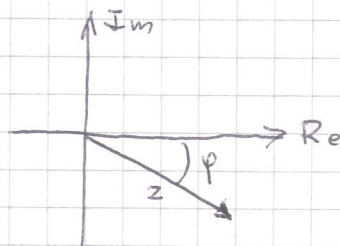


1.11.1

$$a) 18 - j10 = z e^{j\varphi} = 20,6 e^{-j29,05^\circ} = 20,6 \angle -29,05^\circ$$

$$\varphi = \arctan\left(-\frac{10}{18}\right) = -29,05^\circ$$

$$z = \sqrt{18^2 + 10^2} = 20,6$$



$$*b) -0,05 - j0,8 = z e^{j\varphi} = 0,802 \angle -93,58^\circ$$

$$z = \sqrt{(-0,05)^2 + (-0,8)^2} = 0,802$$

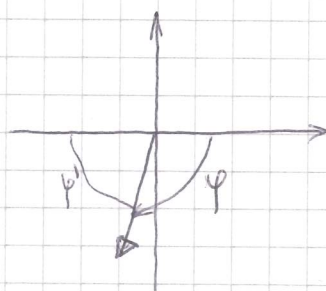
0,05

0,8

 $\Rightarrow 93,58^\circ$

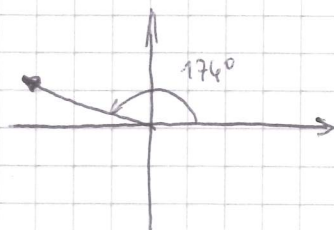
$$\varphi' = \arctan \frac{0,8}{0,05} = 86,42^\circ$$

$$\varphi = 180 - 86,42 = 93,58^\circ$$



$$*c) 0,5 \angle 174^\circ = a + jb$$

$$0,5 \angle 174^\circ = 0,5 (\cos 174^\circ + j \sin 174^\circ) = -0,497 + j0,052$$



$$d) \sqrt{1+j2} = z e^{j\varphi} = \sqrt{2,24} e^{j\frac{63,4}{2}} = 1,5 \angle 31,7^\circ$$

$$z^n = z^n e^{jn\varphi}$$

$$z = \sqrt{1^2 + 2^2} = 2,24$$

$$\varphi = \arctan \frac{2}{1} = 63,4^\circ$$

1.2

a) $\underline{Z}_1 = 10 + j5$

$\underline{Z}_2 = 20 \angle 30^\circ$

$\underline{Z}_1 = 10 + j5 = 11,18 \angle 26,57^\circ$

$\underline{Z}_2 = 20 \angle 30^\circ = 17,32 + j10$

$$\frac{11,18 \angle 26,57^\circ \cdot 20 \angle 30^\circ}{10 + j5 + 17,32 + j10} = \frac{223,6 \angle 56,57^\circ}{27,32 + j15} = \frac{223,6 \angle 56,57^\circ}{31,17 \angle 28,77^\circ} =$$

$$= 7,17 \angle 27,8^\circ = 6,34 + j3,34$$

* b) $\underline{Z}_1 = 5 \angle 45^\circ$

$\underline{Z}_2 = 10 \angle -70^\circ$

$\underline{Z}_1 = 5 \angle 45^\circ = 3,54 + j3,54$

$\underline{Z}_2 = 10 \angle -70^\circ = 3,42 - j9,4$

$$\frac{5 \angle 45^\circ \cdot 10 \angle -70^\circ}{3,54 + 3,42 + j(3,54 - 9,4)} = \frac{50 \angle -25^\circ}{6,96 - j5,86} = \frac{50 \angle -25^\circ}{9,1 \angle -40,1^\circ} =$$

$$= 5,5 \angle 15,1^\circ = 5,31 + j1,43$$

* c) $\underline{Z}_1 = 6 - j2$

$\underline{Z}_2 = 1 + j8$

$\underline{Z}_1 = 6 - j2 = 6,32 \angle -18,43^\circ$

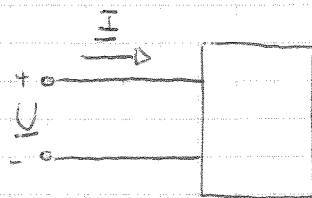
$\underline{Z}_2 = 1 + j8 = 8,06 \angle 82,87^\circ$

$$\frac{6,32 \angle -18,43^\circ \cdot 8,06 \angle 82,87^\circ}{7 + j6} = \frac{50,94 \angle 64,44^\circ}{9,22 \angle 40,6^\circ} = 5,53 \angle 23,84^\circ =$$

$$= 5,06 + j2,24$$

1.2

*
1.3

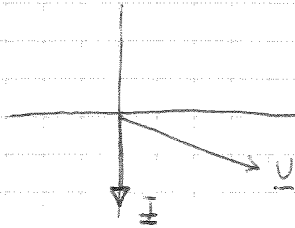


motoriska ref.

1.3

a) $\underline{U} = 200 \angle -30^\circ \text{ V}$

$\underline{I} = 5 \angle -90^\circ \text{ A}$



$$\underline{S} = \underline{U} \underline{I}^* = 200 \angle -30^\circ \cdot 5 \angle 90^\circ = 1000 \angle 60^\circ =$$

$$= 1000 (\cos 60^\circ + j \sin 60^\circ) = 500 + j 866$$

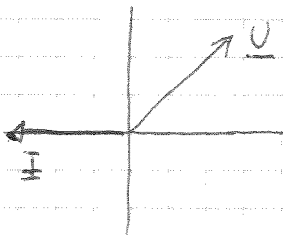
förbrukar (upptar) P o Q

$\downarrow$
P
[W]

$\downarrow$
Q
[VAr]

b) $\underline{U} = 200 \angle 45^\circ \text{ V}$

$\underline{I} = 5 \angle 180^\circ \text{ A}$



$$\underline{S} = \underline{U} \underline{I}^* = 1000 \angle -135^\circ = -707 - j 707$$

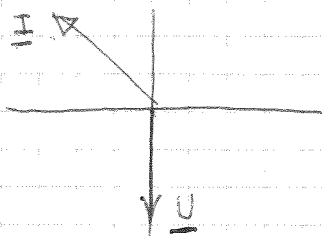
$\downarrow$
P

$\downarrow$
Q

avger P o Q

c) $\underline{U} = 200 \angle -90^\circ \text{ V}$

$\underline{I} = 5 \angle 135^\circ \text{ A}$



$$\underline{S} = \underline{U} \underline{I}^* = 1000 \angle -225^\circ = -707 + j 707$$

$\downarrow$
P

$\downarrow$
Q

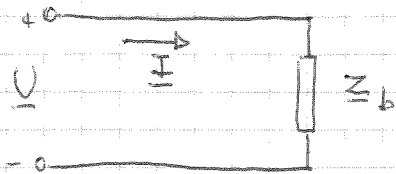
avger

upptar

1.4

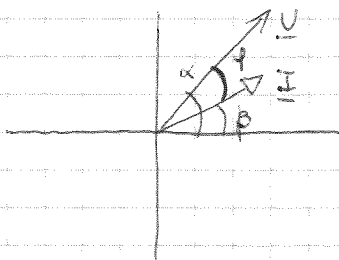
$$U = 250 \text{ V} ; P = 1,5 \text{ kW} ; I = 10 \text{ A}$$

induktiv belastning $\rightarrow$ ström efter sp.



belastning uppstår effekt

$$P = UI \cos \varphi \rightarrow \cos \varphi = \frac{1500}{250 \cdot 10} = 0,6 \rightarrow \underline{\varphi = 53,13^\circ}$$



$$\varphi = \alpha - \beta$$

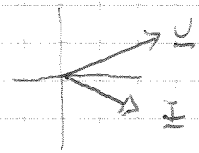
$$\underline{U} = U \angle \alpha ; \underline{I} = I \angle \beta$$

$$\underline{S} = \underline{U} \angle \alpha \cdot \underline{I} \angle -\beta = UI \angle \alpha - \beta = UI \angle \varphi$$

$$\varphi = \alpha - \beta = 53,13^\circ$$

$$a) \underline{U} = 250 \angle 30^\circ \text{ V} \quad \alpha = 30^\circ \rightarrow \beta = \alpha - \varphi = 30^\circ - 53,13^\circ = -23,13^\circ$$

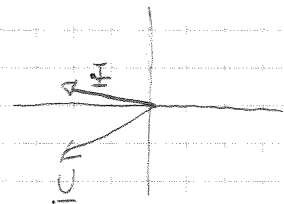
$$\underline{I} = 10 \angle -23,13^\circ \text{ A}$$



$$b) \underline{U} = 250 \angle -145^\circ \text{ V} \quad \alpha = -145^\circ$$

$$\beta = -145^\circ - 53,13^\circ = -198,13^\circ$$

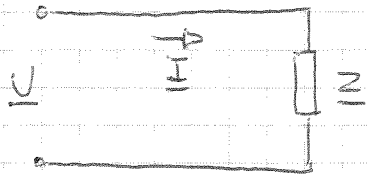
$$\underline{I} = 10 \angle -198,13^\circ \text{ A}$$



1.5

$$U = 220 \text{ V} ; Z = 16 + j12 \text{ }\Omega$$

$$I = ?$$

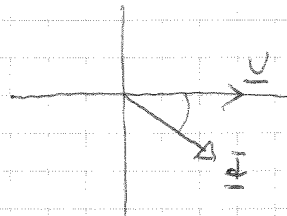


Sp. som referens $\rightarrow U = 220 \angle 0^\circ \text{ V}$

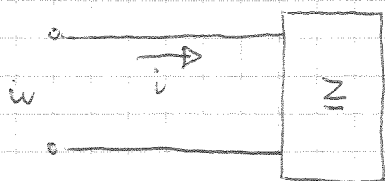
$$U = Z I$$

$$I = \frac{U}{Z} = \frac{U \angle 0^\circ}{16 + j12} = \frac{220 \angle 0^\circ}{20 \angle 36.87^\circ} = 11 \angle -36.87^\circ =$$

$$= \underline{\underline{8.8 - j6.6 \text{ A}}}$$



sp. före ström $\rightarrow$ ind. last

1.6

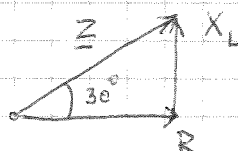
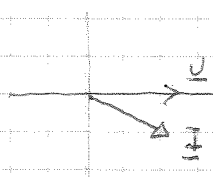
$$u = 311 \cos \omega t$$

$$\rightarrow \underline{U} = \frac{311}{\sqrt{2}} \angle 0^\circ \text{ V (riktfas)}$$

$$i = 23 \cos(\omega t - 30^\circ)$$

$$\rightarrow \underline{I} = \frac{23}{\sqrt{2}} \angle -30^\circ \text{ A}$$

$$\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{311 \angle 0^\circ}{23 \angle -30^\circ} = 13,52 \angle 30^\circ = 11,71 + j6,76 \text{ }\Omega$$



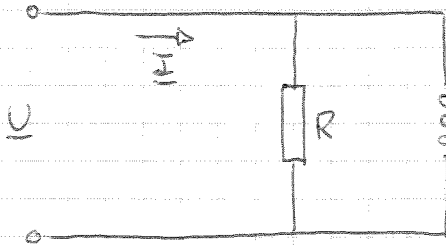
*
1.7

$$U = 220 \text{ V} ; R = 22 \Omega ; X_L = 55 \Omega$$

1.7

$$\underline{I} = I \angle \varphi$$

$$\underline{U} = 220 \angle 0^\circ \text{ V}$$



$$Z = R \parallel jX_L$$

$$Z = \frac{R \cdot jX_L}{R + jX_L} = \frac{22 \cdot 55 \angle 90^\circ}{59,24 \angle 68,2^\circ} = 20,43 \angle 21,8^\circ \Omega$$

$$22 + j55 = 59,24 \angle 68,2^\circ$$

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{220 \angle 0^\circ}{20,43 \angle 21,8^\circ} = \underline{10,77 \angle -21,8^\circ \text{ A}}$$

$$I_R = \frac{U}{R} = \frac{220}{22} = 10 \text{ A}$$

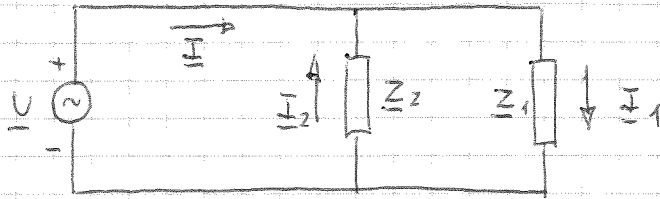
$$I_L = \frac{U}{X_L} = \frac{220}{55} = 4 \text{ A} \Rightarrow 4 \angle -90^\circ$$

ej 14!

$$\Rightarrow 10 + 4 \cos 90^\circ + j4 \sin 90^\circ = 10 + j4 = 10,77 \angle -21,8^\circ \text{ A}$$

1.8

1.8



$$\underline{I}_1 = 20 \angle 0^\circ \text{ A} ; \underline{I}_2 = 12 \angle -150^\circ \text{ A} ; \underline{Z}_1 = 4,33 + j2,5 \Omega$$

a) $\underline{Z}_2 = ? ; \underline{U} = U \angle \alpha ? ; \underline{I} = I \angle \beta ?$

$\underline{I}_1$ - referens

$$\underline{Z}_1 = 4,33 + j2,5 = 5 \angle 30^\circ \Omega$$

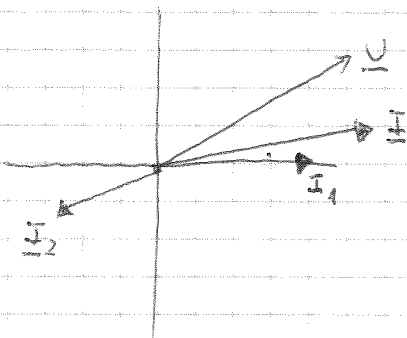
$$\underline{U} = \underline{Z}_1 \cdot \underline{I}_1 = 5 \angle 30^\circ \cdot 20 \angle 0^\circ = \underline{\underline{100 \angle 30^\circ \text{ V}}}$$

$$\underline{U} = - \underline{Z}_2 \underline{I}_2$$

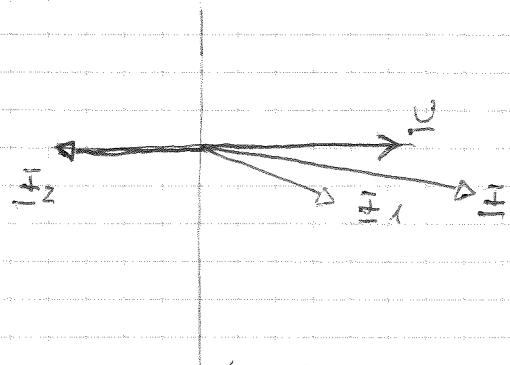
$$\underline{Z}_2 = - \frac{\underline{U}}{\underline{I}_2} = - \frac{100 \angle 30^\circ}{12 \angle -150^\circ} = -8,33 \angle 180^\circ = -8,33(-1 + j0) = \underline{\underline{8,33 \Omega}}$$

$$\underline{I} = \underline{I}_1 - \underline{I}_2 = 20 - (-10,4 - j6) = 30,4 + j6 = \underline{\underline{31 \angle 11,16^\circ \text{ A}}}$$

$\uparrow$
 $12 \angle -150^\circ$



b)



$$\underline{I}_1 = 20 \angle -30^\circ \text{ A}$$

$$\underline{I}_2 = 12 \angle -180^\circ \text{ A } (-12 \angle 0^\circ)$$

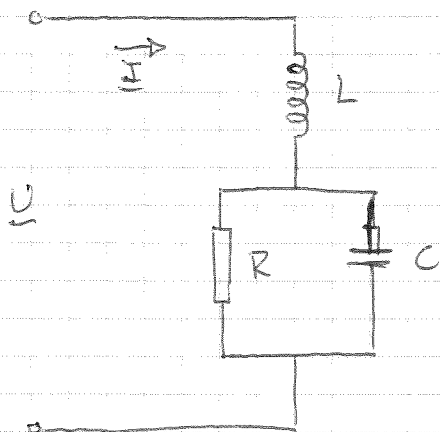
$$\underline{I} = 31 \angle -18,84^\circ \text{ A}$$

1.9

1.9

$$U = 10 \text{ V} ; \omega = 1000 \text{ rad/s} ; L = 0,1 \text{ H} ; C = 10 \mu\text{F} ; R = 5 \Omega$$

$$P_R = ?$$



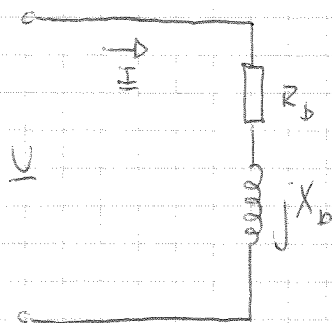
$$jX_L = j\omega L = j1000 \cdot 0,1 = j100 \Omega$$

$$-jX_C = \frac{1}{j\omega C} = -j \frac{10^6}{1000 \cdot 10} = -j100 \Omega$$

$$\underline{Z} = jX_L + \frac{R \cdot (-jX_C)}{R + (-jX_C)} =$$

$$= j100 + \frac{5 \cdot (-j100)}{5 - j100} = \frac{j500 + 10^4 - j500}{5 - j100} =$$

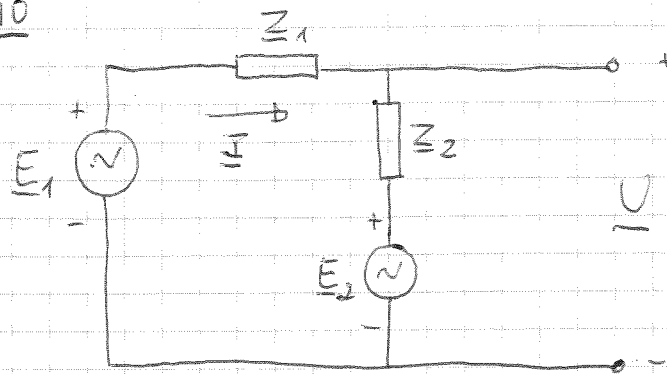
$$= \frac{10^4}{5 - j100} \approx 5 + j100 = R_b + jX_b = 100 / \underline{87,1^\circ} \Omega$$



$$\underline{I} = \frac{10 / 0^\circ}{100 / \underline{87,1^\circ}} = 0,1 / \underline{-87,1^\circ} \text{ A}$$

$$I = 0,1 \text{ A}$$

$$P_R = R_b I^2 = 5 \cdot 0,1^2 = \underline{\underline{50 \text{ mW}}}$$

1.101.10

$$\underline{E}_1 = 100 \angle 0^\circ \text{ V}$$

$$\underline{E}_2 = 100 \angle -30^\circ \text{ V}$$

$$\underline{Z}_1 = 2 + j80 \, \Omega$$

$$\underline{Z}_2 = 4 + j100 \, \Omega$$

$$\underline{I}; \underline{U} = ?$$

$$\underline{E}_2 = 100 \angle -30^\circ = 86,6 - j50 \text{ V}$$

$$\underline{Z}_1 = 2 + j80 = 80 \angle 88,57^\circ \, \Omega$$

$$\underline{Z}_2 = 4 + j100 = 100 \angle 87,7^\circ \, \Omega$$

$$\underline{E}_1 - \underline{Z}_1 \underline{I} - \underline{Z}_2 \underline{I} - \underline{E}_2 = 0$$

$$\underline{I} = \frac{\underline{E}_1 - \underline{E}_2}{\underline{Z}_1 + \underline{Z}_2} = \frac{100 - 86,6 + j50}{2 + j80 + 4 + j100} = \frac{13,4 + j50}{6 + j180} =$$

$$= \frac{51,8 \angle 75^\circ}{180 \angle 88,1^\circ} = \underline{\underline{0,29 \angle -13,1^\circ \text{ A}}}$$

$$\underline{U} = \underline{E}_1 - \underline{Z}_1 \underline{I} = 100 - [80 \angle 88,57^\circ \cdot 0,29 \angle -13,1^\circ] =$$

$$= 100 - 23,2 \angle 75,47^\circ = 100 - 5,82 - j22,46 = 94,18 - j22,46 =$$

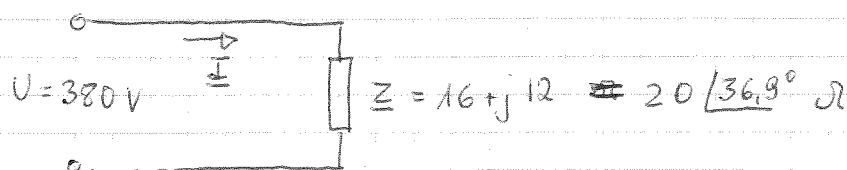
$$= \underline{\underline{96,82 \angle -13,41^\circ \text{ V}}}$$

$$\text{alt. } \underline{E}_2 + \underline{Z}_2 \underline{I}$$

1.11

$$U = 380 \text{ V} ; Z = 16 + j12 \Omega$$

1.11



$$\underline{U} = 380 \angle 0^\circ \text{ V}$$

$$a) \quad \underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{380 \angle 0^\circ}{20 \angle 36,9^\circ} = 19 \angle -36,9^\circ = (15,2 - j11,4) \text{ A}$$

$$\underline{S} = \underline{U} \underline{I}^* = 380 \angle 0^\circ \cdot 19 \angle 36,9^\circ = 7220 \angle 36,9^\circ = (5774 + j4335) \text{ VA}$$

$\downarrow \qquad \qquad \downarrow$
 $P + jQ$

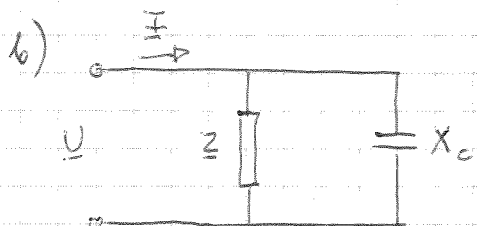
alt.

$$P = UI \cos \varphi = 380 \cdot 19 \cos 36,9^\circ = 5774 \text{ W}$$

$$Q = UI \sin \varphi = 380 \cdot 19 \sin 36,9^\circ = 4335 \text{ VAR}$$

alt.

$$S = \frac{U^2}{Z} = \frac{380^2}{20} = 7220 \text{ VA} \rightarrow \begin{aligned} P &= S \cos \varphi \\ Q &= S \sin \varphi \end{aligned}$$



$$\underline{I} = \underline{I}_{\min}$$

$$S = \sqrt{P^2 + Q_s^2} \rightarrow S_{\min} \text{ om } Q = 0$$

$$\rightarrow I_{\min} (S = UI)$$

$$\underline{Z} = R + jX_L \rightarrow \text{förbrukar } Q_L$$

$$C \rightarrow \text{producerar, alstrar } Q_C$$

$$Q_L = Q_C \rightarrow \underline{I} = \underline{I}_{\min}$$

$$X_L I^2 = \frac{U^2}{X_C} = Q \Rightarrow X_C = \frac{U^2}{Q} = \frac{380^2}{4335} = 33,3 \Omega$$

$$S = UI = 380 \cdot 15,2 = 5776 \text{ VA}$$

$$P = S = 5776 \text{ W}$$

$$Q = 0$$

alt $j11,4 = \frac{U}{-jX_C}$ ritas
 $X_C = \frac{380}{11,4} = 33,3 \Omega$

1.11

1.11

c)

 X_c minskar med 25%

$$X_c' = 0,75 \cdot X_c = 0,75 \cdot 33,3 = 25 \Omega$$

$$Q_c' = \frac{U^2}{X_c'} = \frac{380^2}{25} = 5776 \text{ VAR}$$

$$Q = Q_L - Q_c = 4335 - 5776 = -1441 \text{ VAR}$$

 Q matas ut mot generator; generator upptar 1441 VAR

$$P = 5774 \text{ W}$$

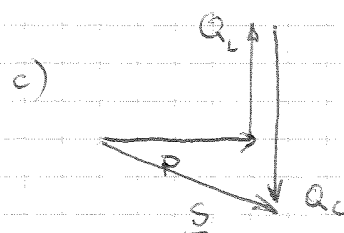
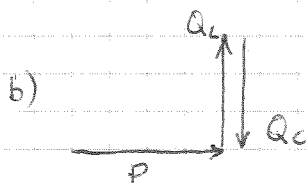
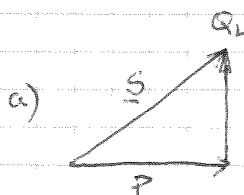
$$S = \sqrt{5774^2 + 1441^2} = 5941 \text{ VA}$$

alt:

$$\begin{aligned} Z_{\text{ekv}} &= Z // X_c \Rightarrow \frac{(16 + j12) \cdot (-j25)}{16 + j12 - j25} = \frac{300 - j400}{16 - j13} = \\ &= \frac{500 \angle -53,1^\circ}{20,62 \angle -39,1^\circ} = 24,25 \angle -14^\circ \Omega \end{aligned}$$

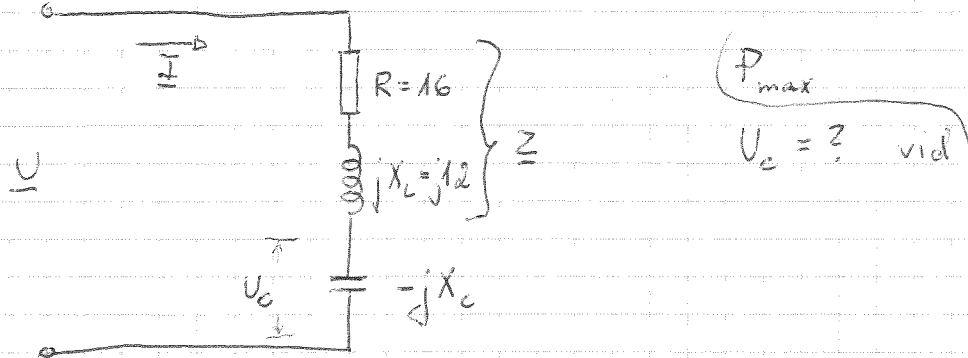
$$I = \frac{U}{Z_{\text{ekv}}} = \frac{380 \angle 0^\circ}{24,25 \angle -14^\circ} = 15,67 \angle 14^\circ \text{ A} \quad \text{kap}$$

$$S = U I^* = 380 \angle 0^\circ \cdot 15,67 \angle -14^\circ = 5955 \angle -14^\circ = (5778 - j1441) \text{ VA}$$



1.111.11

d)

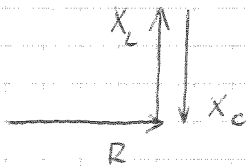


$$P = UI \cos \varphi \rightarrow \text{om } \varphi = 0 \rightarrow P_{\max}$$

$$Z_{\text{ekv}} = \sqrt{R^2 + (X_L - X_C)^2} \quad Z_{\text{ekv}} = Z_{\min} \text{ om } X_L = X_C$$

(då $I_{\max}$)

$$\rightarrow X_C = -j12 \, \Omega \quad \rightarrow Z = R = 16 \, \Omega$$



$$I = \frac{U}{Z} = \frac{380}{16} \, \text{A}$$

$$U_C = \frac{380}{16} \cdot 12 = \underline{\underline{285 \, \text{V}}}$$

1.12

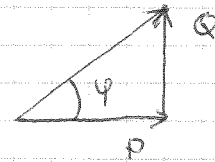
$$U = 220 \text{ V}$$

$$I, \cos \varphi = ?$$

	P(kW)	$\cos \varphi$	Q(kVAR)
1	1,2	1,0	0
2	2,0	0,6	2,67
3	1,2	0,2 kap	-5,88

$$\Sigma P = 4,4$$

$$\Sigma Q = -3,21$$



$$\tan \varphi = \frac{Q}{P}$$

$$S = \sqrt{P^2 + Q^2} = \sqrt{4,4^2 + (-3,21)^2} = 5,45 \text{ kVA}$$

$$I = \frac{S}{U} = \frac{5450}{220} = \underline{\underline{24,8 \text{ A}}}$$

$$\cos \varphi = \frac{\Sigma P}{S} = \frac{4,4}{5,45} = 0,81 \text{ kap} \quad (\Sigma Q - \text{negativ})$$