15 The Smith Chart, Impedance Matching, and Transmission Line Circuits

Errors using inadequate data are much less than those using no data at all

—attributed to Charles Babbage (1791–1871)
Designer of the "difference machine" –
the first programmable computing machine and
predecessor to modern computers

15.1 Introduction

A look back at much of what we did with transmission lines reveals that perhaps the dominant feature in all our calculations is the use of the reflection coefficient. The reflection coefficient was used to find the conditions on the line, to calculate the line impedance, and to calculate the standing wave ratio. Voltage, current, and power were all related to the reflection coefficient. The reflection coefficient, in turn, was defined in terms of the load and line impedances (or any equivalent load impedances such as at a discontinuity). You may also recall, perhaps with some fondness, the complicated calculations which required, in addition to the use of complex variables, the use of trigonometric, harmonic and hyperbolic functions. Thus, the following proposition: Build a graphical chart (or an equivalent computer program) capable of representing the reflection coefficient as well as load impedances in some general fashion and you have a simple method of designing transmission line circuits without the need to perform rather tedious calculations. This has been accomplished in a rather general tool called the Smith chart. The Smith chart is a chart of normalized impedances (or admittances) in the reflection coefficient plane. As such, it allows calculations of all parameters related to transmission lines as well as impedances in open space, circuits, and the like. Although the Smith chart is rather old, it is a common design tool in electromagnetics. Some measuring instruments such as network analyzers actually use a Smith chart to display conditions on lines and networks. Naturally, any chart can also be implemented in a computer program, and the Smith chart has, but we must first understand how it works before we can use it either on paper or on the screen. A computerized Smith chart can then be used to analyze conditions on lines. The examples provided here are solved using graphical tools and a printed Smith chart, rather than the computer program, to emphasize the techniques and approximations involved although some of the numerical results listed were obtained with a computerized Smith chart (**smith-chart.m**) available with this text (see **page xi**).

The Smith chart is an impedance chart. As such it does not provide for direct calculations of voltages, currents, or power. Nevertheless, it is a useful tool in the calculation of voltages and currents as well as power since it provides important information such as the generalized reflection coefficient, standing wave ratio, and the location of voltage and current maxima and minima. With the information available from the Smith chart, the formulas developed in **Chapter 14** can then be used to obtain the required values or conditions.

N. Ida, *Engineering Electromagnetics*, DOI 10.1007/978-3-319-07806-9_15,

15.2 The Smith Chart[1]

To better understand the Smith chart and to gain some insight in its use, we will "build" a Smith chart, gradually, based on the definitions of the reflection coefficient. Then, after all aspects of the chart are understood, we will use the chart in a number of examples to show its utility. In the process, we will also define a number of transmission line circuits for which the Smith chart is commonly used. Consider the circuit in **Figure 15.1**. The line impedance is real and equals Z_0, but the load is a complex impedance $Z_L = R_L + jX_L$, where R_L is the load resistance and X_L the load reactance. The reflection coefficient [see **Eqs. (14.91)** and **(14.92)**] may be written in one of two forms. The first is a rectangular form (i.e., written in complex variables):

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(R_L - Z_0) + jX_L}{(R_L + Z_0) + jX_L} = \Gamma_r + j\Gamma_i \quad (15.1)$$

The reflection coefficient is not modified by normalizing the numerator and denominator by Z_0:

$$\Gamma_L = \frac{(Z_L - Z_0)/Z_0}{(Z_L + Z_0)/Z_0} = \frac{(R_L/Z_0 - 1) + jX_L/Z_0}{(R_L/Z_0 + 1) + jX_L/Z_0} = \frac{(r-1) + jx}{(r+1) + jx} = \Gamma_r + j\Gamma_i \quad (15.2)$$

To obtain this result, we substituted $r = R_L/Z_0$ and $x = X_L/Z_0$ as the normalized resistance and reactance. For much of the remainder of this chapter, we will drop the specific notation for load partly to simplify notation but mostly because the magnitude of the reflection coefficient remains constant along the line and, therefore, the results we obtain apply equally well for any impedance on the line (see **Figure 15.2**). In the latter case, the generalized reflection coefficient is obtained and this can be written in exactly the same form as **Eq. (15.1)** or **(15.2)** by replacing Z_L with $Z(z)$. **Equation (15.2)** defines a complex plane for the reflection coefficient as shown in **Figure 15.3a**. Any normalized impedance (load impedance or line impedance) is represented by a point on this diagram.

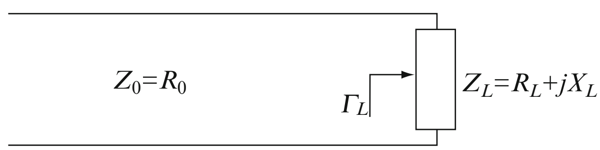


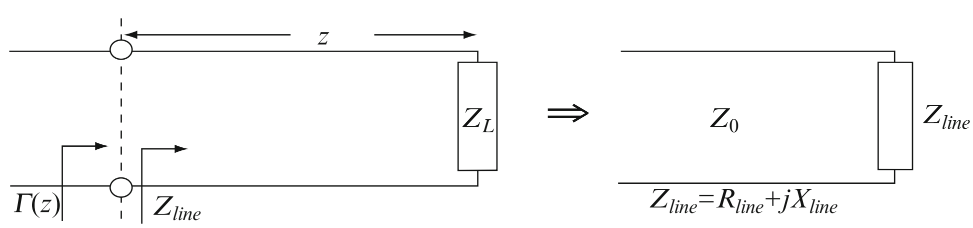
Figure 15.1 A simple transmission line used to introduce the Smith chart

The second form of the reflection coefficient is the polar form. This may be written as

$$\Gamma_L = |\Gamma|e^{j\theta_\Gamma} = |\Gamma|(\cos\theta_\Gamma + j\sin\theta_\Gamma) \quad (15.3)$$

where θ_Γ is the phase angle of the load reflection coefficient as discussed in **Section 14.7.1**. For a given magnitude of the reflection coefficient, the phase angle defines a point on the circle of radius $|\Gamma_L|$. Thus, since $|\Gamma_L| \leq 1$, only that section of the rectangular diagram enclosed by the circle of radius 1 is used, as shown in **Figure 15.3b**. The polar form is more convenient to use than the rectangular form, but we will, for the moment, retain both.

Figure 15.2 Use of an equivalent transmission line to describe the line impedance at a distance z from the load



[1] The Smith chart was introduced by Phillip H. Smith in January 1939. Smith developed the chart as an aid in calculation and called it a "transmission line calculator." In spite of its age, the chart is as useful as ever as a standard tool in analysis either in its printed form, slide-rule form, or, more recently, as computer programs and instrument displays.

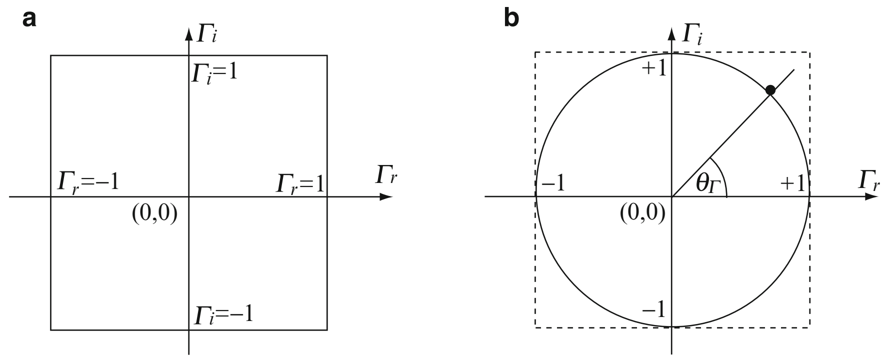


Figure 15.3 The complex plane representation of the reflection coefficient. (**a**) In rectangular form. **(b)** In polar form

We now go back to the rectangular representation and calculate the real and imaginary parts of the reflection coefficient in terms of the normalized impedance. The starting point is **Eq. (15.2)**:

$$\Gamma_r + j\Gamma_i = \frac{(r-1)+jx}{(r+1)+jx} \tag{15.4}$$

Cross-multiplying gives

$$(r+1)\Gamma_r - \Gamma_i x + j\Gamma_i(r+1) + jx\Gamma_r = (r-1)+jx \tag{15.5}$$

Separating the real and imaginary parts and rearranging terms, we get two equations:

$$(\Gamma_r - 1)r - \Gamma_i x = -(\Gamma_r + 1) \tag{15.6}$$

$$\Gamma_i r + (\Gamma_r - 1)x = -\Gamma_i \tag{15.7}$$

We now write two equations: one for r and one for x, by first eliminating x and then, separately, r.
From **Eq. (15.7)** we write

$$x = -\frac{\Gamma_i(r+1)}{\Gamma_r - 1} \tag{15.8}$$

Substituting this into **Eq. (15.6)** we get

$$(\Gamma_r - 1)r + \frac{\Gamma_i^2(r+1)}{\Gamma_r - 1} = -(\Gamma_r + 1) \tag{15.9}$$

Multiplying both sides by $\Gamma_r - 1$ and rearranging terms, this gives
After rearranging terms, this gives

$$\Gamma_r^2(r+1) - 2\Gamma_r r + \Gamma_i^2(r+1) = 1 - r \tag{15.10}$$

Dividing by the common term $(r + 1)$,

$$\Gamma_r^2 - \frac{2\Gamma_r r}{(r+1)} + \Gamma_i^2 = \frac{1-r}{(r+1)} \tag{15.11}$$

Adding $r^2/(r + 1)^2$ to both sides of the equation and rearranging terms, we get

$$\boxed{\left(\Gamma_r - \frac{r}{r+1}\right)^2 + \Gamma_i^2 = \frac{1}{(r+1)^2}} \tag{15.12}$$

Repeating the process, we now eliminate r in **Eq. (15.7)** by first writing from **Eq. (15.6)**:

$$r = -\frac{(\Gamma_r + 1) - \Gamma_i x}{(\Gamma_r - 1)} \tag{15.13}$$

Substituting this back into **Eq. (15.7)**,

$$\Gamma_i \frac{(\Gamma_r + 1) - \Gamma_i x}{\Gamma_r - 1} + (\Gamma_r - 1)x = -\Gamma_i \tag{15.14}$$

Multiplying both sides of **Eq. (15.14)** by $\Gamma_r - 1$ and rearranging terms we get

$$(\Gamma_r - 1)^2 x + \Gamma_i^2 x - 2\Gamma_i = 0 \tag{15.15}$$

The equation now is divided by x:

$$(\Gamma_r - 1)^2 + \Gamma_i^2 - 2\Gamma_i \left(\frac{1}{x}\right) = 0 \tag{15.16}$$

To bring this into a useful form, we add $1/x^2$ to both sides of the equation:

$$(\Gamma_r - 1)^2 + \Gamma_i^2 - 2\Gamma_i \left(\frac{1}{x}\right) + \left(\frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2 \tag{15.17}$$

Rearranging terms we get

$$\boxed{(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2} \tag{15.18}$$

Both **Eqs. (15.12)** and **(15.18)** describe circles in the complex Γ plane.

Equation (15.12) is the equation of a circle, with its center at $\Gamma_r = r/(r + 1)$, $\Gamma_i = 0$ and radius $1/(r + 1)$. The center of the circle is on the real axis and can be anywhere between $\Gamma_r = 0$ for $r = 0$ and $\Gamma_r = 1$ for $r \to \infty$. For example, for $r = 1$, the center of the circle is at $\Gamma_r = 0.5$ and its radius equals 0.5. A number of these circles are drawn in **Figure 15.4a**. The larger the normalized resistance, the smaller the circle. All circles pass through $\Gamma_r = 1$, $\Gamma_i = 0$. The normalized resistance r can only be positive. Should there ever be a need to describe normalized impedances with negative real part, these must be multiplied by -1 before analysis using the Smith chart can commence.

From **Eq. (15.18)**, we obtain a second set of circles for x. Since x can be positive or negative, the circles are centered at $\Gamma_r = 1$, $\Gamma_i = 1/x$ for positive values of x and at $\Gamma_r = 1$, $\Gamma_i = -1/x$ for x negative. These circles are shown in **Figure 15.4b** for a number of values of the normalized reactance x. **Figure 15.5** shows the r and x circles on the Γ plane, truncated at the circle $|\Gamma| = 1$. This is the basic Smith chart. A number of properties of the two sets of circles are immediately apparent:

(1) The circles are loci of constant r or constant x.
(2) x and r circles are orthogonal to each other.
(3) There is an infinite number of circles for r and for x.
(4) All circles pass through the point $\Gamma_r = 1$, $\Gamma_i = 0$.
(5) The circles for x and $-x$ are images of each other, reflected about the real axis.
(6) The center of the chart is at $\Gamma_r = 0$, $\Gamma_i = 0$.
(7) The intersections of the r circles with the real axis, for $r = r_0$ and $r = 1/r_0$, occur at points symmetric about the center of the chart ($\Gamma_r = 0$, $\Gamma_i = 0$).
(8) The intersections of the x circles with the outer circle ($|\Gamma| = 1$) for $x = x_0$ and $x = 1/x_0$ occur at points symmetrically opposite each other.
(9) The intersection of any r circle with any x circle represents a normalized impedance point.
(10) The real part of the normalized impedance, r, can only be positive but x can be negative or positive.

The chart as described above is an impedance chart since we defined all points in terms of normalized impedance. We will see how to use the chart as an admittance chart later.

In addition to the properties of the r and x circles given above, we note the following:

(1) The point $\Gamma_r = 1$, $\Gamma_i = 0$ (rightmost point in **Figure 15.5**) represents $r = \infty$, $x = \infty$. This is the impedance of an open transmission line. This point is therefore the ***open circuit point***.

(2) The diametrically opposite point, at $\Gamma_r = -1, \Gamma_i = 0$ represents $r = 0, x = 0$. This is the impedance of a short circuit and is called the ***short circuit point***.

(3) The outer circle represents $|\Gamma| = 1$. The center of the diagram represents $|\Gamma| = 0$. Any circle centered at the center of the diagram ($\Gamma_r = 0, \Gamma_i = 0$) with radius a is a circle on which the magnitude of the reflection coefficient is constant, $|\Gamma| = a$. Moreover, if we take the intersection between any r and x circles, the distance between this point to the center of the diagram is the magnitude of the reflection coefficient for this normalized impedance. A circle drawn through this point represents the generalized reflection coefficient at different locations on the line for this normalized load impedance. The intersection of the reflection coefficient circle with r and x circles represents line impedances at various locations. These aspects of the use of transmission lines are shown in **Figure 15.5**. For example, point A represents an impedance $r_A + jx_A$ and point B represents an impedance $r_B + jx_B$, but the magnitude of the reflection coefficient is the same. This will later be used to calculate the line impedance as well as voltages and currents on the line.

(4) Any point on the chart represents a normalized impedance, say, $z = r + jx$. The admittance of this point is $y = 1/(r + jx) = (r - jx)/(r^2 + x^2)$. The admittance point corresponding to an impedance point lies on the reflection coefficient circle that passes through the impedance point, diametrically opposite to the impedance point. Thus, if we mark a normalized impedance as z and draw the reflection coefficient circle through point z, this circle passes through the admittance point $y = 1/z$. The admittance point y is found by passing a line through z and the center of the diagram.

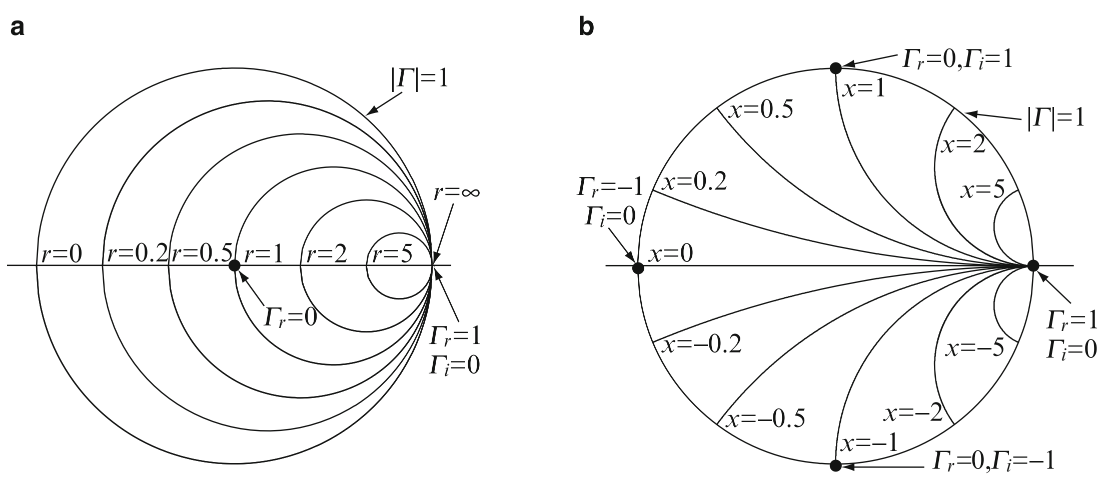


Figure 15.4 The basic components of the Smith chart. (**a**) Circles of constant values of r. (**b**) Circles of constant values of x or $-x$

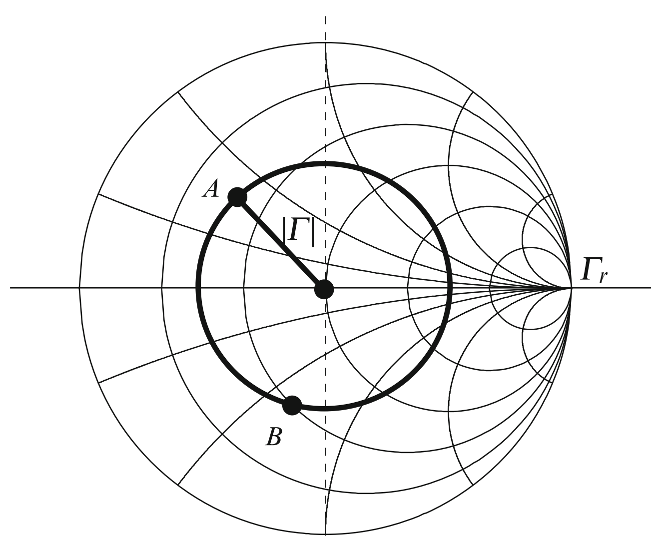


Figure 15.5 The Smith chart. A normalized impedance is a point on the Smith chart defined by the intersection of a circle of constant normalized resistance r and a circle of constant normalized reactance x

The intersection of this line with the reflection coefficient circle is point y. These steps are shown in **Figure 15.6a**. These considerations will later be used to calculate admittances instead of impedances.

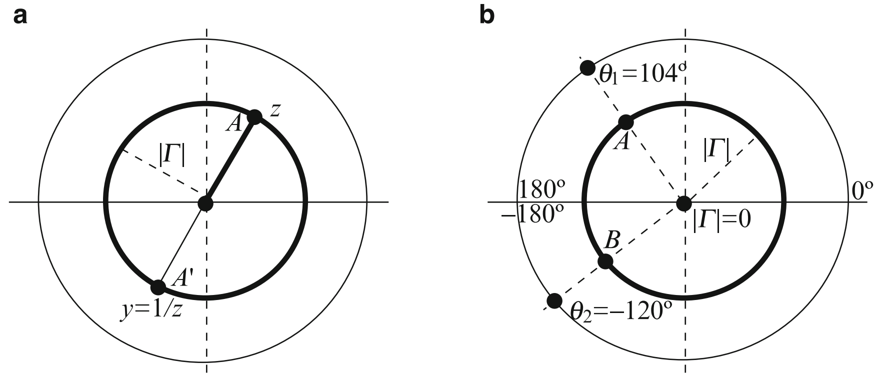


Figure 15.6 (**a**) Normalized impedance, reflection coefficient, and normalized admittance. (**b**) Indication of phase angle of the reflection coefficient on the Smith chart

The Smith chart also provides for calculation of phase angles and lengths of transmission lines. For this purpose, the Smith chart is equipped with a number of scales, marked on the outer periphery of the diagram. These are defined as follows:

(1) For a given impedance, a point on the chart is found. The distance from the center of the chart to the point is the magnitude of the line reflection coefficient. If the line connecting the center of the chart with the impedance point is continued until it intersects the outer ($\Gamma = 1$) circle, the location of intersection gives the phase angle of the reflection coefficient in degrees. This is the first set of values given on the circumference of the Smith chart and is shown in **Figure 15.6b**. Note that the open circuit point has zero phase angle ($\Gamma = +1$) and the short circuit point has either a 180° or −180° phase angle. The difference is in the sign of the imaginary part of the load impedance (below or above the real axis). Intermediate points will vary in phase depending on the distance from the load. For example, for point A in **Figure 15.6b**, the phase angle of the reflection coefficient is 104°, whereas for point B it is −120°.

(2) We recall that the distance between a point of maximum voltage and a point of minimum voltage was found to be $\lambda/4$ in **Section 14.7.3**. In particular, the impedance of a shorted transmission line changes from zero to infinity (or negative infinity) if we move a distance $\lambda/4$ from the short. Thus, the distance between the short circuit and open circuit points is $\lambda/4$. This fact is indicated on the outer circle of the chart, starting at the short circuit point. Since the short (or any other load) can be anywhere on a line, we may wish to move either toward the generator or toward the load to evaluate the line behavior. These two possibilities are indicated with arrows showing the direction toward load and toward generator (**Figure 15.7**). Although the distance is marked from the short circuit point, the distance is always relative: if a point is given at any location on the chart, movement on the chart, a distance $\lambda/4$ represents half the circumference of the chart.

(3) The direction toward the generator is the clockwise direction. If we wish to calculate the line impedance starting from the load, we move in the clockwise direction toward the generator. If, on the other hand, we wish to calculate the line impedance starting from the generator going toward the load or, starting at the load and going away from the generator, we must move in the counterclockwise direction and use the appropriate distance charts (see **Figure 15.7**).

(4) The whole Smith chart encompasses one-half wavelength. This, of course, is due to the fact that all conditions on lines repeat at intervals of $\lambda/2$ regardless of loading or any other effect that may happen on the line. If we need to analyze lines longer that $\lambda/2$, we simply move around the chart as many half-wavelengths as are necessary. Only the remainder length (length beyond any integer numbers of half-wavelengths) needs to be analyzed.

The Smith chart also allows for the calculation of standing wave ratios. The standing wave ratio is calculated from the reflection coefficient as

$$\text{SWR} = \frac{1 + |\Gamma|}{1 - |\Gamma|} \quad \text{[dimensionless]} \tag{15.19}$$

We note that the circle of radius $|\Gamma|$ intersects the positive real axis at $x = 0$. At this point, the normalized impedance is equal to r and the reflection coefficient is given as $\Gamma = (r - 1)/(r + 1)$. Substituting this into the relation for SWR, we get

$$\text{SWR} = r \tag{15.20}$$

Thus, the standing wave ratio equals the value of normalized resistance at the location of intersection of the reflection coefficient circle and the real axis, right of the center of the Smith chart. From property (7) above, the intersection of the reflection coefficient circle with the real axis, left of the center of the chart, is at point $1/r$. Thus, this point gives the value 1/SWR. The two points are shown for the reflection coefficient in **Figure** 15.7.

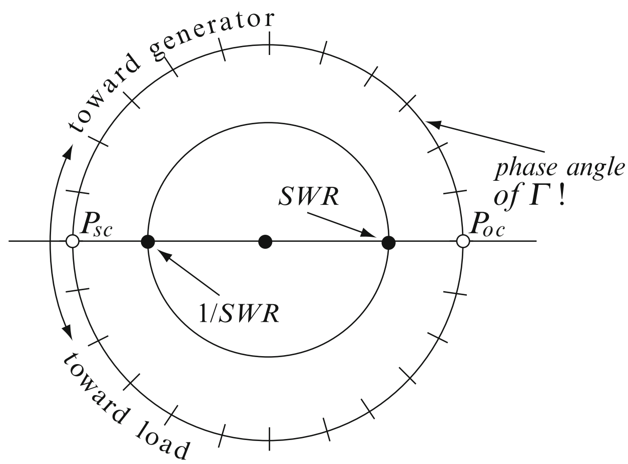


Figure 15.7 Directions on the Smith chart and indication of SWR. The distance between short and open circuit points is $\lambda/4$

Now that we discussed the individual parts making up the Smith chart, it is time to put it all together. The result is the Smith chart shown in **Figure** 15.8. You will immediately recognize the r and x circles as well as the scales discussed. There are, however, a number of other scales given at the bottom of the chart as well as a number of indications on the chart itself which we have not discussed. These have to do with losses on the line (which we have neglected) and the use of the chart as an admittance rather than impedance chart (which we will take up later).

Although the chart is relatively simple, it contains considerable information and can be used in many different ways and for purposes other than transmission lines. To see how the chart is used, we will discuss next a number of applications of the Smith chart to design of transmission lines. Because the chart gives numerical data, the examples must also be numerical, but, in general, the equations in the previous chapter can also be used for this purpose. The main difference in the Smith chart solution and the analytic solution is that the Smith chart uses normalized impedances, whereas in analytic calculations, we tend to use the actual values of the impedance. Also, because it is a graphical chart, the results are approximate and depend on our ability to accurately read the values off the chart. The Smith chart is available commercially as a paper chart as well as computer software. The advantage of a software-based Smith chart is that calculations are exact in addition to the ease of analysis and display of results.

Example 15.1 Calculation of Line Conditions The_Smith_Chart.m

A long line with characteristic impedance $Z_0 = 50\ \Omega$ operates at 1 GHz. The speed of propagation on the line is c and load impedance is $75 + j100\ \Omega$. Find:

(a) The reflection coefficient at the load.
(b) The reflection coefficient at a distance of 20 m from the load toward the generator.
(c) Input impedance at 20 m from the load.
(d) The standing wave ratio on the line.
(e) Locations of the first voltage maximum and first voltage minimum from the load.

Solution:

(a) **(1)** Normalize the load impedance: $z_l = (75 + j100)/50 = 1.5 + j2$. Enter this on the Smith chart at the intersection of the resistance circle equal to 1.5 and reactance circle equal to 2. This is point P_2 in **Figure** 15.9.

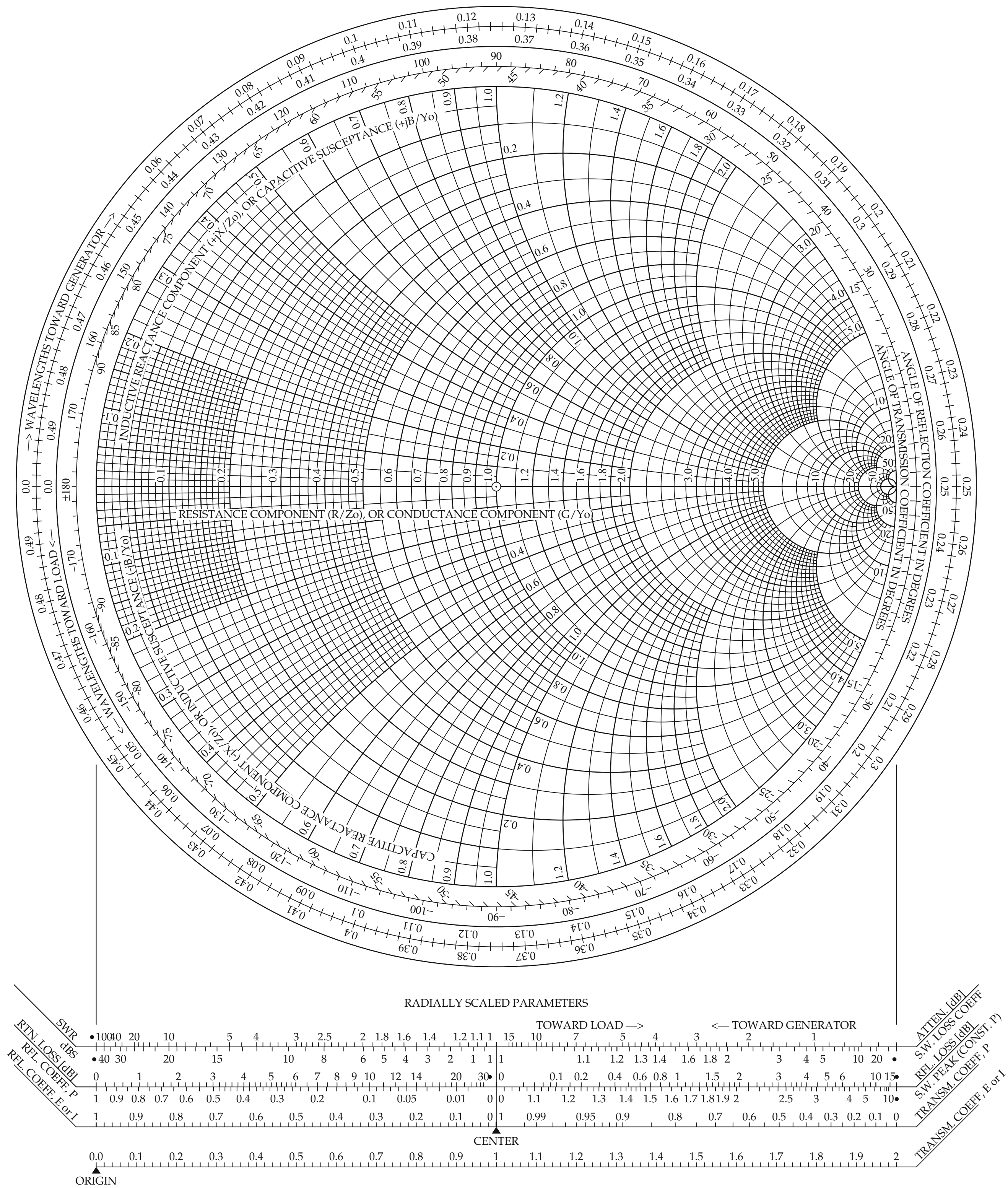


Figure 15.8 The complete Smith chart

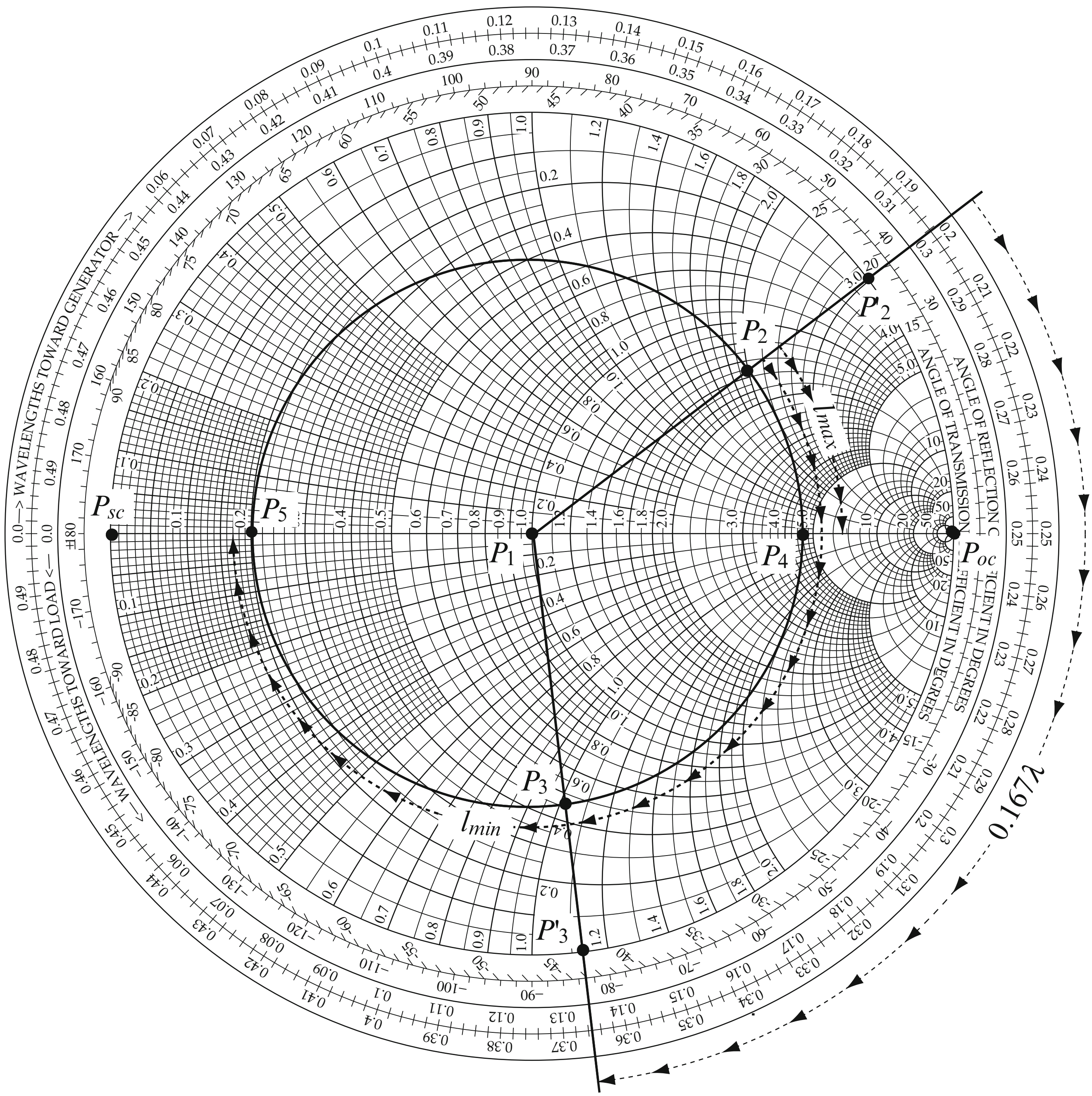


Figure 15.9 The Smith chart for **Example 15.1**

(2) With center at origin (point P_1), draw a circle that passes through point P_2. This circle is the reflection coefficient circle and gives $|\Gamma|$ anywhere on the line. Measure the length of the radius (distance between P_1 and P_2) and divide by the radius of the Smith chart's outer circle (distance between P_1 and P_2'). This gives the magnitude of the reflection coefficient. In this case, $|\Gamma| = 0.6439$.

Note: The radius of the Smith chart should be equal to 1, but to facilitate reading, the size is often different, thus the need to calculate the magnitude of the reflection coefficient.

(3) Draw a straight line between P_1 and P_2 and extend it to the periphery of the chart to point P_2'. The angle (in degrees, on the periphery) is the phase angle of the reflection coefficient at the load. In this case, it is 37.3°. Alternatively, read the "wavelength toward generator" circle. This is equal to 0.198 at point P_2'. To calculate the angle, subtract this value from the value on the real axis (open circuit point) and multiply by 4π: $(0.25 - 0.198) \times 4\pi = 0.208\pi$ radians or 37.3°. Thus, the answer to **(a)** is

$$\Gamma_L = |\Gamma_L| e^{j\theta r} = 0.6439 e^{j0.208\pi} = 0.6439 \angle 37.3^\circ$$

(b) To calculate the reflection coefficient at 20 m from the load, moving toward the generator, we first calculate the wavelength because the chart can only accommodate wavelengths:

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{10^9} = 0.3 \quad [\mathrm{m}]$$

Since the circumference of the Smith chart represents 0.5λ (or 0.15 m), the 20 m distance represent (20/0.15) = 133.3334 half-wavelengths. Thus, we move around the reflection coefficient circle toward the generator 133 times, starting at P_2. This puts us exactly where we started (at point P_2). The remainder is one-third of a half-wavelength or $\lambda/6$ (0.167λ).

We now move from point P_2 along the reflection coefficient circle, a distance of 0.167 wavelengths toward the generator to point P_3. Connecting this point with the center of the chart and with the circumference gives the intersection with the reflection coefficient circle at P_3 and with the circumference at P_3'. This point gives the phase angle of the reflection coefficient as −82.7°.Thus the reflection coefficient at 20 m from the load is

$$\Gamma = 0.6439 \angle -82.7^\circ.$$

(c) The input impedance 20 m from the load is represented at point P_3. The normalized input impedance is

$$z(l = 20\,\mathrm{m}) = 0.468 - j1.02$$

Multiplying by the characteristic line impedance ($Z_0 = 50\ \Omega$), we get the actual line impedance as

$$Z(l = 20\,\mathrm{m}) = 23.4 - j51.1 \quad [\Omega].$$

(d) The reflection coefficient circle intersects the real axis at point P_4. At this point, $r = 4.62$. This is the standing wave ratio: SWR = 4.62. At point P_5 (on the other side of the reflection coefficient circle) r = 1/SWR = 0.217. At point P_4, the line impedance is real and maximum and equals $Z_{max} = Z_0 \times 4.62 = 230.8\ \Omega$ = At point P_5, the impedance is minimum and real and equals $Z_{min} = Z_0/4.62 = 10.83\ \Omega$.

(e) Location of maximum voltage is on the real axis at the same point where SWR = 4.76 since, at this point, the line impedance is maximum (and real). Thus, moving from point P_2 to the positive real axis, we reach a voltage maximum: the distance is the difference in wavelengths between point P_{oc} and point P_2 or $l_{max} = 0.25\lambda - 0.198\lambda = 0.052\lambda$ from the load. The voltage minimum is a quarter-wavelength away (where 1/SWR = 0.21) at point P_5 or $l_{min} = 0.302\lambda$ from the load. In terms of actual distance the first maximum occurs at a distance of $0.052 \times 0.3 = 0.0156$ m, or 15.6 mm from the load. The first minimum occurs at $0.302 \times 0.3 = 0.0906$ m or 90.6 mm from the load.

15.3 The Smith Chart as an Admittance Chart

We mentioned earlier that the Smith chart may be used as an admittance chart. In **Figure 15.6a**, we showed that for any given normalized impedance, the admittance is found by locating the normalized impedance point $z = r + jx$ on the Smith chart, drawing the reflection coefficient circle, and then drawing a straight line that passes through the impedance point, the center of the chart, and then intersects the reflection coefficient circle, again, on a point diametrically opposite the impedance point, at point y. This point represents the normalized admittance of the load. Any normalized impedance may be converted into its equivalent admittance using this simple step.

In addition to this, we note that an infinite normalized impedance (open circuit point on the impedance Smith chart) represents infinite admittance on the admittance Smith chart. Similarly, the short circuit point on the impedance Smith chart represents zero admittance on the admittance Smith chart (see **Figure 15.10**).

The admittance may be written in terms of the impedance at point z as

$$y = \frac{1}{r + jx} = \frac{r}{r^2 + x^2} - j\frac{x}{r^2 + x^2} = g - jb \tag{15.21}$$

Since we use the same chart, the constant resistance circles now become constant conductance circles, and the constant reactance circles become constant susceptance circles. All other aspects of the chart, including phase angles, distances, etc., remain unchanged.

The use of the Smith chart as an admittance chart is shown in **Figure 15.10**, in comparison with the impedance chart.

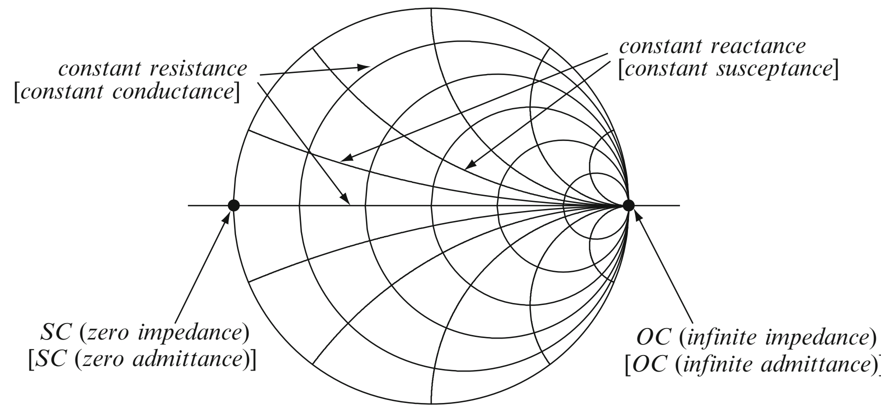


Figure 15.10 Relations between the impedance and admittance Smith charts. Descriptions in *square brackets* are for the admittance chart

Example 15.2 The_Smith_Chart.m

A load, such as an antenna, of impedance $Z_L = 50 - j100\ \Omega$ is connected to a lossless transmission line with characteristic impedance $Z_0 = 100\ \Omega$. The line operates at 300 MHz and the speed of propagation on the line is $0.8c$:

(a) Calculate the input admittance a distance 2.5 m from the load.
(b) Calculate the input impedance a distance 2.5 m from the load.
(c) Suppose the load is shorted accidentally. What is the input admittance at the same point?

Solution: To calculate the input admittance, we first calculate the wavelength on the line. The load is then located on the impedance chart and the admittance is found on the reflection coefficient circle. Then, we move toward the generator a distance 2.5 m (in wavelengths, of course) to find the normalized input admittance. The admittance is found by multiplying with the characteristic admittance of the line. The input impedance can be found from the input admittance by finding the diametrically opposite point on the reflection coefficient circle.

(a) The normalized load impedance is

$$z_L = \frac{50 - j100}{100} = 0.5 - j1$$

This is marked on the chart as point P_2 in **Figure 15.11**. The reflection coefficient circle is drawn around point P_1, with a radius equal to the distance between P_2 and P_1. The admittance point is P_3. The normalized load admittance is

$$y_L = 0.4 + j0.8$$

The wavelength on the line is $\lambda = 0.8c/f = 2.4 \times 10^8/3 \times 10^8 = 0.8$ m. The given distance represents $2.5/0.8 = 3.125$ wavelengths. To find the input admittance, we move from the load admittance point toward the generator a distance of 0.125λ (the three wavelengths mean simply moving six times around the chart to get to

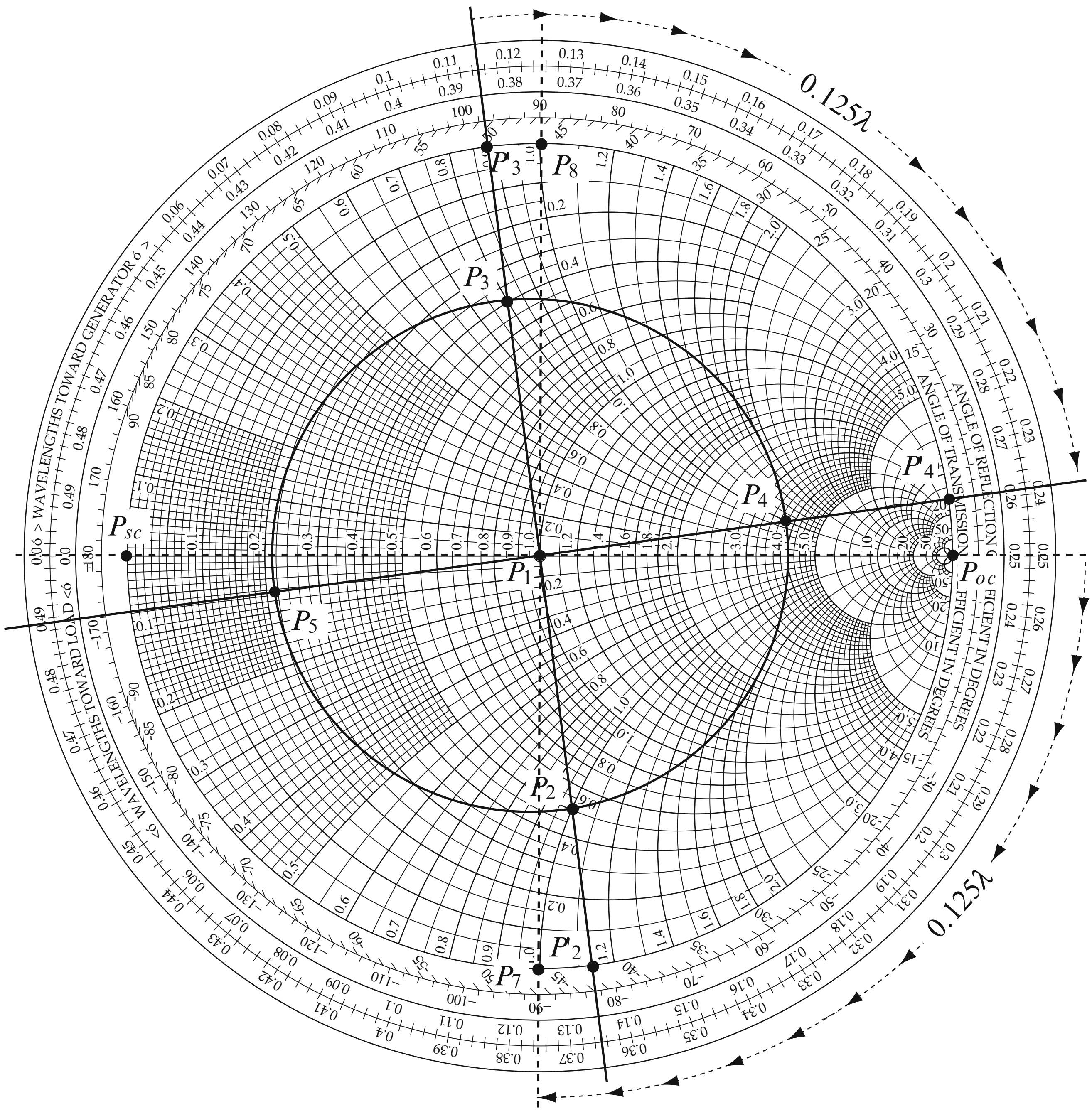


Figure 15.11 Use of the Smith chart as an admittance chart **(Example 15.2)**

the initial point). Moving from point P_3' a distance 0.125λ brings us to point P_4' ($0.114\lambda + 0.125\lambda = 0.239\lambda$). Connecting this point with P_1 intersects the reflection coefficient circle at point P_4. The normalized input line admittance is

$$y_{in} = 4.0 + j1.0$$

The input line admittance is the normalized input line admittance above multiplied by the characteristic line admittance, which equals 0.01:

$$Y_{in} = 0.04 + j0.01 \quad [1/\Omega].$$

(b) The normalized input impedance is found by locating point P_5, which is the diametrically opposite point to P_4, on the reflection coefficient circle. The normalized line impedance at this point is $0.235 - j0.059$. The line impedance is found by multiplying this normalized impedance by the characteristic impedance of the line:

$$Z_{in} = 23.5 - j5.9 \quad [\Omega].$$

(c) If the load is shorted, the load impedance is zero and the line admittance is infinite. This is represented at point P_{oc} on the admittance chart. From here, we move 0.125 wavelengths toward the generator on the outer circle, since for shorted loads, $|\Gamma| = 1$. This point is shown as P_7. The normalized input line admittance is $-j1$. The line admittance is, therefore, $-j0.01$ (line impedance is $j100$, at point P_8).

15.4 Impedance Matching and the Smith Chart

15.4.1 Impedance Matching

When connecting a transmission line to a generator, a load, or another transmission line, the impedances are, in general, mismatched and the result is a reflection coefficient at the load, generator, or discontinuity, which, in turn, generates standing waves on the line. The effect of this reflection was discussed at some length in **Chapter** 14. It is often necessary to match a transmission line to a load or to a generator, for the purpose of eliminating standing waves on the line. Similarly, if a discontinuity exists, such as the connection of an unmatched line section, it is often necessary to eliminate this mismatch before the line can be used. The result of mismatch on a line can be disastrous: large amounts of reactive power may travel along the line which can easily damage circuitry, especially generators.

A transmission line is matched to a load if the load impedance is equal to the characteristic impedance. Similarly, if the line impedance is equal to the generator impedance, the two are matched. To match a load to a line (or a generator for that matter), a matching network is connected between the line and the load, as shown in **Figure 15.12**.

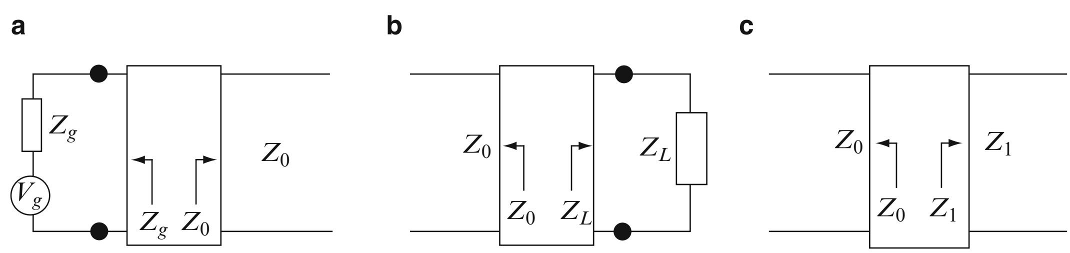


Figure 15.12 Matching networks at (**a**) generator side, (**b**) load side, (**c**) arbitrary location on the line

The location of the matching network depends on the application. If we wish to reduce the standing waves on the line, the matching network should be located as closely as possible to the mismatched impedance. If, however, the line can operate with standing waves, then a more convenient location, at some distance away, can be found. The latter approach is possible since all conditions on the line repeat at intervals of $\lambda/2$. Thus, if a matching network has been designed to be located at a given point on the line, the network can now be moved a distance $\lambda/2$ without affecting the line conditions.

There are two types of impedance matching networks that are particularly useful. One is the so-called stub matching, which makes use of properties of shorted (or open) transmission lines. In this type of network, the impedance on the line is altered by connecting shorted or open transmission lines in parallel or in series with the line to adjust the impedance. The second method of impedance matching is based on the properties of transformers. In effect, we build a transformer which then can match two impedances in a manner similar to that discussed in **Section 10.7.1**.

The following sections discuss these methods and develop the relations required to design matching networks. We use the Smith chart in the design of matching networks for two reasons: First, in many cases, the design is greatly simplified by the use of the Smith chart. Second, and more importantly, the Smith chart is routinely used for this type of application.

15.4.2 Stub Matching

The idea of stub matching is to connect open- or short-circuited sections of transmission lines, either in parallel or in series with the transmission line as shown in **Figure 15.13**. The impedance of the stub and/or location on the line is chosen such that the combined impedance of line and stubs is equal to the characteristic impedance of the line. The details of design of the stubs for the three methods in **Figure 15.13** are discussed next.

Consider first the matching network in **Figure 15.13a**. Assuming a characteristic impedance Z_0 (or admittance Y_0) and a line admittance $Y_0 + jB_0$, at a distance d_1 from the load, the two can be matched by adding a stub in parallel, at distance d_1 from the load, such that the admittance of the stub is $-jB_0$. The distance d_1 defines the imaginary part of the line admittance from **Eq. (14.102)**. l_1 is then that length of the shorted transmission line stub that cancels the imaginary part of the line admittance at the location of the stub. The choice of l_1 and d_1 is not unique, but any practical combination that satisfies the above conditions can be used.

Although a single stub may be used to match any load (except for a purely imaginary load) to any line which has real characteristic impedance, sometimes the physical conditions of the line do not allow perfect matching with a single stub because of physical constraints. In such cases, two stubs, at two fixed locations, may be used. This method is similar to the single stub method, but now we must design the lengths l_1 and l_2 whereas d_1 and d_2 are fixed as shown in **Figure 15.13b**.

In the series matching method in **Figure 15.13c**, the idea is the same as in single stub matching: we must choose a stub length l_1 and place it a distance d_1 from the load so that the sum of the line impedance at that point with that of the stub equals Z_0.

To summarize, in the single stub matching method, we choose the length and position of the stub. In the double stub matching method, we choose the lengths of two stubs whereas their positions are fixed and often prescribed by the device being matched. It is also possible to match loads and other devices by more than two stubs, but we will not discuss these here.

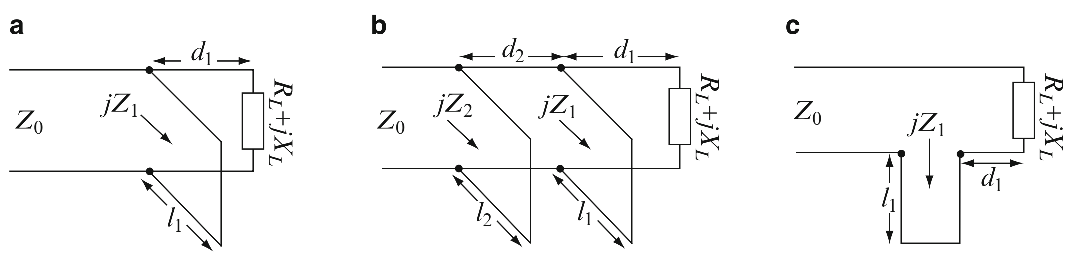


Figure 15.13 (**a**) Single stub matching. (**b**) Double stub matching. (**c**) Series stub matching

15.4.2.1 Single Stub Matching

The idea of single stub matching relies on the fact that the line impedance varies along the line and a parallel or series stub changes only the reactive part of the line impedance. To see how this is accomplished, consider a load impedance $Z_L = R_L + jX_L$ connected on a line of characteristic impedance Z_0. For the load to be matched, its impedance must be changed so that $Z_L' = Z_0$. This is done as follows:

(1) Move along the line from the load (**Figure 15.13a**) and find a point at which $Z(z) = Z_0 + jX(z)$. Note that Z_0 does not have to be real, but in most cases, it will be.
(2) At this point (a distance d_1 from the load), connect a shorted or open transmission line of length l_1 such that the term $jX(z)$ cancels. As a result, the line sees a total impedance equal to Z_0 and the new load (which now is the whole line section to the right of the location of the stub) is matched.

These steps are implemented with the use of the Smith chart with the following differences:

(1) The impedance is first normalized to conform with the requirements of the Smith chart.
(2) If the stub is connected in parallel (**Figure 15.13a**), it is easier to work with admittances. Therefore, the normalized load admittance is first located on the chart.
(3) If the stub is connected in series (**Figure 15.13c**), it is easier to work with normalized impedances.

The stubs will be assumed to have the same characteristic impedance as the line, but this is not a necessary condition. The following two examples show the steps and details involved in parallel and series single stub matching.

Example 15.3 Application: Single Parallel Stub Matching at an Antenna The_Smith_Chart.m

An antenna operates at a wavelength of 2 m and is designed with an impedance of 75 Ω. However, because of mistakes in design, the antenna is badly mismatched. The measured impedance after installation is $15 + j60\ \Omega$. The antenna is connected to a 75 Ω line as shown in **Figure 15.14**. Calculate:

(a) The required shorted stub and its location on the line to match the antenna to the line. The line and stub have the same characteristic impedance.
(b) The shortest required open circuit stub that will accomplish the same purpose as the short circuit stub in **(a)**.

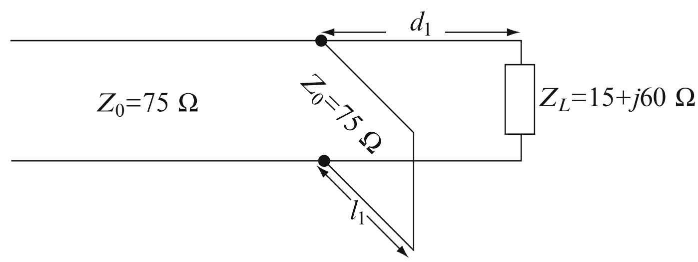


Figure 15.14 Mismatched antenna connected to a line and a stub designed to match the antenna to the line

Solution: First, we find a location on the line at which the real part of the line admittance is equal to the characteristic admittance of the line; that is, find $Z(d_1)$ such that $Y(d_1) = Y_0 + jB(d_1)$. Now, we connect a shorted stub in parallel with the line at this point and of a length such that the imaginary part of the line admittance is canceled. The open circuit stub in **(b)** is placed at the same location and its length is that of the short circuit stub $\pm\ \lambda/4$.

(a) In this case, it is simpler to use the Smith chart as an admittance chart. To do so, we first calculate the normalized load impedance:

$$z_L = \frac{15 + j60}{75} = 0.2 + j0.8.$$

(1) We mark this point as P_2 on the Smith chart in **Figure 15.15**, using the chart as an impedance chart. The reflection coefficient circle is now drawn around the center of the chart, with the radius equal to the distance between P_2 and P_1.
(2) To find the load admittance, we draw a straight line from P_2 through P_1 and extend this line to the periphery of the chart. The line intersects the reflection coefficient circle at point P_3. This point is the normalized load admittance:

$$y_L = 0.294 - j1.176.$$

(3) As we move around the reflection coefficient circle, the line admittance changes. To match the load, we must find the location at which the real part of the line admittance equals the characteristic admittance. Since we are working with normalized admittances, this happens when $\mathrm{Re}\{y_L\} = 1$. This happens at the locations at which the reflection coefficient circle intersects the circle $g = 1$. The two possible points are P_4 and P_5. The line admittance at these points is
At P_4,

$$y_a = 1 + j2.53$$

at P_5,

$$y_b = 1 - j2.53$$

Each one of these points provides one possible solution.

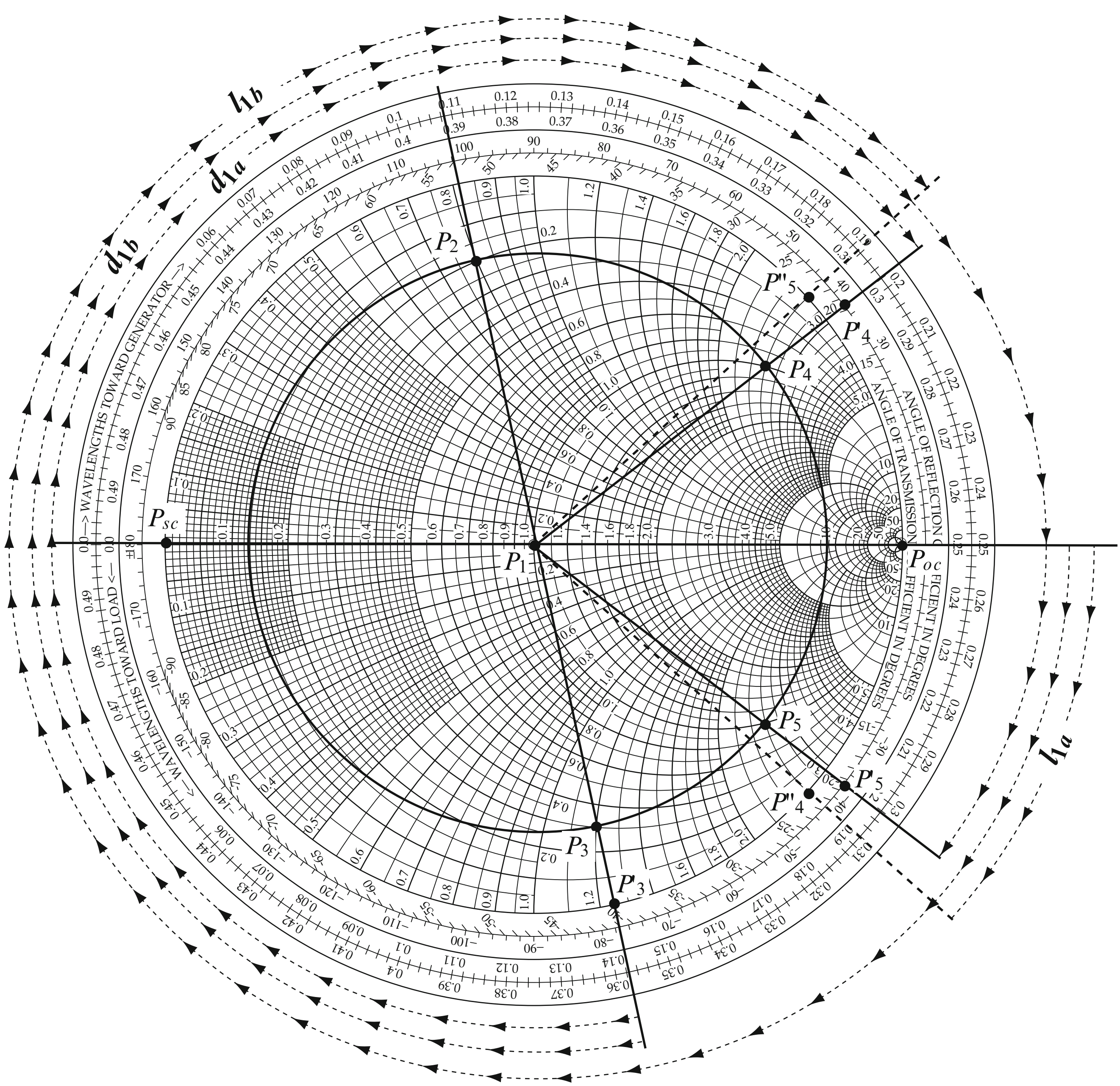


Figure 15.15 Smith chart for **Example 15.3**

(4) Solution No. 1: Point P_4. The distance d_1 for this solution is the distance traveled from point P_3 to point P_4, on the reflection coefficient circle. The distance in wavelengths is the difference in readings between point P_3' and P_4' moving from P_3' to P_4' toward the generator. First, we move a distance of $0.5\lambda - 0.358\lambda = 0.142\lambda$ up to the short circuit point. Then, we move an additional 0.198λ to point P_4'. The total distance is $d_{1a} = 0.142\lambda + 0.198\lambda = 0.34\lambda$.

The normalized line susceptance at this point is 2.6. The stub must, therefore, have a normalized susceptance of -2.6. This point is shown as point P_4'. The length of the stub is the distance from the open circuit point P_{oc} (infinite admittance) to point P_4'' (moving toward the generator). This is $l_{1a} = 0.308\lambda - 0.25\lambda = 0.058\lambda$. Thus, the first possible solution is

$$d_{1a} = 0.337, \quad l_{1a} = 0.058 \quad [\lambda]$$

Since we know the wavelength ($\lambda = 2$ m), we can write the solution in actual lengths:

$$d_{1a} = 0.674, \quad l_{1a} = 0.116 \quad [\text{m}].$$

(5) Solution No. 2. Point P_5. The distance d_1 at this point is the distance between points P_3 and P_5. Again, we move a distance of 0.142λ up to the short circuit point and then a distance of 0.302λ from the short circuit point to point P_5. Thus, $d_{1b} = 0.444\lambda$. The line susceptance at P_5 is -2.6. The stub susceptance must be +2.6. This is marked as point P_5''. The distance from P_{oc} to point P_5'', moving toward the generator, is $l_{1b} = 0.25\lambda + 0.192\lambda = 0.442\lambda$. The second solution is therefore

$$d_{1b} = 0.444\lambda, \quad l_{1b} = 0.442 \quad [\lambda] \quad \text{or} \quad d_{1b} = 0.888, \quad l_{1b} = 0.884 \quad [\text{m}].$$

(b) Because an open line behaves as a shorted line at a distance of $\lambda/4$ from the short, the lines in **(a)** can be replaced by open circuit lines by either shortening the stubs by $\lambda/4$ or lengthening them by $\lambda/4$. Taking in each case the shortest possible stub length (lengthening l_{1a} and shortening l_{1b}), the solutions for open circuit stubs are

$$\begin{matrix} d_{1a} = 0.337\,\lambda = 0.674 \quad [\text{m}], & l_{1a} = 0.308\,\lambda = 0.616 \quad [\text{m}] \\ d_{1b} = 0.444\,\lambda = 0.888 \quad [\text{m}], & l_{1b} = 0.192\,\lambda = 0.384 \quad [\text{m}] \end{matrix}.$$

Exercise 15.1 Suppose that in **Example 15.3**, part **(a)**, it is not physically possible to connect the stub at either location found. The nearest location at which a stub may be connected is 1 m from the load:

(a) What are the solutions for d_1 and l_1?
(b) Are these solutions unique?

Answer 1 m = 0.5λ. The solutions are:

(a)

$$\begin{matrix} d_{1a} = (0.337 + 0.5)\,\lambda = 1.674 \quad [\text{m}], & l_{1a} = 0.442\,\lambda = 0.884 \quad [\text{m}] \\ d_{1b} = (0.444 + 0.5)\,\lambda = 1.888 \quad [\text{m}], & l_{1b} = 0.058\,\lambda = 0.116 \quad [\text{m}] \end{matrix}.$$

(b) No. The addition of any integer number of half-wavelengths to d_1 or l_1 or both is also acceptable solutions.

Example 15.4 Application: Series Stub Matching at an Antenna The_Smith_Chart.m

Consider again the transmission line and load in **Example 15.3**. The load has an impedance of $15 + j60\ \Omega$ and the line impedance is 75 Ω, as shown in **Figure 15.14**. However, now it is required to match the load using a shorted, series stub similar to that shown in **Figure 15.13c**. Calculate the required length of a series shorted circuit stub and its distance from the load to match the antenna to the line. The line and stub have the same characteristic impedance.

Solution: The solution is similar to that in **Example 15.3**. To match the load, we seek a location d_1 and a stub length l_1 as shown in **Figure 15.13c**. Since the stub's reactance is in series with the line impedance at d_1, the sum of the line impedance and stub reactance must be equal to the line resistance. Therefore, we should now use the Smith chart as an impedance chart. We move a distance d_1 from the load at which location the normalized line impedance is $z_l(d_1) = 1 + jx$. Then, we find a stub length l_1 such that $z_s(l_1) = -jx$. The sum of the two gives the correct match at d_1.

(1) The normalized load impedance is $z_L = 0.2 + j0.8$. This is marked at point P_2 (**Figure 15.16**).

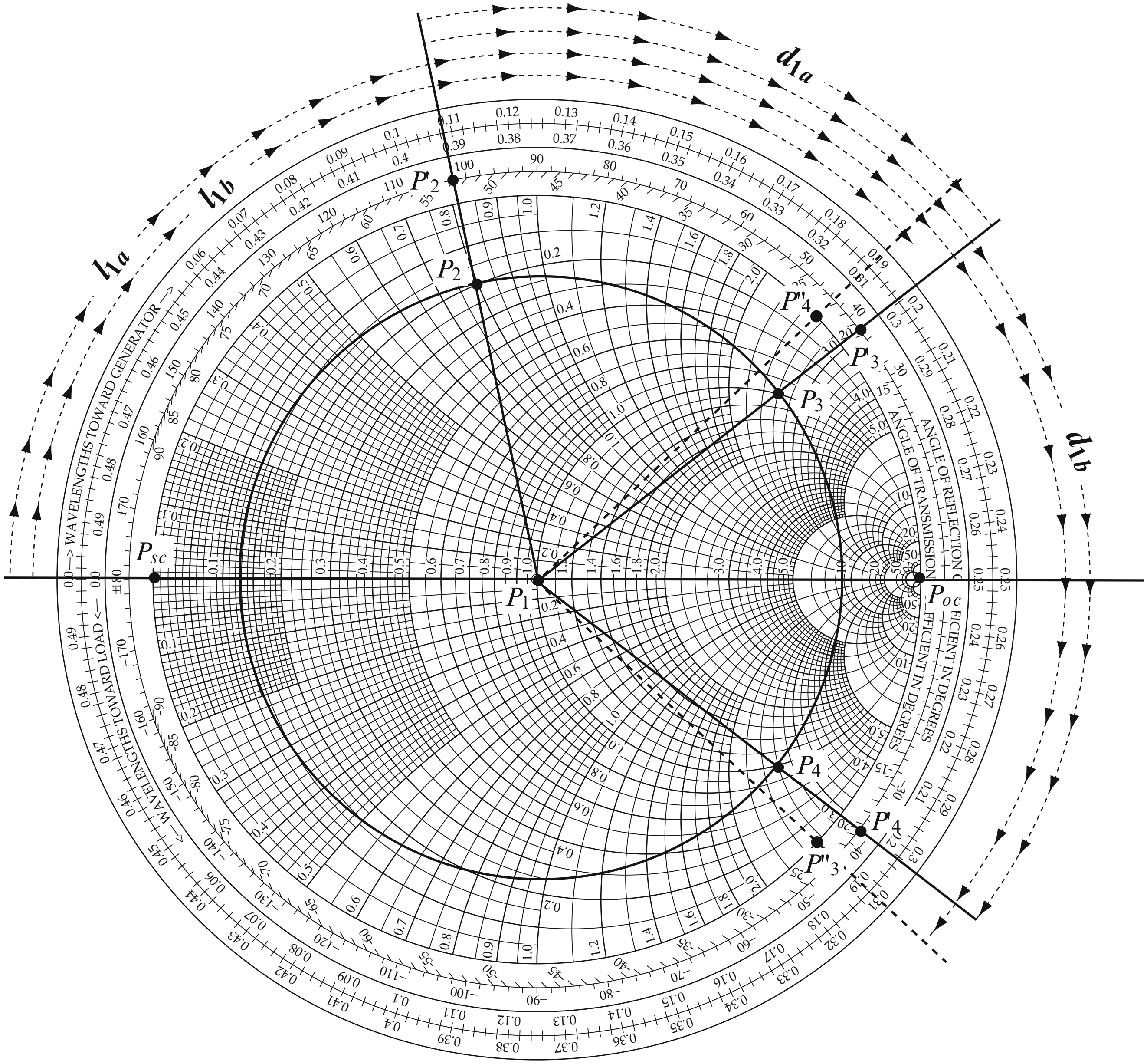


Figure 15.16 Smith chart for **Example 15.4**

(2) Now, we move toward the generator on the reflection coefficient circle until we intersect the $r = 1$ circle at points P_3 and P_4. Connection of P_1 to P_3 and P_1 to P_4 and extending the lines to the circumference gives points P'_3 and P'_4. Each of these is a possible solution.

(3) Solution No. 1: The distance between P'_3 and P'_2 is the first possible solution for d_1. In this case, we moved a distance $d_{1a} = 0.198\lambda - 0.109\lambda = 0.089\lambda$.

The normalized line reactance at point P_3 is $j2.53$. The stub length must be such that its normalized input impedance is $-j2.53$. This required impedance is marked as point P''_3. The distance between the short circuit point P_{sc} and P''_3 moving toward the generator is the stub length necessary. This distance is 0.31λ. Thus, the first solution (with $\lambda = 2$ m) is

$$d_{1a} = 0.087\,\lambda = 0.174 \quad [\text{m}], \quad l_{1a} = 0.31\,\lambda = 0.62 \quad [\text{m}].$$

(4) Solution No. 2: This occurs at point P_4. The distance d_1 now is the distance between point P'_4 and P'_2 or $d_{1b} = 0.31\lambda - 0.109\lambda = 0.201\,\lambda$.

The normalized line reactance at point P_4 is $-j2.53$. The stub normalized impedance must be $+j2.53$. This impedance is marked at point P_4''. The distance l_{1b} is the distance between the short circuit point to point P_4'': $l_{1b} = 0.190\lambda$. The second solution is therefore

$$d_{1b} = 0.201\,\lambda = 0.402 \quad [\mathrm{m}], \quad l_{1b} = 0.190\,\lambda = 0.38 \quad [\mathrm{m}]$$

Either solution is correct, but perhaps in practical terms, the closest stub to the load (solution no. 1) may be chosen.

15.4.2.2 Double Stub Matching

As mentioned earlier, double stub matching takes a different approach than single stub matching. There are now two stubs at fixed locations d_1 and d_2, as shown in **Figure 15.13b**. Matching is achieved by adjusting the two stub lengths l_1 and l_2. To see how this is accomplished, it is best to look at the process in reverse. Suppose that we have already accomplished matching. From the results for single stub matching, we know that when the load is matched, we must be on a point on the unit circle ($g = 1$). In fact, we know that there will be two points at which matching can be accomplished, but, for clarity, only point P_1 is shown in **Figure 15.17**. The point shown represents the load impedance at a distance $d_1 + d_2$ from the load. Now, we move from P_1 toward the load a distance d_2. For any of the points on the unit circle, this means moving on its reflection coefficient circle. The locus of all points on the unit circle, moved toward the load a distance d_2, is a shifted unit circle, as shown in **Figure 15.17**. This shifted unit circle represents the equivalent load impedance at a distance d_1 from the load (this equivalent load impedance is due to the line impedance and the stub at this point). Point P_1' is the equivalent impedance at the location of stub (2) corresponding to the matched point P_1. Stub (1) only adds a susceptance to the line admittance. Therefore, to get to the load admittance point, we must first remove this susceptance by moving along the circles of constant conductance. This brings us to point P_1'' marked on the chart in **Figure 15.17**. In addition we must move a distance d_1 from P_1' toward the load (not shown on the chart). Note, also, that the difference in susceptance between points P_1' and P_1'' is the susceptance stub (1) must add to the line whereas the susceptance of stub (2) is the imaginary part of the admittance at point P_1.

Of course, when matching a load, we will start with the load impedance, but the above process is more instructive because it explains the need for the shifted unit circle and what the contribution of each stub is. In effect, we may say that the purpose of the first stub (the stub closer to the load) is to modify the line susceptance so that the second stub can then take the line admittance to the unit circle. The following two examples show the steps and the details of double stub matching.

Example 15.5 Double Stub Matching The_Smith_Chart.m

A line with characteristic impedance $Z_0 = 300\ \Omega$ and load impedance $Z_L = 150 + j225\ \Omega$ is given. Design a double stub matching network such that the two stubs are 0.1λ apart as shown in **Figure 15.18**.

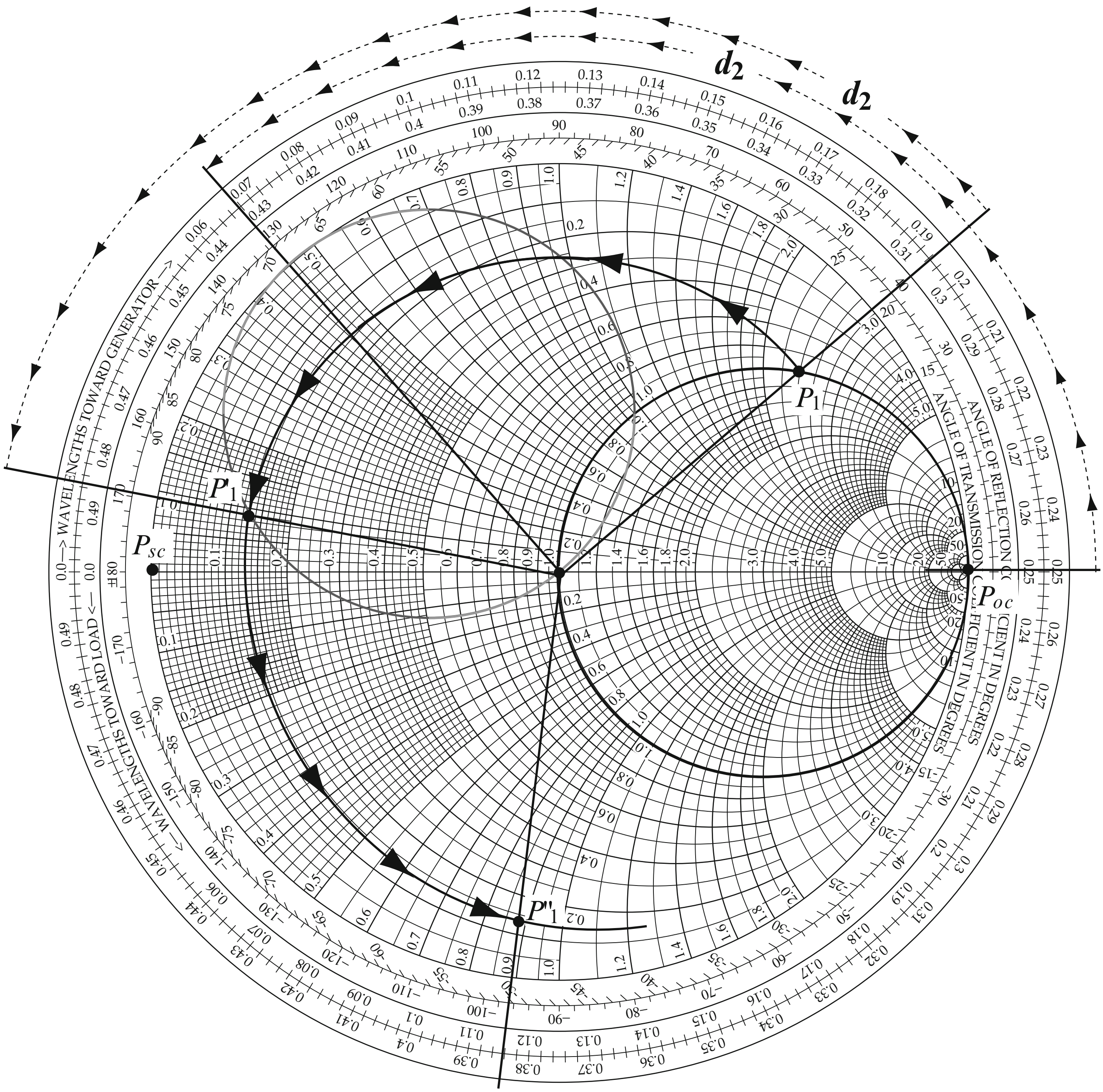


Figure 15.17 Smith chart for **Example 15.4**

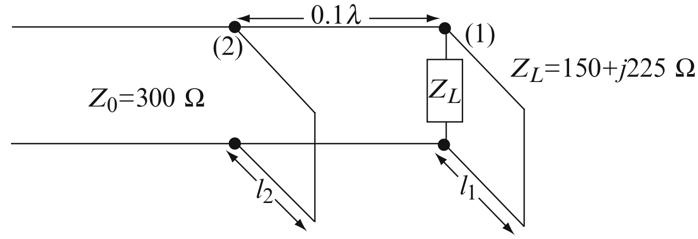


Figure 15.18 A load impedance matched to the line with two stubs

Solution: After calculating the normalized load impedance, we draw the reflection coefficient circle and find the normalized load admittance since, as with the single stub, the Smith chart is used as an admittance chart. In the single stub case, matching consisted of finding the intersection of the reflection coefficient circle with the $g = 1$ circle. The same principle is used here, but the actual matching is at the second stub from the load (stub (2)) since we want to match the load to the line. Thus, stub (1) is represented by its own unit circle, which is shifted a distance 0.1λ from the $g = 1$ circle toward the load. Now, we start at the load (P_3) and move from the admittance point on the constant conductance circle at the load until we intersect the unit circle for stub (1). The intersection points represent the reflection coefficients of the combined load and stub (1). The combined impedance of the load and stub represent a new, modified load with a stub a distance 0.1λ away, toward the generator. This modified line is a load with a single stub; therefore, its treatment is the same as for the single stub matching in that the length of the stub is chosen to cancel the susceptance for each of the two stubs possible at the load. Stubs (1) and (2) refer to the notation used in **Figures 15.13b** and **15.18**, with stub (1) at the load.

(1) The normalized load impedance (without stubs) is $z_L = 0.5 + j0.75$ and is shown at point P_2 in **Figure 15.19**. The normalized load admittance is at point P_3 and is $y_L = 0.615 - j0.923$.

(2) In preparation for the calculation of the stubs, we draw the two unit circles. The unit circle for stub (2) is the $g = 1$ circle of the chart. The unit circle for stub (1) is the same circle, shifted toward the load a distance of 0.1λ, as shown in **Figure 15.19**.

(3) Now, we add the stub at the load. The stub's impedance is purely imaginary. Therefore, it can only change the susceptance of the combined stub and load while the conductance remains the same. To find the combined admittance on the unit circle for stub (1), we move on the constant conductance circle, starting from P_3 (load admittance). This path is shown (gray line) in **Figure 15.19**. The path intersects unit circle (1) at two points, marked P_4 and P_5. The admittances at P_4 and P_5 are

$$y_{P4} = 0.615 + j0.192, \quad y_{P5} = 0.615 + j2.56.$$

(4) In moving from the load admittance point P_3 to points P_4 and P_5, the change in admittance is only due to the susceptance contributed by stub (1). Subtracting the load admittance from the admittances at points P_4 and P_5 gives the susceptance stub (1) must contribute to the impedance at these points:

At P_4:

$$y_{1a} = y_{P4} - y_L = 0.615 + j0.192 - 0.615 + j0.923 = j1.115$$

At P_5:

$$y_{1b} = y_{P5} - y_L = 0.615 + j2.56 - 0.615 + j0.923 = j3.483$$

These two values are shown at points P_4'' and P_5''. The possible stub lengths are found by moving from the short circuit admittance point (P_{oc}) toward the generator, to points P_4'' and P_5''. For point P_4, the susceptance of the stub must be 1.115. Starting at P_{oc} and moving, in turn, to point P_4'' and P_5'' (always toward the generator) gives the two possible lengths for stub (1):

$$l_{1a} = 0.25\lambda + 0.133\lambda = 0.383\ \lambda \quad \left(\text{at } P_4''\right)$$
$$l_{1b} = 0.25\lambda + 0.205\lambda = 0.455\ \lambda \quad \left(\text{at } P_5''\right).$$

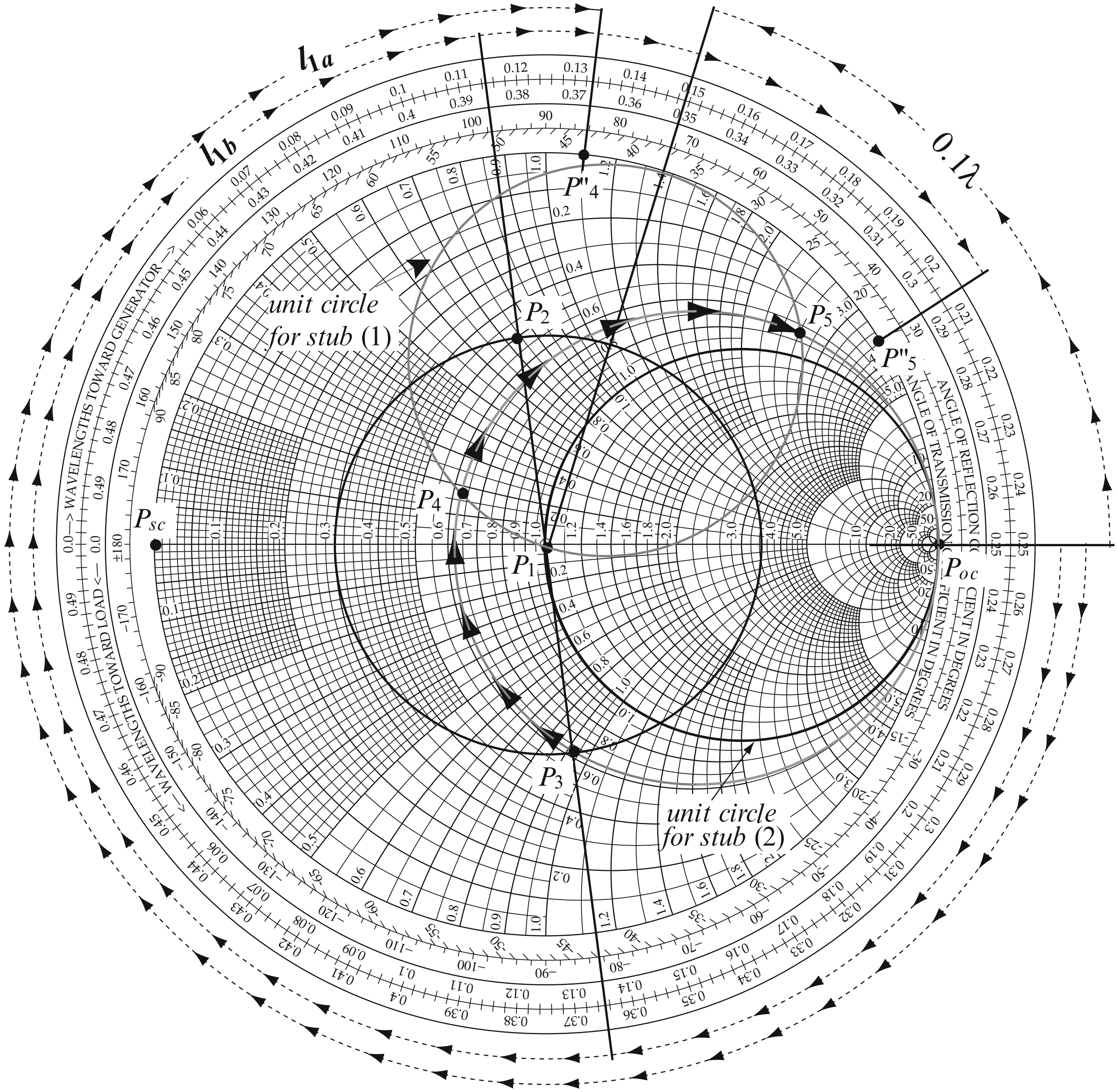


Figure 15.19 Smith chart for the line in **Figure 15.18**

(5) Now, we consider the admittances at P_4 and P_5 as the new load admittances as shown in **Figure 15.20**. From here on, we treat the problem as a single stub matching for each of these admittances and with the distance between load and stub (2) known and equal to 0.1λ. We start with y_{P4} and use **Figure 15.21**, on which the unit circle has been marked. We draw the reflection coefficient circle for the admittance y_{P4}. As we move on the reflection coefficient circle, starting at P_4, toward the generator, and move 0.1λ, we intersect the unit circle at point P_6. Although we cut the unit circle at another point, symmetrically located about the real axis, this intersection cannot be used since the stub must be a distance 0.1λ from the load. At P_6, the line admittance is $1 + j0.55$. Thus, the stub must have admittance $-j0.55$ so that the line susceptance is canceled at the location of stub (2). The latter is marked as point P_6''. The stub length that will accomplish this is the distance between the short circuit point and P_6''. This is

$$l_{2a} = 0.42\lambda - 0.25\lambda = 0.17\,\lambda$$

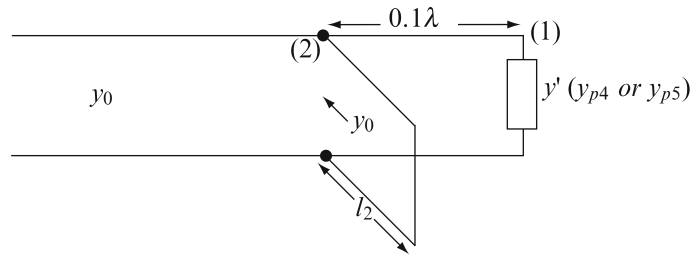


Figure 15.20 The equivalent condition at the location of stub (2) after the load and stub (1) in **Figure 15.18** were taken into account

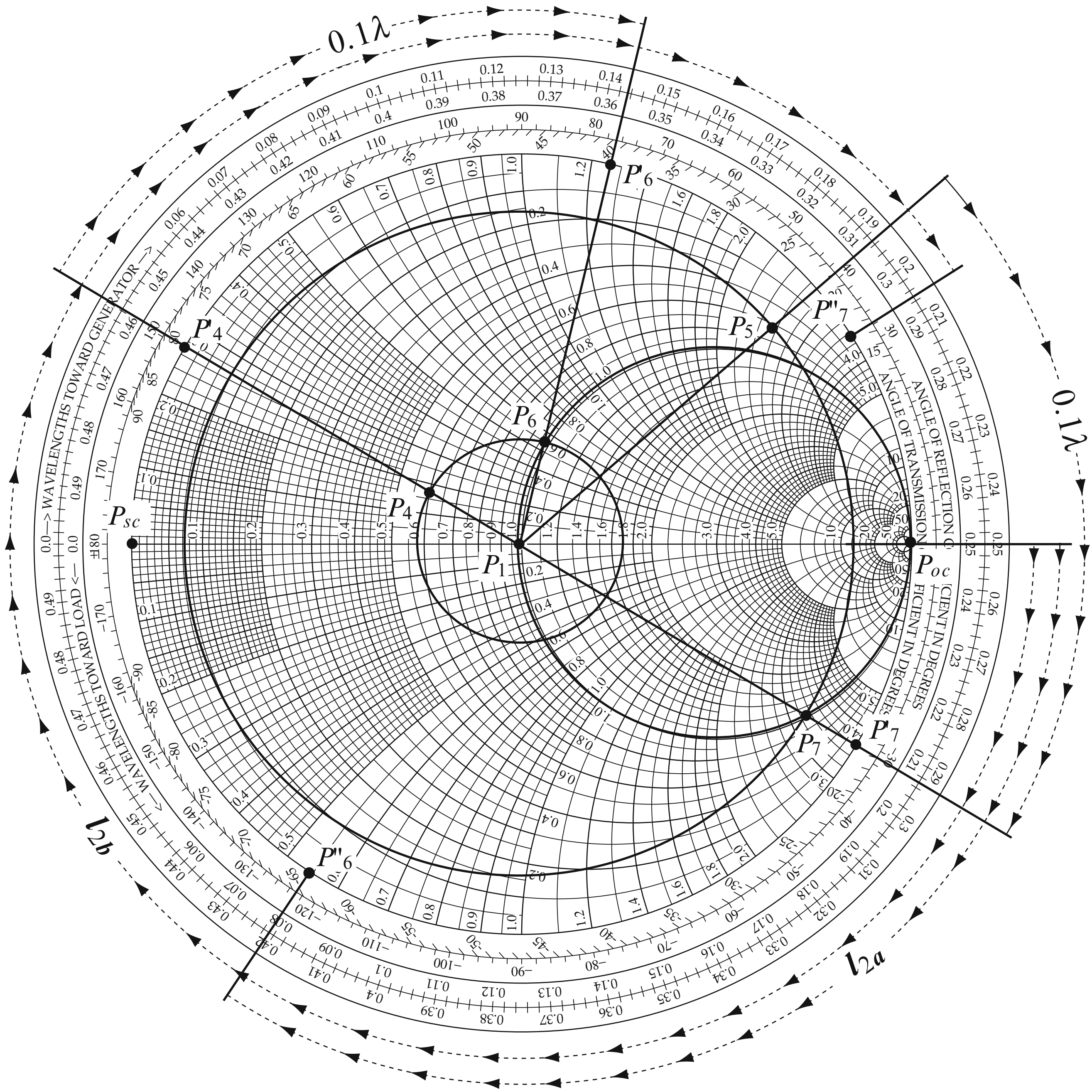


Figure 15.21 Smith chart for the line in **Figure 15.20**

Similarly, for point P_5, we draw the reflection coefficient circle and move 0.1λ toward the generator, to point P_7. The line admittance at P_7 is $1 - j3.4$. The required stub admittance is $+j3.4$, which is marked as point P_7''. The stub length is the distance between P_{oc} and this point:

$$l_{2b} = 0.25\lambda + 0.203\lambda = 0.453\ \lambda$$

Therefore, the two possible solutions are

$$l_1 = 0.383\ \lambda, \quad l_2 = 0.17\ \lambda \quad \text{or} \quad l_1 = 0.455\ \lambda, \quad l_2 = 0.453\ \lambda.$$

Example 15.6 The_Smith_Chart.m

A transmission line and load are given in **Figure 15.22a**. It is required to calculate the lengths of the stubs so that the load is matched to the line.

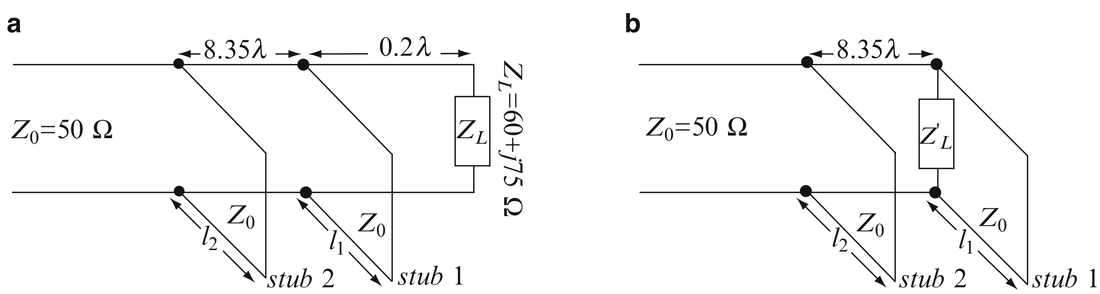


Figure 15.22 (**a**) The double stub matching network for **Example 15.6**. (**b**) Equivalent network after the load impedance has been moved to the location of stub (1)

Solution: The steps in the solution are as follows:

(1) First, we normalize the load impedance:

$$z_L = \frac{Z_L}{Z_0} = \frac{60 + j75}{50} = 1.2 + j1.5$$

This is marked as point P_2 in **Figure 15.23**. The reflection coefficient circle can now be drawn. The point opposite P_2 is P_3. This is the load admittance:

$$y_L = 0.325 - j0.406.$$

(2) The line admittance at the location of the first stub is found by moving from point P_3 toward the generator a distance of 0.2λ. This brings us to point P_4. The admittance at P_4 (without the stub) is

$$y_{p4} = 0.56 + j0.94$$

that is, the normalized line impedance at the location of stub (1), before the stub is added, is

$$z_L' = z_{P5}'' = 0.47 - j0.78$$

Note: This is at point P_5, which is the opposite point to point P_4. Since P_4 represents the normalized admittance, P_5 represents the normalized impedance. Now the line and stubs appear as in **Figure 15.22b**. The new load admittance y_L' is marked as P_4 in **Figure 15.24**.

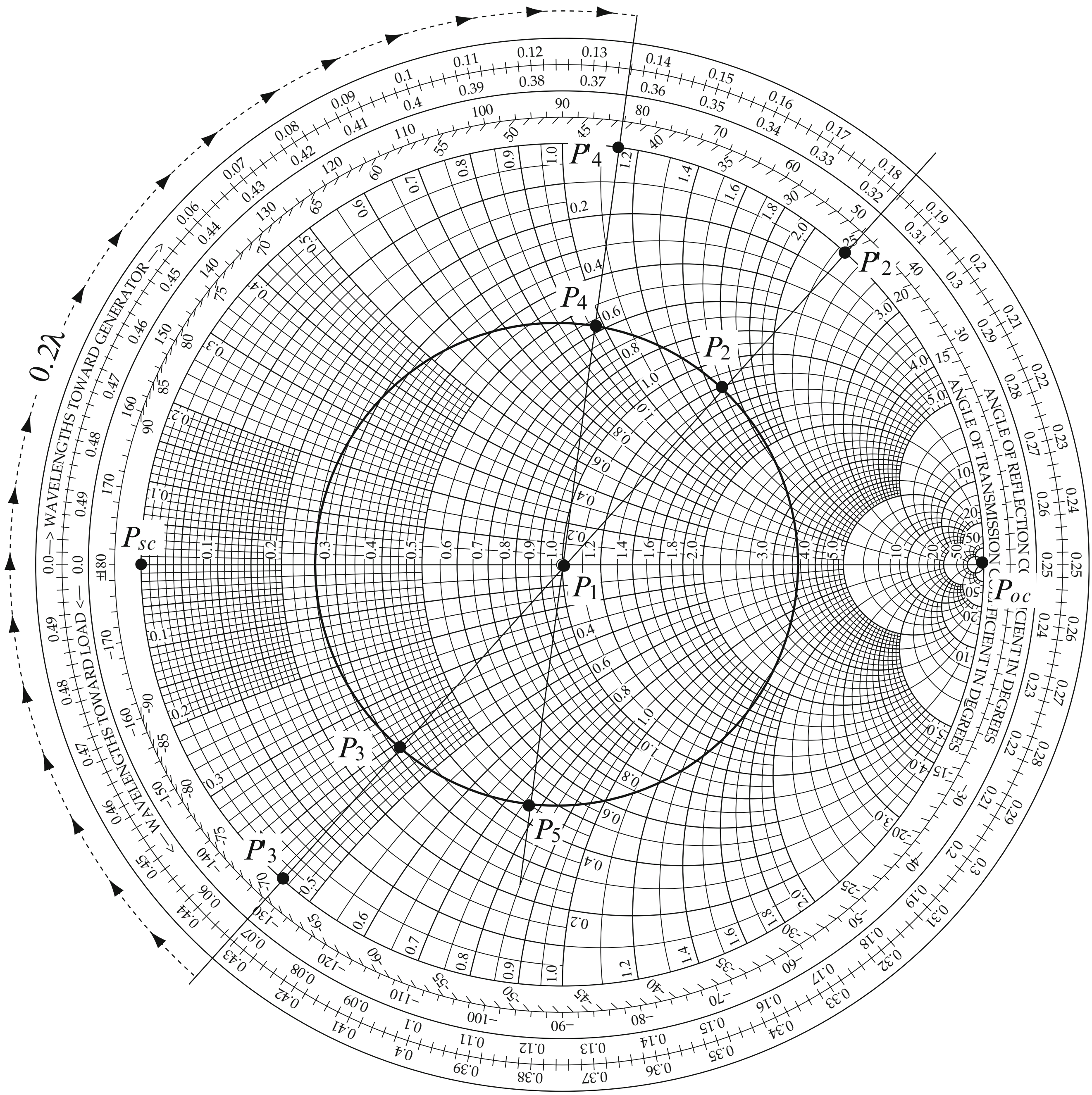


Figure 15.23 Smith chart for the configuration in **Figure 15.22a.** Calculation of the equivalent load impedance at the location of stub (1)

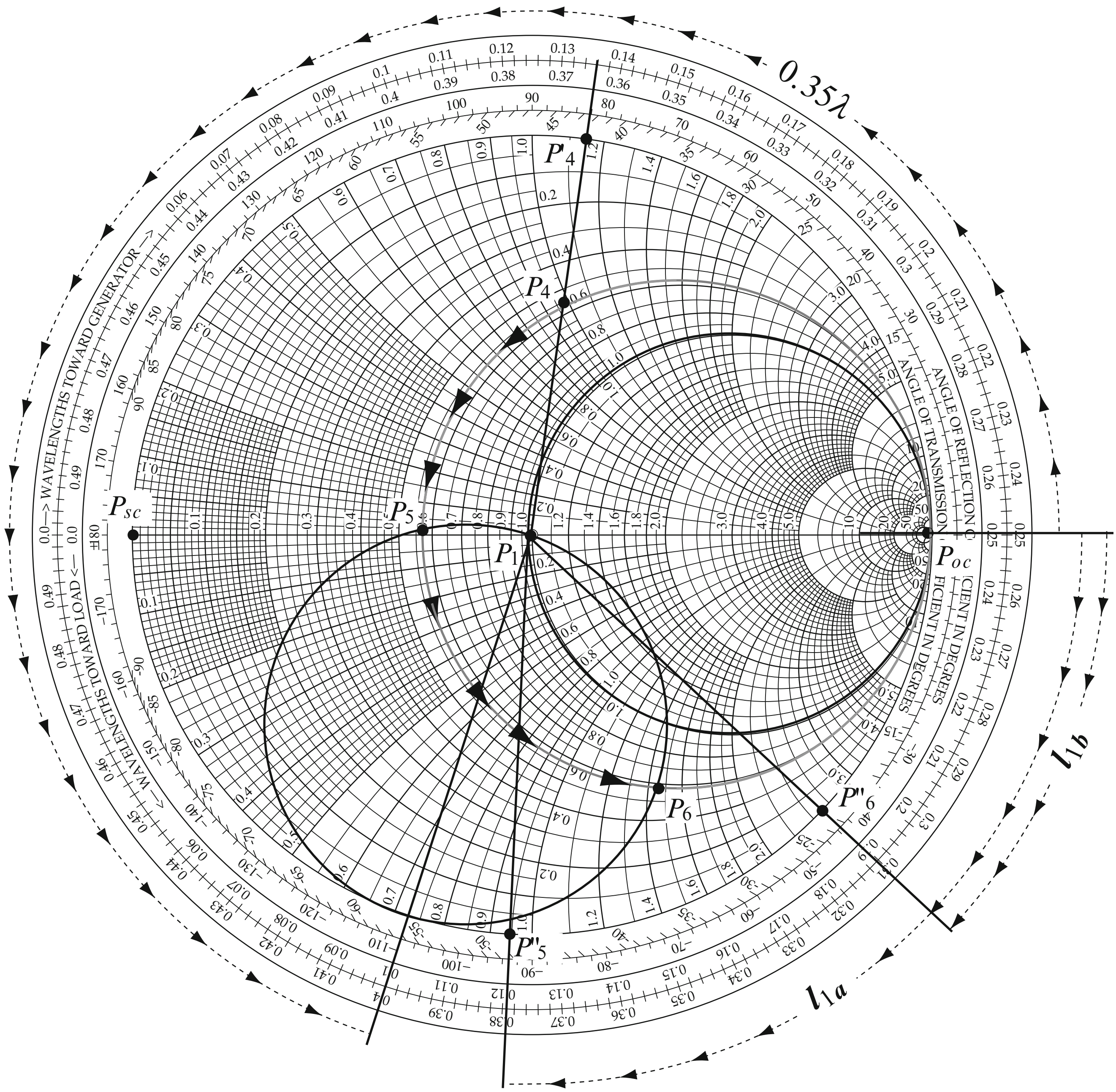


Figure 15.24 Smith chart for **Example 15.6** (continuation)

(3) The distance between the two stubs is 8.35λ. Stub (2) is calculated to match the line. Stub (1) must be calculated for a unit circle ($g = 1$) that has been moved toward the load a distance of 8.35λ. **Figure 15.24** shows the actual unit circle and the shifted circle after moving it 8.35λ toward the load (i.e., from stub (1) to stub (2)). Note that this is the same as moving the circle 0.35λ toward the load.

Now, we move on the conductance circle that passes through point P_4 ($g = 0.56$) until the shifted unit circle is intersected at points P_5 and P_6. The $g = 0.56$ circle is shown as a gray line.

(4) The normalized admittances at point P_5 and P_6 are

$$y_{P5} = 0.56 + j0.01, \quad y_{P6} = 0.56 - j1.463$$

To find the length of stub (1), we argue as follows: moving from P_4 to P_5 or P_6, we have changed the imaginary part of admittance only. This change is
From P_4 to P_5:

$$y_{1a} = y_{P5} - y_{P4} = (0.56 + j0.01) - (0.56 + j0.94) = -j0.95$$

From P_4 to P_6:

$$y_{1b} = y_{P6} - y_{P4} = (0.56 - j1.463) - (0.56 + j0.98) = -j2.43$$

The admittances y_{1a} and y_{1b} are the admittances added to the load by the two possible choices for stub (1). The two admittances required are shown as points P_5'' and P_6''. Thus, the length of stub (1) is the distance between the short circuit point and P_5'' or P_6''. For $y_{1a}(P_5)$, the input stub admittance must be equal to $-j0.95$. We move from the infinite admittance point (point P_{oc} on the chart), toward the generator, on the outer circle of the Smith chart up to point P_5''. The total distance traveled is the length of the stub:

$$l_{1a} = 0.38\lambda - 0.25\lambda = 0.13\,\lambda$$

Similarly, the stub admittance for point P_6 is $y_{1b} = -j2.43$. The stub length is the distance between P_{oc} and P_6'':

$$l_{1b} = 0.31\lambda - 0.25\lambda = 0.06\,\lambda.$$

(5) For each one of these solutions, we have an equivalent admittance point: P_5 and P_6. The problem now is that of an equivalent admittance y_{P5} or y_{P6}, and a single stub a distance 8.35λ toward the generator. To avoid confusion, we use the new chart in **Figure 15.25**. Points P_5 and P_6 as well as the unit circle for stub (2) are shown. We now draw the reflection coefficient circles for each of these two admittance points starting with point P_5. From P_5, we move 0.35λ toward the generator. This intersects the unit circle at point P_5''. The line admittance at this point (before connecting stub (2)) is $1 - j0.56$. The admittance of the stub must be $+j0.56$, a value shown at point P_7. The length of stub (2) corresponding to the point is the distance between P_{oc} and P_7, moving toward the generator:

$$l_{2a} = 0.25\lambda + 0.082 = 0.332\,\lambda$$

Starting with point P_6 and moving 0.35λ toward the generator, we reach point P_6''. The line admittance at this point is $1 + j2.2$. The stub admittance must be $-j2.2$, shown at point P_8. The stub length is therefore

$$l_{2b} = 0.322\lambda - 0.25\lambda = 0.072\,\lambda$$

The two possible solutions are therefore

$$l_{1a} = 0.13\,\lambda, \quad l_{1b} = 0.06\,\lambda \quad \text{or} \quad l_{2\mathrm{a}} = 0.332\,\lambda, \quad l_{2b} = 0.072\,\lambda$$

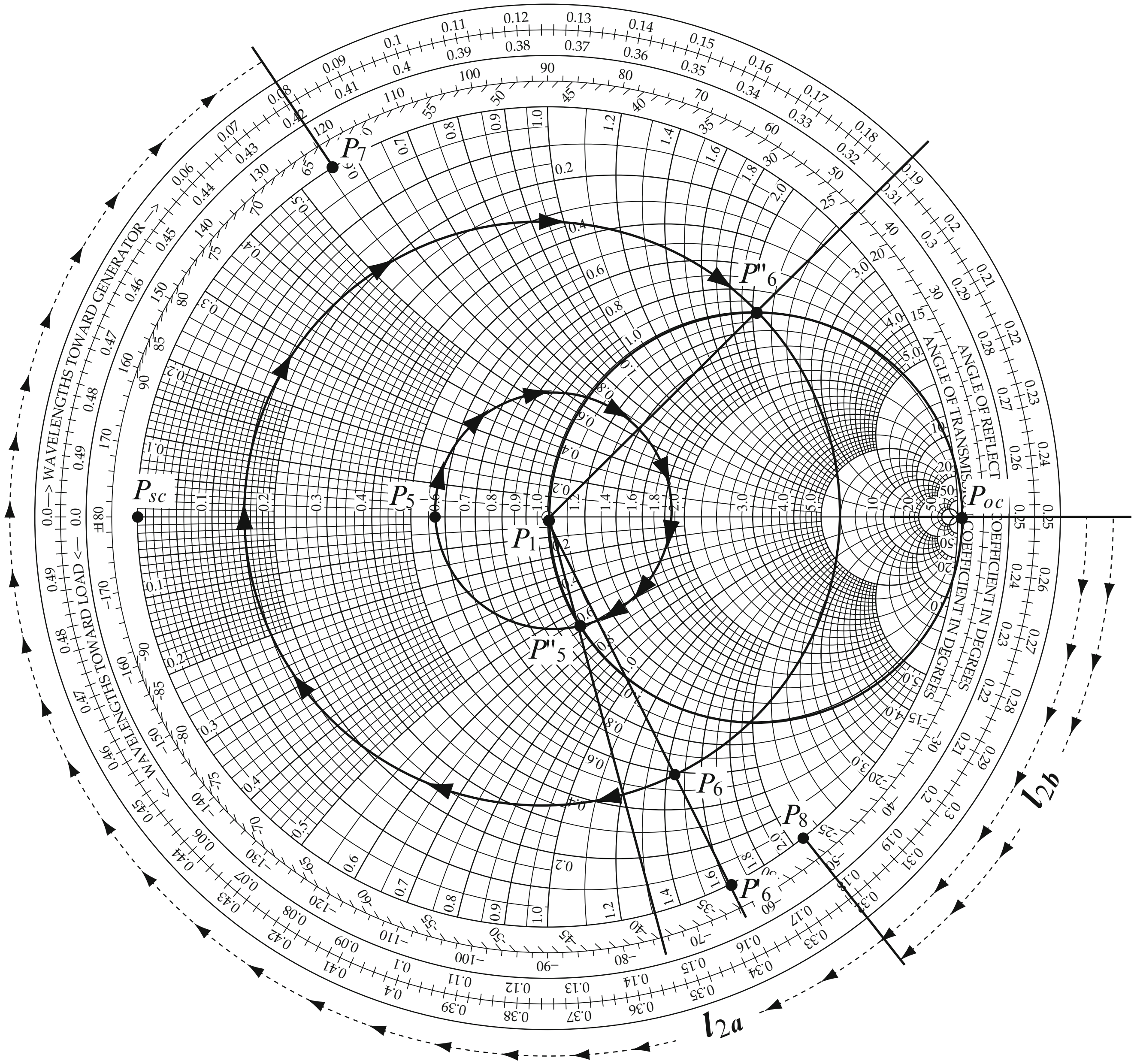


Figure 15.25 Smith chart for **Example 15.6** (continuation)

Exercise 15.2 In **Figure 15.26**, the load impedance is 0.2λ from the first stub (stub (1)) and the distance between the two stubs is 0.1λ. Calculate the lengths of the two stubs to match the load to the line.

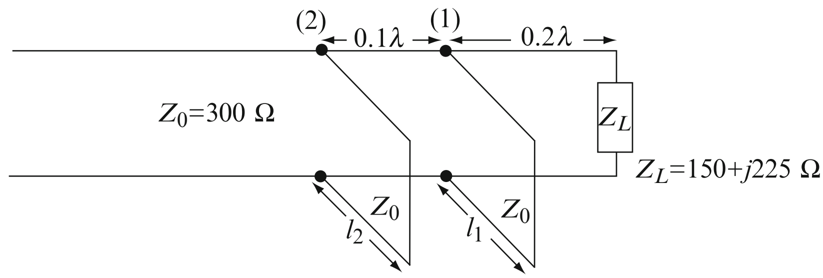


Figure 15.26

Answer $l_{1a} = 0.257\ \lambda,\ \ l_{1b} = 0.104\ \lambda$ or $l_{2a} = 0.424\ \lambda,\ \ l_{2b} = 0.461\ \lambda$.

15.5 Quarter-Wavelength Transformer Matching

Stub matching, in effect, is capable of removing a mismatch for any load (except a purely reactive load), but it is not an impedance transformer. If different lines must be matched, a transmission line transformer can be used, as in **Figure 15.27**.

From **Eq. (14.102)**, the line impedance Z_{in} of a lossless transmission line of characteristic impedance Z_0, at a distance z_0,

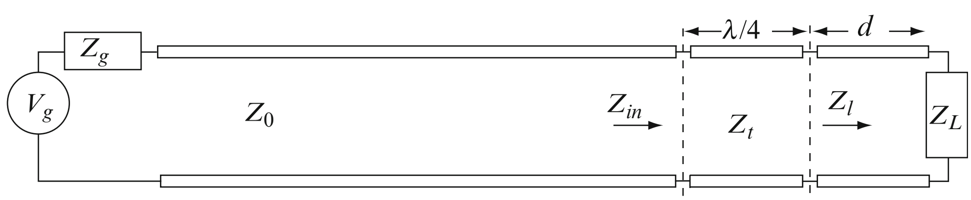


Figure 15.27 A quarter-wavelength transformer located at distance d from load

from the load may be viewed as the input impedance of the line section between z_0 and the load:

$$Z_{in} = Z_0 \frac{[Z_L \cos\beta z_0 + jZ_0 \sin\beta z_0]}{[Z_0 \cos\beta z_0 + jZ_L \sin\beta z_0]} \quad [\Omega] \tag{15.22}$$

Now, suppose we chose a transmission line section, with characteristic impedance Z_t, cut it so it is $\lambda/4$ long, and connect it to a load impedance Z_l. Setting $z_0 = \lambda/4$ and $\beta z_0 = \beta\lambda/4 = (2\pi/\lambda)(\lambda/4) = \pi/2$ and replacing Z_l by Z_l and Z_0 by Z_t in **Eq. (15.22)**, we get for the input impedance of the $\lambda/4$ section

$$Z_{in} = Z_t \frac{\left[Z_l \cos\frac{\pi}{2} + jZ_t \sin\frac{\pi}{2}\right]}{\left[Z_t \cos\frac{\pi}{2} + jZ_l \sin\frac{\pi}{2}\right]} = \frac{Z_t^2}{Z_l} \quad [\Omega] \tag{15.23}$$

Referring now to **Figure 15.27**, where Z_l is the line impedance at a distance d from the load, we get the condition for matching using the quarter-wavelength transformer shown:

$$\boxed{Z_t = \sqrt{Z_{in} Z_l} \quad [\Omega]} \tag{15.24}$$

Thus, two different transmission lines or any two impedances may be matched, provided a transformer of proper characteristic impedance Z_t can be found. The quarter-wavelength transformer is normally connected at a point of maximum or minimum voltage since the line impedance is real at that point. The line impedance at a point of minimum voltage is

$$Z_l = \frac{Z_0}{\text{SWR}} \quad [\Omega] \tag{15.25}$$

where Z_0 is the characteristic impedance of the line, and the standing wave ratio on the line is given as

$$\text{SWR} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} \tag{15.26}$$

The location of the minimum voltage on the line for a general load is at a distance [see **Eqs. (14.122)** and **(14.123)**]

$$d_{\min} = \frac{\lambda}{4\pi}(\theta_\Gamma + \pi) + n\frac{\lambda}{2} \quad [\lambda] \tag{15.27}$$

from the load, where n is any integer, including zero. For a resistively loaded line, the location of minimum voltage is either at the load (if $R_L < Z_0$) or at a distance λ/4 (if $R_L > Z_0$). Thus, the transformer can be located at any of the points in **Eq. (15.27)**. If the characteristic line impedance is Z_0, the characteristic impedance of a transformer located at a point of minimum voltage must be

$$\boxed{Z_t = Z_0\sqrt{\frac{1}{\text{SWR}}} \quad [\Omega]} \tag{15.28}$$

Similarly, if the transformer is located at a point of maximum voltage [by moving it a quarter-wavelength in either direction of any of the points in **Eq. (15.27)**], the characteristic impedance of the transformer for matching is

$$\boxed{Z_t = Z_0\sqrt{\text{SWR}} \quad [\Omega]} \tag{15.29}$$

How can we use the Smith chart to design a quarter-wavelength transformer and, therefore, match two lines or a line and a load? First, we note that two parameters are important in this design. The first is the standing wave ratio SWR. The second is the location of the minimum (or maximum) voltage on the line. For any given load, these are obtainable from the Smith chart. Once the SWR and location of minimum or maximum are found, the transformer impedance is found from **Eq. (15.28)** or **(15.29)**, depending on where the transformer is placed. The following examples discuss the design sequence.

Example 15.7 Application: Matching of Two Different Lines A student has found out that he/she is out of money and cannot pay his/her cable TV bill. He/she decides to cancel the service and go back to the old rooftop antenna. However, the TV input is 75 Ω, while the cable coming down from the antenna is 300 Ω. Design a matching network to match the two lines assuming that the antenna is matched to the 300 Ω line and the TV is matched to the 75 Ω line. Where should the matching network be placed?

Solution: A quarter-wavelength transformer can be used, although, because TV reception is in a range of frequencies, the lines will only be matched at the frequency at which the transformer is exactly one-quarter wavelength. The characteristic impedance of the transformer must be

$$Z_t = \sqrt{Z_{in}Z_l} = \sqrt{75 \times 300} = 150 \quad [\Omega]$$

The transformer may be placed anywhere between the antenna and TV because one line is matched to the TV and the second to the antenna and, therefore, the impedance anywhere on each line equals its characteristic impedance. The only important point is that the line between the transformer and the TV must be a 75 Ω line, and between the transformer and the antenna the line must be a 300 Ω line (**Figure 15.28**).

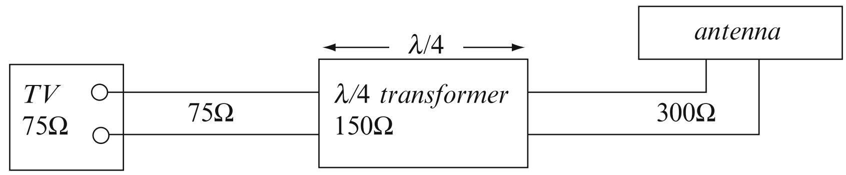


Figure 15.28 A quarter-wavelength transformer used to match two different transmission lines

Example 15.8. Matching a Load to a Line The_Smith_Chart.m

A load $Z_L = 45 - j60\ \Omega$ is connected to a line with characteristic impedance $Z_0 = 50\ \Omega$. Design a quarter-wavelength transformer to match the load to the line. It is required to connect the transformer as close to the load as possible. Find the required characteristic impedance of the transformer and its location.

Solution: There are two methods to solve this problem. The most obvious is to use **Eqs. (15.22)** through **(15.29)**. The second is to use the Smith chart instead. We will do both, starting with the Smith chart method.

Method A: The Smith Chart

(1) Find the normalized impedance of the load and mark it on an impedance Smith chart. The normalized load impedance is $z_L = (45 - j60)/50 = 0.9 - j1.2$ (point P_2 in **Figure 15.29**).

(2) Find the first extremum in impedance from the load (minimum or maximum). This is done by moving on the reflection coefficient circle, toward the generator, from point P_2, until the real axis of the chart is met. This happens at point P_3 at a point of minimum impedance and is a distance of $0.5\lambda - 0.336\lambda = 0.164\lambda$ from the load. At point P_3, the value on the axis is $1/\text{SWR} = 0.3$. Thus, $\text{SWR} = 3.33$.

(3) From **Eq. (15.28)**, the characteristic impedance of the transformer is

$$Z_t = Z_0\sqrt{\frac{1}{\text{SWR}}} = 50\sqrt{0.3} = 27.39 \quad [\Omega].$$

Method B: Direct Calculation First, we find the load reflection coefficient, its magnitude, and its phase angle:

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{45 - j60 - 50}{45 - j60 + 50} = \frac{-5 - j60}{95 - j60} = 0.2475 - j0.475$$

$$|\Gamma_L| = \sqrt{(0.2475)^2 + (0.475)^2} = 0.536$$

$$\theta_L = \tan^{-1}\frac{-j0.475}{0.2475} = -62.48^\circ \rightarrow \theta_L = -1.09 \quad [\text{rad}]$$

The standing wave ratio is

$$\text{SWR} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} = \frac{1 + 0.536}{1 - 0.536} = 3.310$$

The location of the first minimum from the load [$n = 0$ in **Eq. (15.27)**] is

$$d_{\min} = \frac{\lambda}{4\pi}(\theta_L + \pi) = \frac{\lambda}{4\pi}(-1.09 + \pi) = 0.163\,\lambda$$

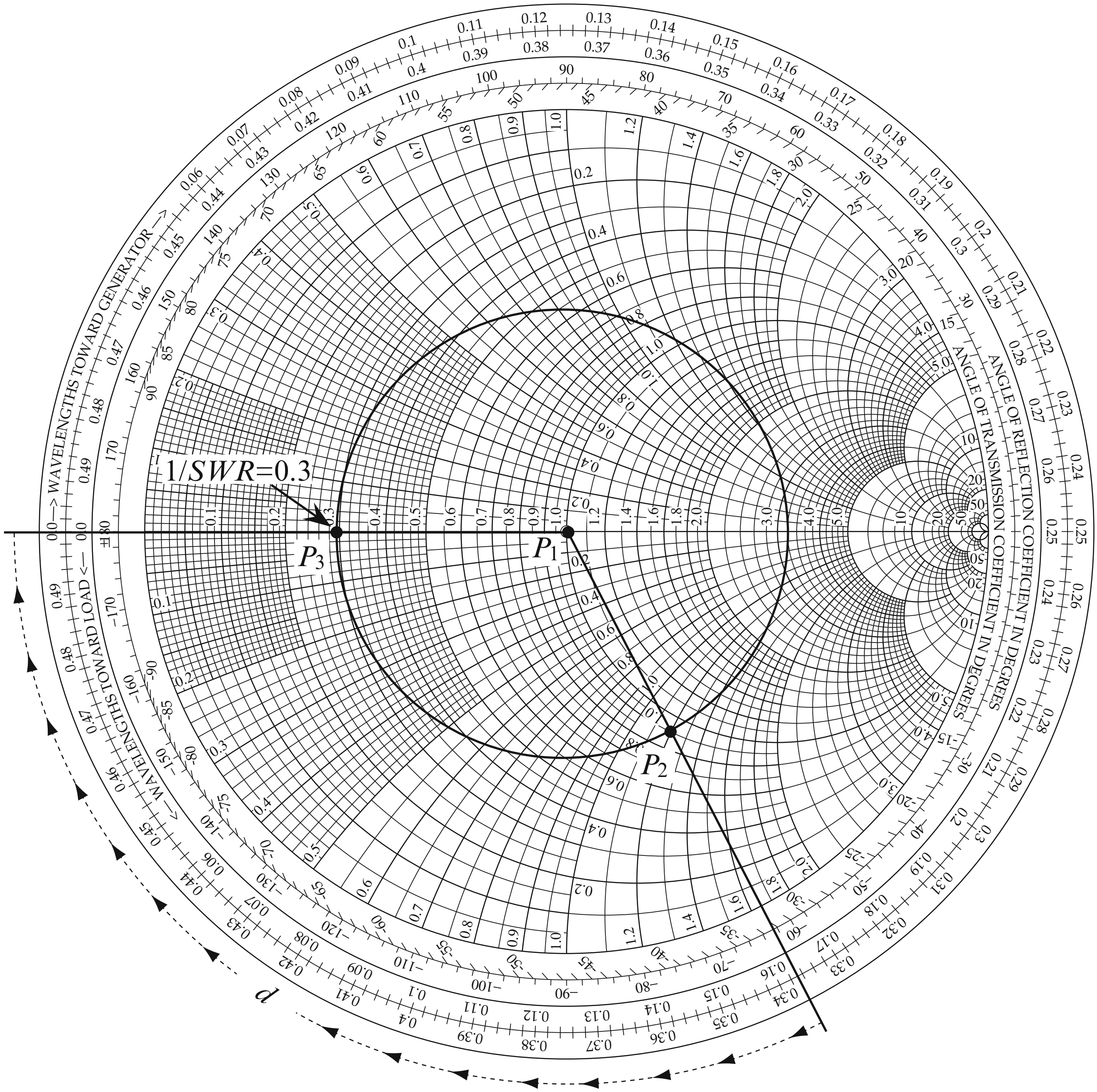


Figure 15.29 Smith chart for **Example 15.8**

The transformer's intrinsic impedance is

$$Z_t = Z_0\sqrt{\frac{1}{\text{SWR}}} = 50\sqrt{\frac{1}{3.310}} = 27.48 \quad [\Omega]$$

Note that the two solutions are not identical although they are close. This, of course, is due to the nature of the chart: the precision depends on accuracy of reading the values on the chart. Much of this difficulty is solved with computerized Smith charts since these charts use the actual mathematical relations involved.

Exercise 15.3 Find the location and characteristic impedance of the quarter-wavelength transformer in **Example 15.8** if the transformer is connected at the first voltage maximum.

Answer $d = 0.414\lambda,\ Z_t = 90.97\ [\Omega]$.

15.6 Experiments

Experiment 1 (Demonstrates: Matching Using Shorted Stubs) Take a 2–3 m length of a 300 Ω transmission line: You can use an antenna down wire (two-conductor flat cable) or a simple two-conductor wire such as the wires used to connect speakers. Leave one end open and connect the other end in parallel to the antenna lugs on your TV while the external antenna is connected as well, and tune a low-frequency channel (VHF channel 2 has a frequency of 54–60 MHz; channel 3, 60–66 MHz; channel 4, 66–73 MHz). With a needle, short the line by piercing through the insulation at different locations on the wire. Note the locations of the short that produce the best and worst receptions. Measure the distance between two peaks and two minima in reception. What can you say about these locations? Can you relate this distance with the frequency received?
Note: Do not perform this experiment on amplified antenna systems or cable TV connections: only on TVs with portable, passive antennas.

Experiment 2 (Demonstrates: Matching Using Open Stubs) Repeat experiment 1 by cutting small sections from the free end of the transmission line. Cut only about 1 cm at a time and make sure that the conductors are not shorted after cutting.

15.7 Summary

The Smith chart is a common tool in transmission line calculations and design. It is based on the properties of the load and generalized reflection coefficient. Because of that it allows calculation of impedances, SWR, magnitudes and phase of the reflection coefficient, as well as other conditions. The Smith chart does not calculate voltages and currents but can be used as an aid in their calculation.

Smith Chart We assume a lossless line with real characteristic impedance Z_0 (but these are not necessary conditions). Given a load impedance $Z_L = R + jX$ and load reflection coefficient Γ_L the Smith chart defines circles of normalized real and imaginary values, r, x so that the normalized load impedance is $z = (R + jX)/Z_0 = r + jx$ (see **Figure 15.4**). The circles are defined as follows:

$$\left(\Gamma_r - \frac{r}{r+1}\right)^2 + \Gamma_i^2 = \frac{1}{(r+1)^2} \qquad (15.12)$$

$$(\Gamma_r + 1)^2 + \left(\Gamma_i - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2 \qquad (15.18)$$

Properties

(1) The circles are loci of constant r or constant x.
(2) x and r circles are orthogonal to each other.
(3) All circles pass through the point $\Gamma_r = 1, \Gamma_i = 0$.
(4) The circles for x and $-x$ are images of each other, reflected about the real axis.
(5) The center of the chart is at $\Gamma_r = 0, \Gamma_i = 0$.
(6) The intersections of the r circles with the real axis, for $r = r_0$ and $r = 1/r_0$, occur at points symmetric about the center of the chart ($\Gamma_r = 0, \Gamma_i = 0$).
(7) The intersections of the x circles with the outer circle ($|\Gamma| = 1$) for $x = x_0$ and $x = 1/x_0$ occur at points symmetrically opposite each other.

(8) The intersection of any r circle with any x circle represents a normalized impedance point.
(9) The point $\Gamma_r = 1, \Gamma_i = 0$ (rightmost point in **Figure 15.7**) represents infinite impedance ($r = \infty, x = \infty$); hence, it is called the ***open circuit point***.
(10) The diametrically opposite point, at $\Gamma_r = -1, \Gamma_i = 0$, represents zero impedance ($r = 0, x = 0$); hence, it is the ***short circuit point***.
(11) The outer circle represents $|\Gamma| = 1$. The center of the diagram represents $|\Gamma| = 0$.
(12) Any circle centered at the center of the diagram ($\Gamma_r = 0, \Gamma_i = 0$) with radius a is a circle on which the magnitude of the reflection coefficient is constant, $|\Gamma| = a$.
(13) A circle drawn through a point representing a normalized load impedance describes the reflection coefficient at different locations on the line (generalized reflection coefficient).
(14) Any point on the chart represents a normalized impedance, $z = r + jx$. The admittance of this point is $y = (r - jx)/(r^2 + x^2)$. The admittance point corresponding to an impedance point lies on the reflection coefficient circle that passes through the impedance point, diametrically opposite of the impedance point **(Figure 15.6a)**.
(15) Motion toward the generator—clockwise. Toward the load—counterclockwise.
(16) Motion around the chart changes the phase but not the magnitude of the reflection coefficient [**Eq. (14.99)**].
(17) A full circle represents $\lambda/2$.
(18) All distances on the Smith chart are in wavelengths, phases are in degrees.

A common use of the Smith chart is for purposes of impedance matching.

Stub Matching Stub matching uses the admittance chart for parallel stubs, impedance chart for series stub. The sequence for parallel stub matching is as follows (see **Figure 15.13**):

(1) A shorted (sometimes open) stub, typically of the same characteristic impedance as the line, is placed at a distance d_1 from the load in parallel with the line.
(2) Normalize the load impedance and place the normalized value on the chart. Draw the reflection coefficient circle through that point (P_2).
(3) Find the normalized admittance by drawing a line from P_2 through the center of the chart until it intersects the reflection coefficient circle on the opposite side (P_3).
(4) Identify the points at which the reflection coefficient circle intersects the $r = 1$ circle.
(5) Find the length of the stub, l_1, which when connected in parallel to the line at a distance d_1 from the load cancels the imaginary part of the normalized admittance (susceptance) at the two points in (4). This provides two possible solutions.
(6) The length of the shorted stub is found by starting from the point of infinite admittance on the chart and moving clockwise until the desired susceptance is found.
(7) Use of open stubs is possible with the appropriate change in (5) and (6) (see **Example 15.3**).
(8) Series stub matching follows the same process but step (3) is skipped, and all steps are done in terms of impedance rather than admittance (see **Example 15.4**).

Double Stub Matching

(1) In this method, two shorted stubs are placed on the line, at any desired location (typically at the load or close to it). The distance between the two stubs is fixed **(Figure 15.13b)**.
(2) Draw a unit circle, shifted from the $r = 1$ circle toward the load (counterclockwise) a distance in wavelengths equal to the distance between the two stubs (**Figure 15.17**)
(3) Place the normalized load impedance on the chart and draw the reflection coefficient circle.
(4) The normalized load admittance is found diagonally opposite the impedance point.
(5) If the load is not at the stub (i.e., if $d_1 \neq 0$) move along the reflection coefficient circle a distance d_1 to the starting point (see **Example 15.6**).
(6) Move on the constant conductance circle from the load admittance point toward the generator until the shifted unit circle is intersected at two possible points. The difference in susceptance between the two points is due to stub **(1)**.
(7) Find the length l_1 of the stub that will add the necessary susceptance at that point as indicated in **(6)**. There are two possible solutions.
(8) Now consider each of the two points found in **(7)** as a load to the line. Repeat the process for single stub matching for each point to find the two possible solutions for l_2.

Notes

(1) Single stub matching guarantees a match for any line and any load except a purely imaginary load.
(2) Double stub matching does not guarantee a solution for all conditions, but it is often more practical because the matching section can be prefabricated and included with the load (such as an antenna).
(3) Adding any number of half-wavelengths to any stub or to the position of a stub on the line has no effect on the matching conditions.
(4) Matching in transmission lines means the two impedances are equal. It does not mean maximum power transfer, which requires conjugate matching.

λ/4 Transformer (Figure 15.27) A section of transmission line, $\lambda/4$ in length loaded with an impedance Z_l, has input impedance:

$$Z_{in} = Z_t^2/Z_l \quad [\Omega] \tag{15.16}$$

We place this section at a distance d from the load so that Z_l at the location of the transformer is real (maximum or minimum voltage point on the line). To ensure matching, select the characteristic impedance of the transformer section, Z_t so that

$$Z_t = \sqrt{Z_{in}Z_l} \quad [\Omega] \tag{15.24}$$

In practical terms, the $\lambda/4$ transformer is placed at the location of voltage maximum or voltage minimum:
At the maximum impedance point

$$Z_t = Z_0\sqrt{SWR} \quad [\Omega] \tag{15.29}$$

At the minimum impedance point

$$Z_t = Z_0/\sqrt{SWR} \quad [\Omega] \tag{15.28}$$

Any number of half-wavelengths may be added to the transformer length or to the location of the transformer without change in the matching conditions.

Problems

General Design Using the Smith Chart

15.1 Line Properties Using the Smith Chart. A long line with characteristic impedance $Z_0 = 100\ \Omega$ operates at 1 GHz. The speed of propagation on the line is c [m/s] and the load impedance is $260 + j180\ \Omega$. Find:

(a) The reflection coefficient at the load.
(b) The reflection coefficient at a distance of 20 m from the load toward the generator.
(c) Standing wave ratio.
(d) Input impedance at 20 m from the load.
(e) Location of the first voltage maximum and first voltage minimum from the load.

15.2 Calculation of Voltage/Current Along Transmission Lines. A transmission line with a characteristic impedance of 100 Ω and a load of $50 - j50\ \Omega$ is connected to a matched generator. The line is very long and the voltage measured at the load is 50 V. Calculate using the Smith chart:

(a) Maximum voltage on the line (magnitude only).
(b) Minimum voltage on the line (magnitude only).
(c) Location of maxima and minima of voltage on the line (starting from the load).

15.3 Impedance of Composite Line. A transmission line is made of two segments, each 1 m long (**Figure 15.30**). Calculate the input impedance of the combined line using a Smith chart if the speed of propagation on line (1) is 3×10^8 m/s and on line (2) 1×10^8 m/s. The lines operate at 300 MHz.

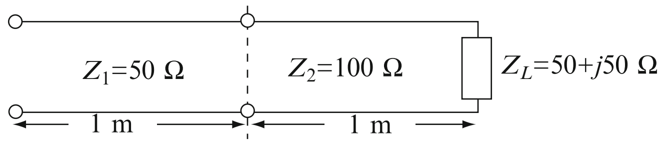


Figure 15.30

15.4 Line Properties. A lossless transmission line has characteristic impedance $Z_0 = 300\ \Omega$, is 6.3 wavelengths long, and is terminated in a load impedance $Z_L = 35 + j25\ \Omega$. Find:

(a) The input impedance on the line.
(b) The standing wave ratio on the main line.
(c) If the load current is 1 A, calculate the input power to the line.

15.5 Line Properties. A lossless transmission line has characteristic impedance $Z_0 = 50\ \Omega$ and its input impedance is $50 - j25\ \Omega$. The line operates at a wavelength of 0.45 m and is 3.85 m long. Calculate:

(a) The load impedance connected to the line.
(b) The location of the voltage minima and maxima on the line, starting from the load.
(c) The reflection coefficient at the load (magnitude and angle) and the standing wave ratio on the line.

15.6 Application: Design of Transmission Lines. It is required to design a load of $75 - j50\ \Omega$ to simulate a device operating at 100 MHz. It is proposed using a section of a 50 Ω line and connecting to its end a lumped resistance R [Ω]. The line's phase velocity is $c/3$ [m/s].

(a) Calculate the length of line and the required resistance R that will accomplish this.
(b) Is the solution unique? Explain and find all possible solutions if the solution is not unique.

15.7 Line Properties Using the Smith Chart. An unknown load is connected to a 75 Ω lossless transmission line. To find the load, two measurements are performed: (1) The location of the first voltage minimum is found at 0.18 λ from the load. (2) The SWR is measured as 2.5. Find using the Smith chart:

(a) The load impedance.
(b) The load reflection coefficient (magnitude and angle).

Stub Matching

15.8 SWR on Line. The transmission line in **Figure 15.31** is given. A general load ($Z_L = Z_0 + jX_L$ [Ω]) is connected as shown in **Figure 15.31**. The shorted section is made of a different line with a different characteristic impedance Z_1 [Ω].

(a) Assuming the generator is matched, calculate the standing wave ratio on the line.
(b) What must be the length of the shorted line to ensure matching of the load (no reflection). Are there any other conditions that must be satisfied for this to happen?

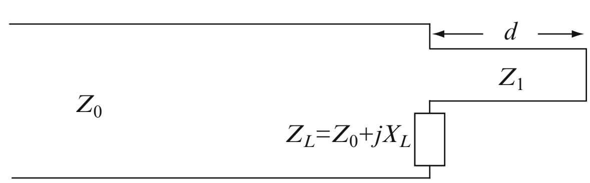


Figure 15.31

15.9 Matching with Shorted/Open Loads. The transmission line network in **Figure 15.32** is given. The shorted transmission line and the open transmission line are part of the network. Show that no stub network will match the two line sections to the main line.

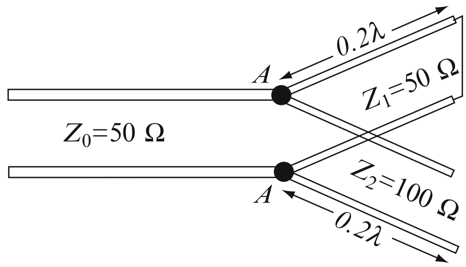


Figure 15.32

15.10 Application: Series Stub Matching. A transmission line of characteristic impedance $Z_0 = 50\ \Omega$ is loaded with an impedance $Z_L = 100 + j80\ \Omega$ (**Figure 15.33**). An open transmission line is connected in series with the line as shown. The open line has the same characteristic impedance. Find the length of the open line and the location (closest to the load) it should be inserted to match the load to the line.

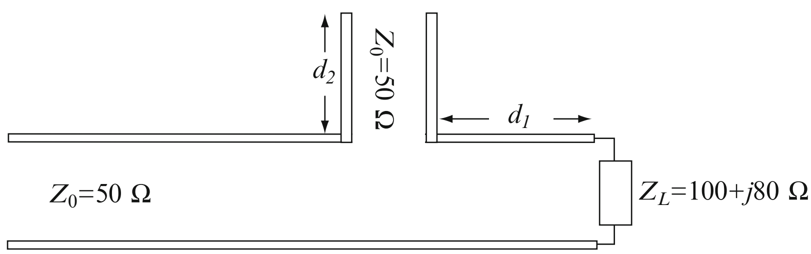


Figure 15.33

15.11 Application: Single Stub Matching. A transmission line is loaded as in **Figure 15.34**. If the wavelength on the line equals 5 m, find a shorted parallel stub (location and length of stub) placed to the left of points *A–A* to match the load to the line.

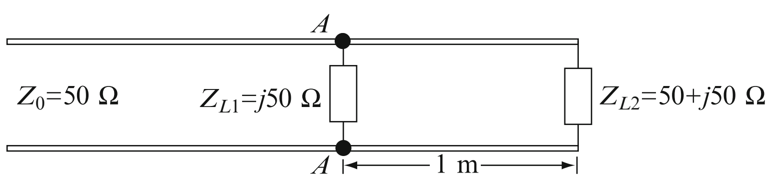


Figure 15.34

15.12 Application: Series Stub Matching. A load is connected to a transmission line as shown in **Figure 15.35**. It is required to match the load to the line (which has a characteristic impedance of 75 Ω). Find the location and length of a stub to match the line. The stub is open as shown in **Figure 15.35**.

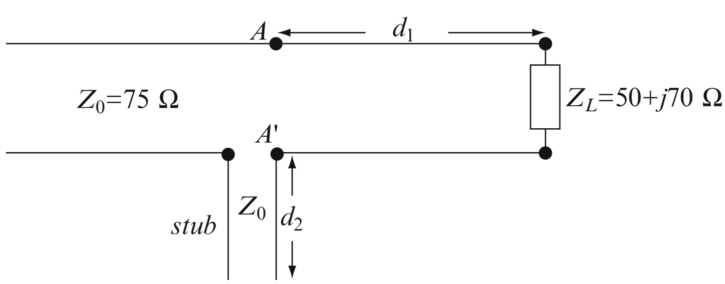


Figure 15.35

15.13 Application: Single Stub Matching. A 75 Ω TV cable is used to connect to a TV. The load is matched to the line **(Figure 15.36a)**. A second TV must be connected 10 m from the first TV, again with a matched section of the same cable **(Figure 15.36b)**. Assuming the phase velocity on the line is $c/2$ [m/s], calculate:

(a) The reflection coefficient at the location of connection of the two lines.
(b) The standing wave ratio on the main line.
(c) Design a single stub (its location to the left of the discontinuity and its length) to match the line for TV channel 3 (63 MHz). Use the same line impedance for the stubs.
(d) For the design in **(c)** calculate the reflection coefficient to the left of the stub for channel 2 (57 MHz). What is your conclusion from this calculation as far as stub matching across a range of frequencies?

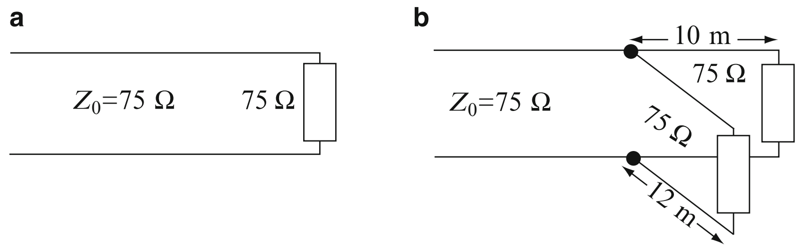


Figure 15.36

15.14 Application: Double Stub Matching. Two stubs are used on a transmission line as shown in **Figure 15.37**. Calculate stub lengths d_1 and d_2 (in wavelengths) to match the load to the line. Is this arrangement of stubs a good arrangement? Why?

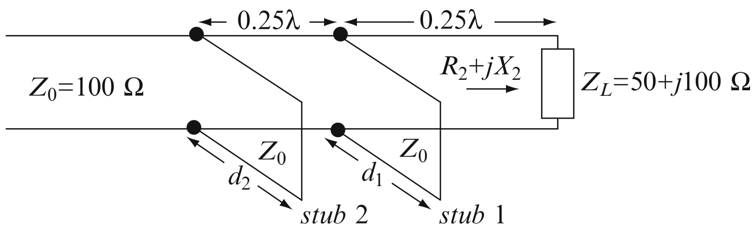


Figure 15.37

15.15 Application: Double Stub Matching. An antenna has an impedance of $68 + j100$ Ω. The antenna needs to be connected to a 75 Ω line. Because the antenna goes on a mast, the design engineer decided to fabricate a matching section as shown in **Figure 15.38**. The matching section is then hoisted and connected to the antenna during installation. At the required frequency, the section is 0.3 λ and the two stubs are made of the same line. Calculate the lengths of the stubs, if the antenna is connected at $A - A$.

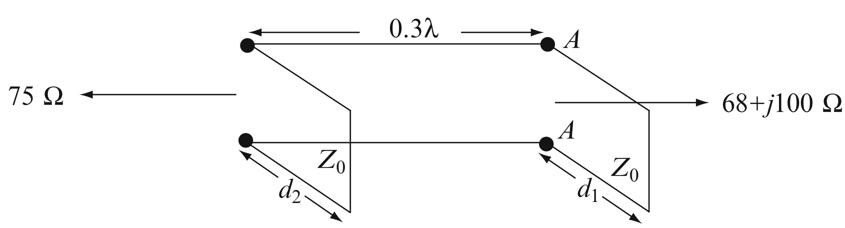


Figure 15.38

Transformer Matching

15.16 Application: λ/4 Transformer. Show that two lines with any characteristic (real) impedances Z_1 [Ω] and Z_2 [Ω] may be matched with a quarter-wavelength line. What is the characteristic impedance of the matching section?

15.17 Application: 3λ/4 Transformer. A lossless transmission line with characteristic impedance Z_0 [Ω] transfers power to a load Z_L [Ω] (real). To match the line, a matching section is connected as shown in **Figure 15.39**. At what distance d (in wavelengths) from the load must the line be connected (minimum distance) and what must the characteristic impedance of the matching section be?

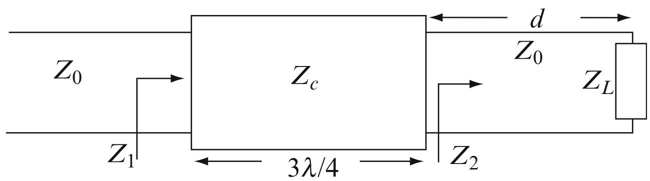


Figure 15.39

15.18 Application: λ/4 Transformer. A transmission line is given as shown in **Figure 15.40**. If the characteristic impedance of the quarter-wavelength transformer must be real, find the location of the transformer (distance d in the figure, in wavelengths) and the intrinsic impedance of the transformer Z_t [Ω].

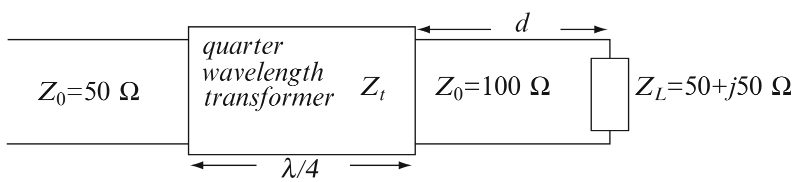


Figure 15.40

15.19 Application: λ/4 Transformer. A two-wire transmission line has characteristic impedance of 300 Ω and connects to an antenna. The line is long and the antenna has an impedance of 200 Ω and operates at a wavelength of 3.8 m. To match the line and load, a quarter-wavelength transformer is connected on the line, but the location at which the transformer may be connected is 10 m from the antenna or larger. Calculate:

(a) The closest location at which the transformer may be connected.
(b) For the result in **(a)**, the characteristic impedance of the transformer section.
(c) The standing wave ratios on the sections of line between the transformer and antenna and between transformer and generator.