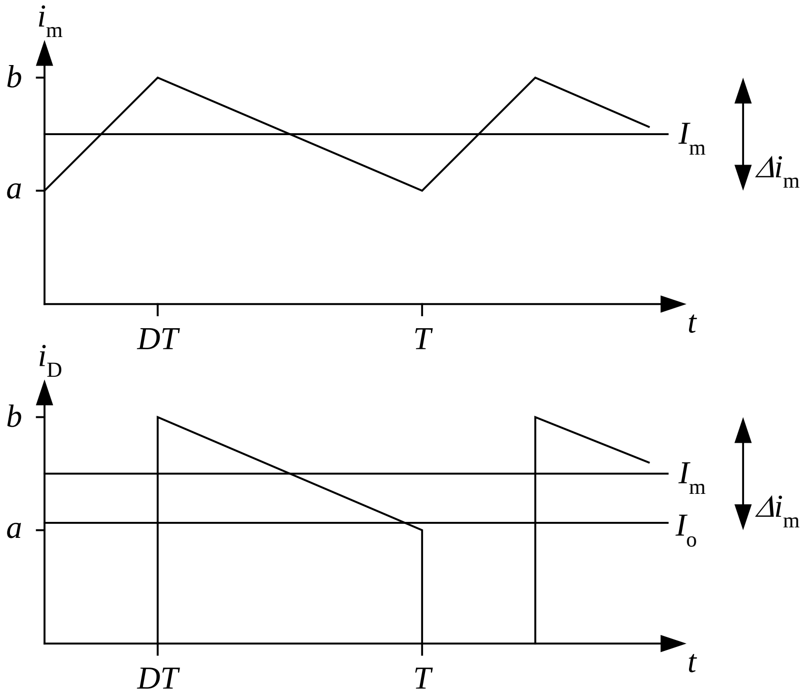
$$\frac{N_3}{N_1}\frac{D}{1-D} \le \frac{N_3}{N_2} \quad \Rightarrow \quad D \le \frac{1}{1+\frac{N_2}{N_1}}$$

If the duty cycle is higher than the value stated above, the converter cannot operate in steady-state as there is no enough time to discharge the energy stored during the on period back to the source. This continuously will increase the magnetizing current resulting in saturation of the transformer.

So far, we have assumed that the converter is in CCM, this should be confirmed through calculation of the average magnetizing current. For CCM, the peak-to-peak ripple magnetizing current ($\Delta \boldsymbol{i_m}$) is calculated from the on period as

$$\boldsymbol{v_1} = \boldsymbol{L_m}\frac{\Delta \boldsymbol{i_m}}{\boldsymbol{DT_{sw}}} = \boldsymbol{V_d} \Rightarrow \Delta \boldsymbol{i_m} = \frac{\boldsymbol{DV_d}}{\boldsymbol{L_m f_{sw}}}$$

Now we have to calculate the average magnetizing current (I_m). Assuming the maximum voltage limitation above (with only diode $\boldsymbol{D}$ conducting), we draw the magnetizing current (i_m) and the diode current (i_D) for CCM:



From the figure we see that the average magnetizing current can be calculated as:

$$\boldsymbol{I_m} = \frac{1}{T_{sw}}\int_0^{T_{sw}} i_m(t)dt = \frac{1}{T_{sw}}\int_0^{DT_{sw}} i_m(t)dt + \frac{1}{T_{sw}}\int_{DT_{sw}}^{T_{sw}} i_m(t)dt = \frac{a+b}{2T_{sw}}DT_{sw} + \frac{a+b}{2T_{sw}}(1-D)T_{sw} = \frac{a+b}{2}$$

$$\boldsymbol{I_o} = \frac{V_o}{R_{load}} = \frac{1}{T_{sw}}\int_0^{T_{sw}} i_D(t)dt = \frac{1}{T_{sw}}\int_{DT_{sw}}^{T_{sw}} i_m(t)dt = \frac{a+b}{2T_{sw}}(1-D)T_{sw} = \frac{a+b}{2}(1-D)$$

$\Rightarrow \quad \boldsymbol{I_o} = \boldsymbol{I_m}(1-D)\frac{N_1}{N_3}$ If the turns ratio is also included.

This is a simple relation between the average output current and the average magnetizing current.

The border between CCM and DCM can now be calculated as:

$$\frac{\Delta i_m}{2} \leq I_m \quad => \quad \frac{DV_d}{2L_m f_{sw}} \leq \frac{I_o}{1-D}\frac{\boldsymbol{N_3}}{\boldsymbol{N_1}} \quad => \quad (\boldsymbol{1}-\boldsymbol{D})^2 \leq \left(\frac{\boldsymbol{N_3}}{\boldsymbol{N_1}}\right)^2 \frac{\boldsymbol{2L_m f_{sw}}}{\boldsymbol{R_{load}}}$$

If the above equation is fulfilled, the converter is operating in CCM. Otherwise the assumption of CCM is wrong and the calculation of DCM should be performed. Observing the various factors ($\boldsymbol{R_{load}}, \boldsymbol{f_{sw}}, \boldsymbol{L_m}, \boldsymbol{D}$), the impact of them on the operation mode of the converter can be investigated for example as

- Keeping all other factors the same, increasing $\boldsymbol{f_{sw}}$, $\boldsymbol{L_m}$ or $\boldsymbol{D}$ leads to CCM whereas, increasing $\boldsymbol{R_{load}}$ leads to DCM. The opposite happens for decreasing a parameter at a time.

Case 2: DCM

Switch-on => from previous analysis, the diodes ***D*** and $\boldsymbol{D_2}$ are both off and we have

$\boldsymbol{v_1 = v_d}$

Switch-off => from previous analysis, the diodes ***D*** or $\boldsymbol{D_2}$ will be conducting. Assuming that the output voltage is less than the maximum value of $\boldsymbol{v_{0,max} = [N_3/N_2]v_d}$, the diode ***D*** will be conducting and we have

$\boldsymbol{v_3 = -v_o \Rightarrow v_1 = (N_1/N_3)\, v_3 = -(N_1/N_3)\, v_o}$

However, part of the off-period now includes the zero magnetizing current region where the magnitude of voltage in all the windings become zero.

$\boldsymbol{v_1 = 0}$

Similarly to the CCM case, the average output voltage can be obtained by using the average voltage over the mutual inductance is zero. This however needs a calculation of the time period where the magnetizing current is zero. Another option is to use the energy conservation principle and assume all the energy stored in the transformer during on-period is transferred to the load during off-period. This gives, the total stored energy during the on-period as

$$W_{load} = W_{Lm} = \frac{1}{2} L_m \Delta i_m^2$$

We can calculate Δi_m by analyzing the voltage over the magnetizing inductance during the on-period of the switch as

$$\Delta i_m = \frac{V_d D}{L_m f_{sw}} \Rightarrow W_{load} = \frac{L_m}{2}\left(\frac{V_d D}{L_m f_{sw}}\right)^2$$

The output power in steady state is given by

$$P_o = \frac{V_o^2}{R_{load}} = W_{load} f_{sw}$$

$$\Rightarrow \frac{V_o^2}{R_{load}} = \frac{L_m}{2}\left(\frac{V_d D}{L_m f_{sw}}\right)^2 f_{sw} \Rightarrow \frac{V_o^2}{V_d^2} = \frac{1}{2}\frac{D^2}{L_m f_{sw}} R_{load} \Rightarrow \frac{V_o}{V_d} = \sqrt{\frac{R_{load}}{2L_m f_{sw}}} D$$

The restriction for the above equation is that

$$\boldsymbol{\frac{V_o}{V_d} = \sqrt{\frac{R_{load}}{2L_m f_{sw}}} D \leq \frac{N_3}{N_2}}$$

If the above restriction is not fulfilled due to the choice of the turns ratio or the duty cycle, the output voltage will be limited to $(N_3/N_2)v_d$. In this case, once the output voltage is at the maximum value, only the diode $\boldsymbol{D_2}$ will be conducting returning the energy stored during the switch-on period back to the source through the protective winding.