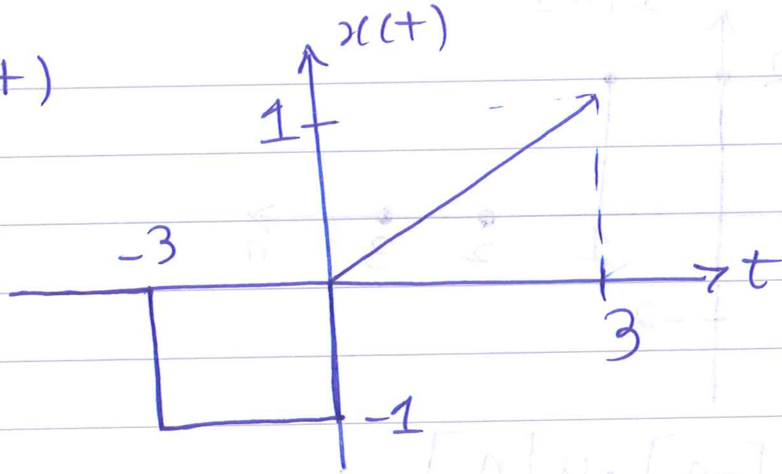
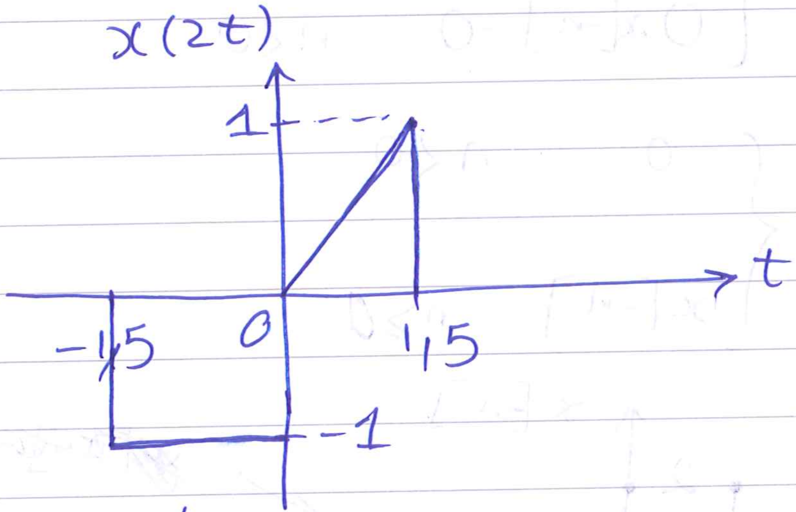
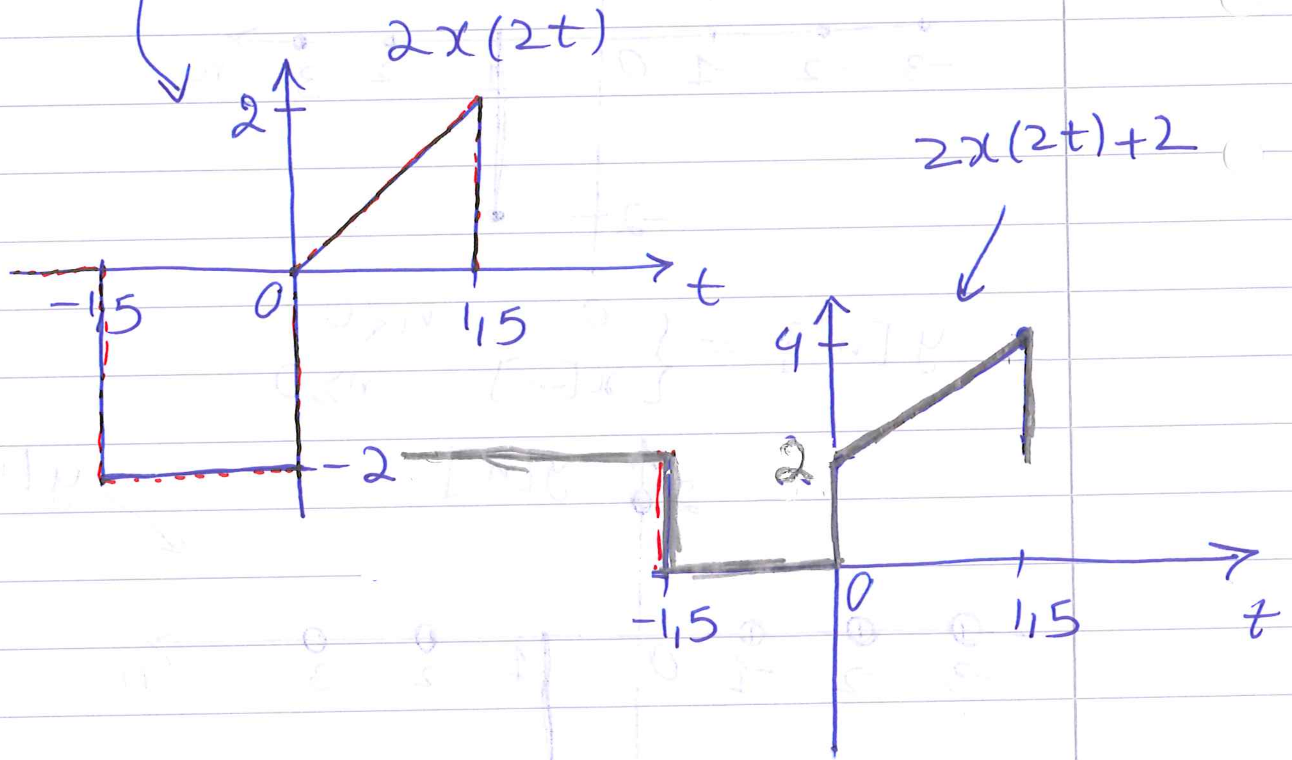
1

$x(t)$

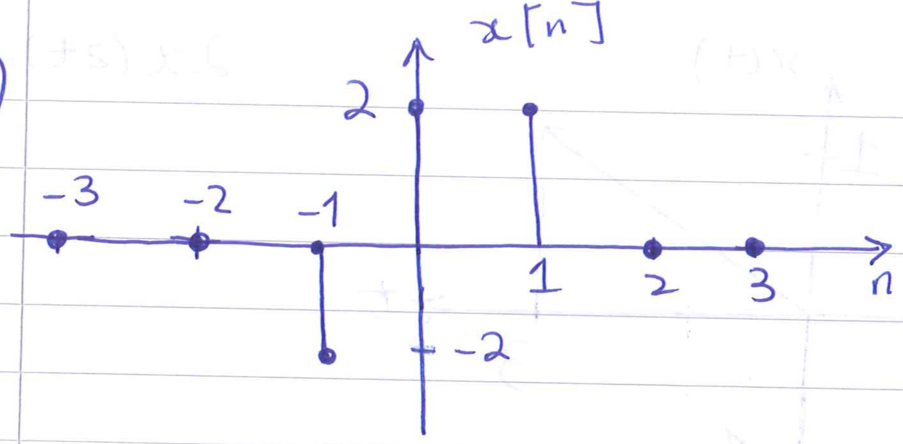
$2x(2t)+2$







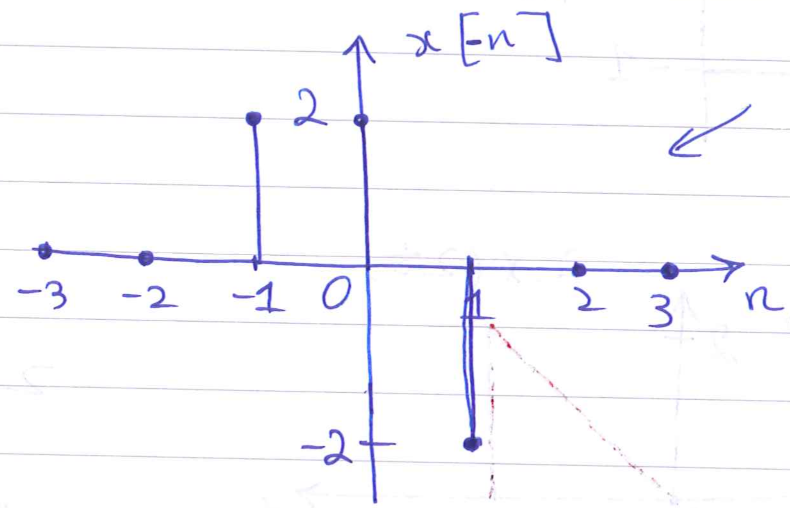
(2)



$$y[n] = x[-n] \cdot u[n]$$

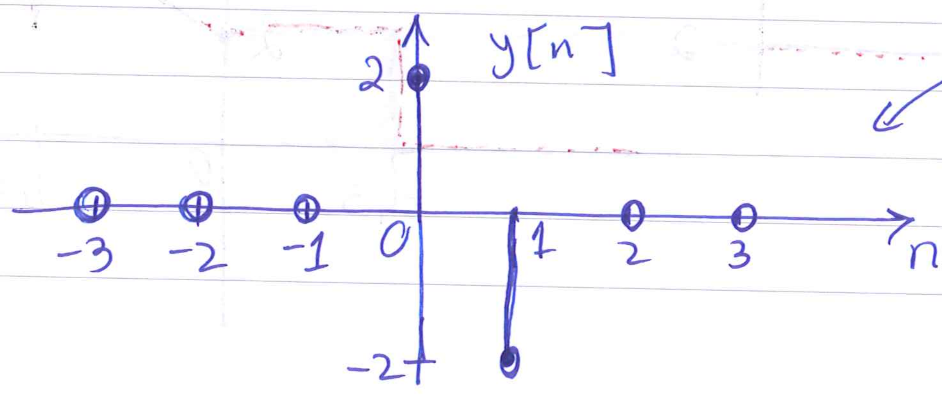
$$y[n] = \begin{cases} x[-n] \cdot 1 = x[-n] & n \geq 0 \\ 0 \cdot x[-n] = 0 & n < 0 \end{cases}$$

$$y[n] = \begin{cases} 0 & n < 0 \\ x[-n] & n \geq 0 \end{cases}$$



x[-n]

$$y[n] = \begin{cases} 0 & n < 0 \\ x[-n] & n \geq 0 \end{cases}$$



y[n]

3) $h[n] = 0,7^n$ Stegsvar?

Vi skall bestämma systemets utsignal då dess insignal $\{h[n]\}$ är ett enhetssteg $\{\sigma[n]\}$.

Insignalen Z-transf är $X(z) = \frac{z}{z-1}$

Impulssvaret $\{h[n]\} = \{0,7^n\}$ med för att

ÖFK $H(z)$ blir z transf of impulssvaret

alltså $H(z) = \frac{Y(z)}{X(z)} \Rightarrow$

$$Y(z) = \frac{z}{z-0,7} \cdot \frac{z}{z-1} = z \frac{z}{(z-0,7)(z-1)}$$

$$Y(z) = z \cdot \frac{T(z)}{N(z)} \rightarrow \text{poler i } \begin{matrix} z = 0,7 \\ z = 1 \end{matrix}$$

$$N'(z) = 1 \cdot (z-1) + (z-0,7) \cdot 1$$

$$A = \left[\frac{T(z)}{N'(z)}\right]_{z=0,7} = \frac{0,7}{0,7-1+0} = \frac{0,7}{-0,3} = -2,33$$

$$B = \left[\frac{T(z)}{N'(z)}\right]_{z=1} = \frac{1}{0+0,3} = 3,33$$

$$\left\{y[n]\right\}_{n=0}^{\infty} = \left\{-2{,}33 \cdot 0{,}7^{n} + 3{,}33\sigma[n]\right\}_{n=0}^{\infty}$$

$$= 3{,}33 - 2{,}33 \cdot 0{,}7^{n} \quad , n \geq 0$$

4) $$x(t) = \frac{\sin(4000\pi t)}{\pi t} + 1 + \cos(2021\pi t)$$

$$\omega_{max} = 4000\pi \ \text{rad/s}$$

$$f_{max} = 2000 \quad \left(\omega = 2\pi f \rightarrow f = \frac{\omega}{2\pi}\right)$$

Theorem: $f_s > 2 \cdot f_{max_{signal}}$

$$f_s > 2 \cdot 2000 = 4000 \text{ Hz}$$

$$f_s > 4000 \ \#$$

ك

5) FT of $x(t) = e^{-3t}\left[u(t) - u(t-4)\right]$

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases} \qquad u(t-4) = \begin{cases} 1 & t \geq 4 \\ 0 & t < 4 \end{cases}$$

$$u(t) - u(t-4) = \begin{cases} 0 - 0 = 0 & t < 0 \\ 1 - 0 = 1 & 0 \leq t < 4 \\ 1 - 1 = 0 & t \geq 4 \end{cases}$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt = \int_{-\infty}^{0} \cancel{0 \cdot e^{-3t} \cdot e^{-j\omega t}}_{0}\, dt + $$

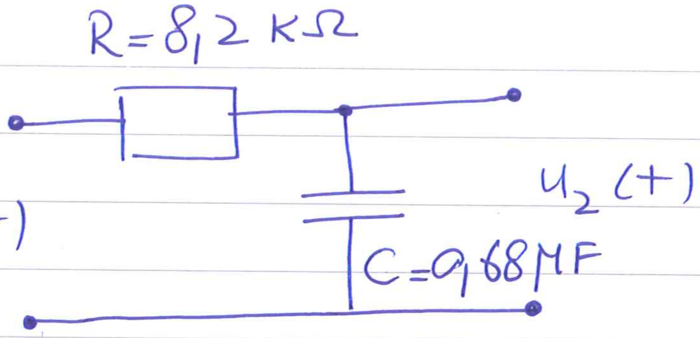
$$\int_{0}^{4} 1 \cdot e^{-3t} \cdot e^{-j\omega t}\, dt + \int_{4}^{\infty} \cancel{0 \cdot e^{-3t} \cdot e^{-j\omega t}}_{0}\, dt$$

$$= \int_{0}^{4} e^{-3t} \cdot e^{-j\omega t}\, dt = \int_{0}^{4} e^{-(3+j\omega)t}\, dt$$

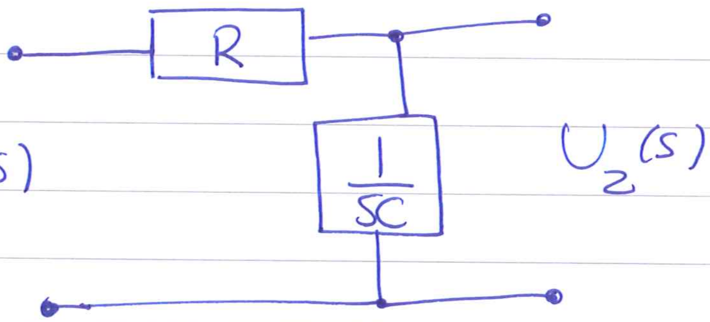
$$= \frac{1}{-(3+j\omega)}\, e^{-(j\omega+3)t}\Big]_{0}^{4} \Rightarrow$$

$$X(\omega) = \frac{1}{-(3+j\omega)}\left(e^{-(j\omega+3)\cdot 4} - 1\right) = \frac{1 - e^{-12-4j\omega}}{3+j\omega}$$

6) $H(s) = ?$



u_1 : Insignalen, u_2 : Utsignalen.



$$U_{out}(s) = \frac{\frac{1}{sC}}{\underbrace{\frac{1}{sC} + R}_{Z_{tot}}} \cdot U_{in} \qquad \frac{1}{sC} = Z_i$$

$R = 8{,}2\,k\Omega$

$C = 0{,}68\,\mu F$

$$= \frac{1}{1 + sRC} \cdot U_{in} \Rightarrow \quad H(s) = \frac{U_{out}}{U_{in}}$$

$$H(s) = \frac{1}{1+sRC} = \frac{1}{1 + (8{,}2 \cdot 10^3 \cdot 0{,}68 \cdot 10^{-6})s}$$

$$H(s) = \frac{1}{1 + 5{,}6 \cdot 10^{-3} s} = \frac{1000}{1000 + 5{,}6 s}$$

type of Filter ⇝ Lowpass filter (LP-filter)

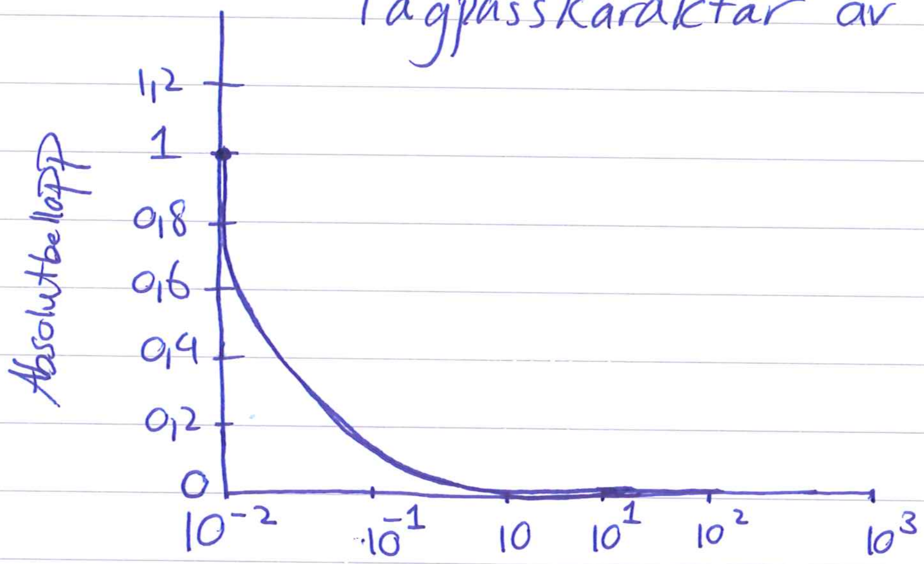
b) Type of Filter:

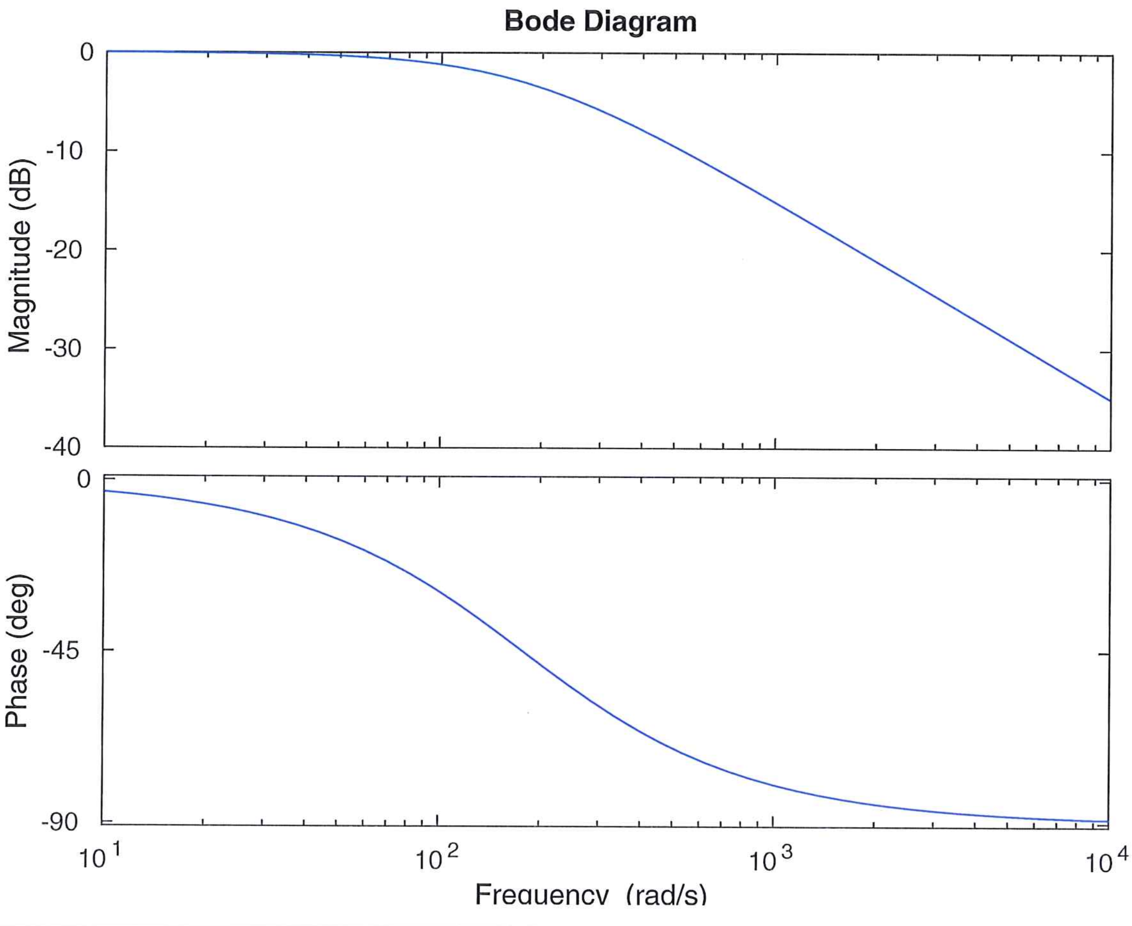
$$H(s) = \frac{1000}{1000 + 5{,}6s}$$

$$|H(j\omega)| = \left| \, |H(s)|_{s=j\omega} \right| \quad \text{så att}$$

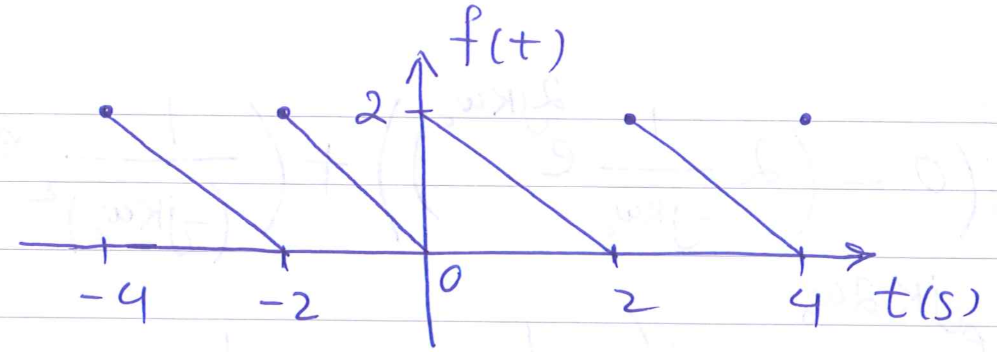
$$|H(\omega)| = \left| \frac{1000}{1000 + 5{,}6 j\omega} \right| = \frac{1000}{|1000 + 5{,}6 j\omega|}$$

$H(0) = 1$, ampeltudfunktionen $\rightarrow 0$ då $\omega \rightarrow \infty$ samt har värdet 1 vid $\omega = 0$. Filterets lågpasskaraktär av Fig.




Bode Diagram
0
-10
-20
-30
-40
Magnitude (dB)
0
-45
-90
Phase (deg)
10 1
10 2
10 3
10 4
Frequency (rad/s)

8)



$$T = 2s \quad , \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi \ \frac{rad}{s} \quad , \omega_0 T = 2\pi$$

$$f(t) = at + b \rightarrow f(0) = 0 + b \rightarrow b = 0$$

$$a = \frac{2-0}{-2-0} = -1 \rightarrow f(t) = -t$$

$$C_0 = \frac{1}{T}\int_{1\,period} f(t)\,dt = \frac{1}{T}\int_{-2}^{0} -t\,dt = \frac{1}{T}\left(-\frac{t^2}{2}\right)\Bigg]_{-2}^{0}$$

$$= \frac{1}{T}\left(0 - \frac{(-(-2)^2)}{2}\right) = \frac{1}{2}\cdot 2 = 1$$

$$\boxed{C_0 = 1}$$

$$C_K = \frac{1}{T}\int_{1\,period} f(t)\,e^{-jK\omega_0 t}\,dt = \frac{1}{T}\int_{-2}^{0} -t\,e^{-jK\omega_0 t}\,dt$$

$$C_K = \frac{1}{2}\int_{-2}^{0} -t\,e^{-jK\omega_0 t}\,dt = \left[\begin{array}{ll} u = t & du = dt \\ dv = e^{-jK\omega_0 t} & v = \frac{e^{-jK\omega_0 t}}{-jK\omega_0} \end{array}\right.$$

$$= \frac{1}{2}\left[-t\cdot\frac{1}{-jK\omega_0}e^{-jK\omega_0 t}\right]_{-2}^{0} + \int_{-2}^{0} 1\cdot\frac{1}{-jK\omega_0}e^{-jK\omega_0 t}\,dt$$

Don't forget to take into accnt $-$ of $-t$!

$$c_K = \frac{1}{2}\left[\left(0 - \left(2\frac{1}{-jK\omega_0}e^{2jK\omega_0}\right)\right) + \left(\frac{1}{(-jK\omega_0)^2}e^{-jK\omega_0 t}\right)\right]_{-2}^{0}$$

$$c_K = \frac{e^{jK2\omega_0}}{jK\omega_0} + \frac{1}{2}\left(\frac{1}{K^2\omega_0^2} - \frac{1}{K^2\omega_0^2}e^{+jK2\omega_0}\right)$$

$$c_K \text{:} = \left\{\begin{matrix}\omega_0 T = \{T=2\} = 2\omega_0 = 2\pi \\ \omega_0 = \pi\end{matrix}\right\} = \frac{e^{jK2\pi}}{jK\pi} + \frac{1}{2}\left(\frac{1}{K^2\omega_0^2} - \frac{e^{jK2\pi}}{K^2\omega_0}\right)$$

Now we note $e^{jK2\pi} = 1$ så

$$c_K = \frac{1}{jK\pi} = \frac{-j}{K\pi}, \quad K \neq 0$$

Reel: $f(t) = a_0 + \sum_{K=1}^{\infty}\left(a_K \cos K\omega_0 t + b_K \sin K\omega_0 t\right)$

$$a_0 = c_0 = 1, \quad a_K = 2\,\mathrm{Re}\, c_K = 0$$

$$b_K = -2\,\mathrm{Im}\, c_K = \frac{2}{K\pi}$$

Så $f(t) = 1 + \sum_{K=1}^{\infty} \frac{2}{K\pi}\sin(K\pi t)$ #

(9) Bestäm LT $F(s)$ för $f(t) = 4e^{-2t}\cos 3t$

$$\mathcal{L}T\left\{4e^{-2t}\cos 3t\right\} = \mathcal{L}T\left\{4e^{-2t}\,g(t)\right\} \quad \text{where}$$

$$g(t) = \cos 3t \xrightarrow{\mathcal{L}T} G(s) = \frac{s}{s^2+9}$$

$$\mathcal{L}T\left\{e^{-\alpha t}\cdot g(t)\right\} = G(s+\alpha)$$

Så $\mathcal{L}T\left\{4e^{-2t}\cos 3t\right\} = 4\,G(s+2)$

$$G(s) = \frac{s}{s^2+9} \rightsquigarrow G(s+2) = \frac{s+2}{(s+2)^2+9}$$

$$G(s+2) = \frac{s+2}{s^2+4s+4+9} = \frac{s+2}{s^2+4s+13}$$

$$\mathcal{L}T\left\{f(t) = 4e^{-2t}\cos 3t\right\} = \frac{4(s+2)}{s^2+4s+13}$$

(10) Z-transform for $\left\{0{,}3^n \sigma[n-2]\right\}_{n=0}^{\infty} = x[n]$

Note: $\sigma[n-3] = \begin{cases} 1 & n \geq 2 \\ 0 & n < 2 \end{cases}$

$$X(z) = \sum_{n=0}^{\infty} x[n]\, z^{-n} =$$

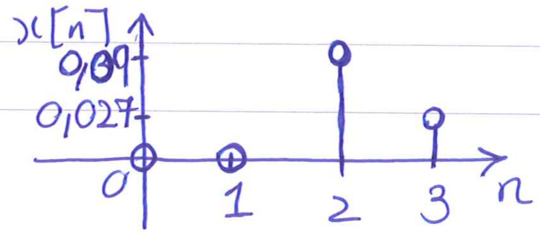
$$X(z) = \underbrace{0{,}3^0 \cdot 0 \cdot z^{-0}}_{x[0]} + \underbrace{0{,}3^1 \cdot 0 \cdot z^{-1}}_{x[1]} + \sum_{n=2}^{\infty} z^{-n} \cdot 0{,}3^n \cdot 1 = \sum_{n=2}^{\infty} 0{,}3^n z^{-n}$$

$$X(z) = \sum_{n=2}^{\infty} 0{,}3^n z^{-n} = \sum_{n=0}^{\infty} 0{,}3^n z^{-n} - \left(0{,}3^0 z^0 + 0{,}3 z^{-1}\right)$$

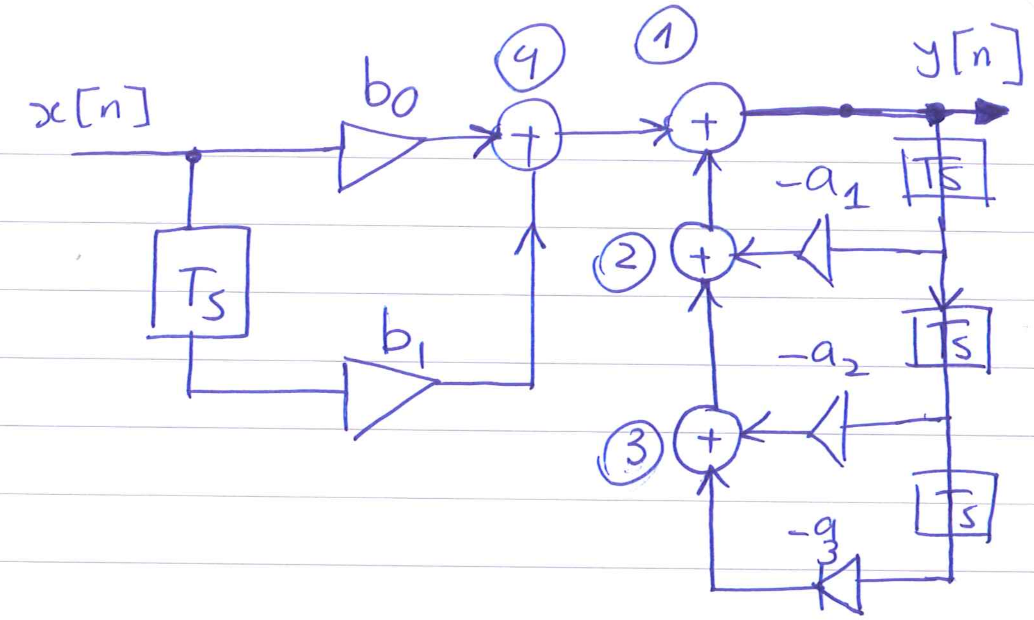
$$X(z) = \sum_{n=0}^{\infty} 0{,}3^n z^{-n} - \left(1 + \frac{0{,}3}{z}\right) = \frac{z}{z-0{,}3} - \frac{z+0{,}3}{z}$$

$$X(z) = \frac{z^2 - \left(z^2 - 0{,}3^2\right)}{z(z-0{,}3)} \; \#$$

Rita: $x[n] = \begin{cases} 0{,}3^n & n \geq 2 \\ 0 & n < 2 \end{cases}$ $\qquad X(z) = \dfrac{0{,}09}{z^2 - 0{,}3z}$



11



At node 3 $\quad -a_3 y[n-3] + -a_2 y[n-2]$

At node 2 $\quad -a_1 y[n-1] + (+)_3$

Så node 2 : $\quad -a_1 y[n-1] - a_3 y[n-3] - a_2 y[n-2]$

at 4 : $\quad b_0 x[n] + b_1 x[n-1]$

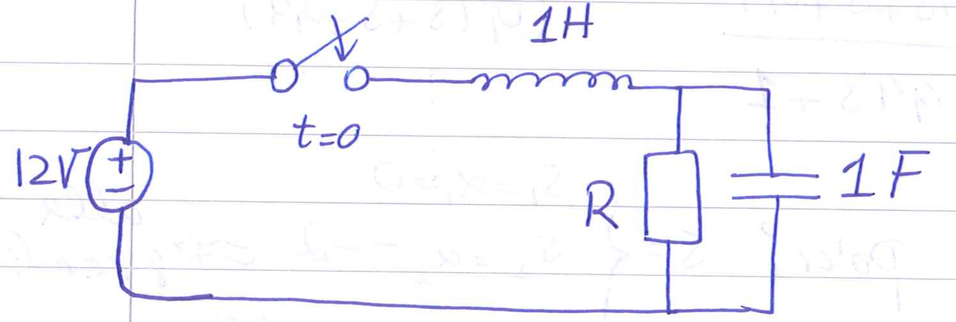
at 1 : $\quad (+)_4 + (+)_2 = y[n]$

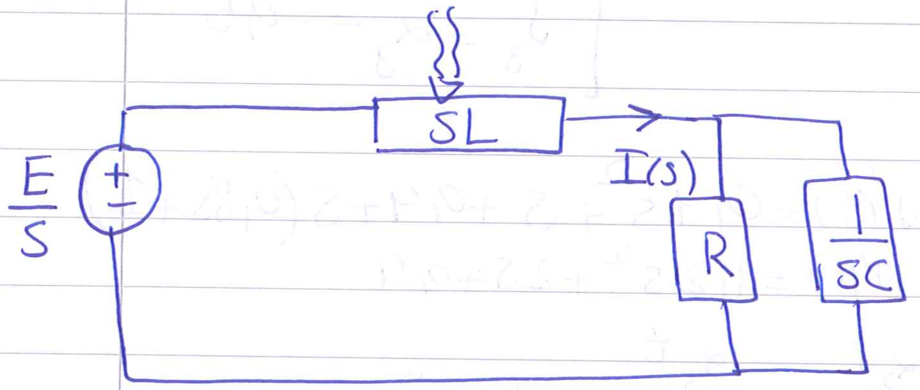
Så $\quad y[n] = b_0 x[n] + b_1 x[n-1] - a_1 y[n-1] - a_3 y[n-3] - a_2 y[n-2]$ #

$$Y(z) = b_0 X(z) + b_1 z^{-1} X(z) - a_1 z^{-1} Y(z) - a_3 z^{-3} Y(z) - a_2 z^{-2} Y(z) \Rightarrow H(z) = \frac{Y(z)}{X(z)} \Rightarrow$$

$$H(z) = \frac{b_0 + b_1 z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3}}$$ #

12) Strömmen $i(t)$ för $t \geq 0$, $R = 0{,}4\,\Omega$.





Ingen begynnelseenergi! För $t \geq 0$.

Strömmen $I(S)$ erhålles som $I(S) = \dfrac{E/s}{Z(S)}$, där

$Z(s)$ är kretsens totala impedens.

$$Z(S) = SL + \underbrace{R // \frac{1}{SC}}_{\text{parallel}} = SL + \frac{R\frac{1}{SC}}{R + \frac{1}{SC}} \Rightarrow$$

$$= \frac{R}{RSC + 1} + SL = \frac{R + RCLS^2 + SL}{RSC + 1} = \begin{Bmatrix} R = 0{,}4\Omega \\ C = 1F \\ L = 1H \end{Bmatrix}$$

$$= \frac{0{,}4 + 0{,}4S^2 + S}{0{,}4S + 1} = \frac{0{,}4S^2 + S + 0{,}4}{0{,}4S + 1} = Z_{tot}(S)$$

Så $I(s) = \dfrac{\dfrac{12}{s}}{\dfrac{0{,}4s^2+s+0{,}4}{0{,}4s+1}} = \dfrac{12(0{,}4s+1)}{s(0{,}4s^2+s+0{,}4)}$

Inverstransformera! poler: $s = \begin{cases} s_1 = \alpha_1 = 0 \\ s_2 = \alpha_2 = -2 \\ s_3 = \alpha_3 = -0{,}5 \end{cases} \Rightarrow$ alla enkel & reel

$I(s) = \dfrac{T(s)}{N(s)}$, $N'(s) = 0{,}4s^2 + s + 0{,}4 + s(0{,}8s+1)$
$N'(s) = 1{,}2s^2 + 2s + 0{,}4$

$i(t) = Ae^{\alpha_1 t} + Be^{\alpha_2 t} + Ce^{\alpha_3 t}$ where:

$A = \left[\dfrac{T(s)}{N'(s)}\right]_{s=0} = \left[\dfrac{12(0{,}4s+1)}{1{,}2s^2+2s+0{,}4}\right]_{s=0} = \dfrac{12}{0{,}4} = 30$, $\boxed{A=30}$

$B = \left[\dfrac{T(s)}{N'(s)}\right]_{s=-2} = \left[\dfrac{12(0{,}4s+1)}{1{,}2s^2+2s+0{,}4}\right]_{s=-2} = 2$, $\boxed{B=2}$

$C = \left[\dfrac{T(s)}{N'(s)}\right]_{s=-0{,}5} = \left[\dfrac{12(0{,}4s+1)}{1{,}2s^2+2s+0{,}4}\right]_{s=-0{,}5} = -32$, $\boxed{C=-32}$

Så $i(t) = 30 - 32e^{-0{,}5t} + 2e^{-2t}$ (A)

12 b) $Z_{tot} = SL + \dfrac{R}{RSC + 1} = S + \dfrac{R}{SCS + 1}$

$$= S + \frac{1}{SC + 1/R} = \frac{S^2 + 1/R\, S + 1}{S + 1/R}$$

$$I = \frac{12/S}{Z_{tot}} = \frac{12(S + 1/R)}{S(S^2 + \frac{1}{R}S + 1)}$$

Svängande Karaktär hos $i(t) \Longleftrightarrow$

Continuos termer $\Longleftrightarrow$ Komplex poler

$$S^2 + \frac{1}{R}S + 1 = 0 \rightsquigarrow S = -\frac{1}{2R} \pm \sqrt{\frac{1}{(2R)^2} - 1}$$

$$\Rightarrow \frac{1}{4R^2} - 1 < 0 \Rightarrow 4R^2 > 1 \rightsquigarrow R^2 > \frac{1}{4} \rightsquigarrow$$

$$\boxed{R > \frac{1}{2}}$$

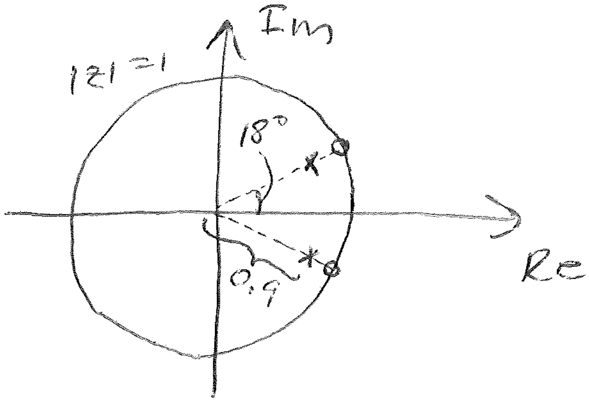
13. Om samplingsfrekvensen $f_s = 1$ kHz, och vi ska filtrera bort en frekvens $f = 50$ Hz, motsvarar detta den normerade vinkelfrekvensen

$$\Omega = 2\pi \frac{f}{f_s} = 2\pi \frac{50}{10^3} = 0.3142 = 18°$$

Vi får alltså följande nollställe/pol par:

$$\begin{cases} z_{noll}^1 = e^{j18°} \\ z_{pol}^1 = 0.9e^{j18°} \end{cases}, \quad z_{noll}^2 = e^{-j18°}, \quad z_{pol}^2 = 0.9e^{-j18°}.$$

Dessa visas i diagrammet under



Överföringsfunktionen blir

$$H(s) = K \frac{(z - z_{noll}^1)(z - z_{noll}^2)}{(z - z_{pol}^1)(z - z_{pol}^2)}.$$

K väljs så att $H(1) = 1$:

$$H(1) = K \frac{(1 - \cos 18° - j\sin 18°)(1 - \cos 18° + j\sin 18°)}{((1 - 0.9\cos 18°) - j0.9\sin 18°)(1 - 0.9\cos 18° + j0.9\sin 18°)} =$$

$$= K \frac{(1 - \cos 18°)^2 + \sin^2 18°}{(1 - 0.9\cos 18°)^2 + 0.9^2 \sin^2 18°} = K \cdot 0.9978$$

$$\Longrightarrow \boxed{K = 1.0022}$$

7. $A \longleftrightarrow 3$ eftersom komplexkonjugerat polpar leder till exponentiellt dämpad svängning enligt inverteringssatsen. Dämpningsfaktorn ges av polernas realdel, vilken är 0 här.

$B \longleftrightarrow 4$ eftersom en rent reell pol ger en exponentiell term i inverstransformen på formen e^{at}, där a är nollstället. (här är $a<0$, så vi får en avtagande exp).

$C \longleftrightarrow 1$ av samma anledning som för A ovan. Realdelen av polparet är negativ, så svängningen blir dämpad med en faktor e^{at}, $a<0$.

$D \longleftrightarrow 2$ av samma anledning som B. Realdelen a är här positiv, så exponentialfunktionen e^{at} blir växande.

$\boxed{14}$ Last question:

$$H(s) = \frac{s+3}{s^2+6s+5} \qquad \text{Transf. funk.}$$

Impulssvar: $Y(s) = H(s) \cdot \underbrace{X(s)}_{=1} = H(s)$

$$Y(s) = H(s) = \frac{s+3}{s^2+6s+5}$$

$$Y(s) = \frac{s+3}{s^2+6s+5} = \frac{T(s)}{N(s)}$$

poles: $N(s) = 0 \Rightarrow (s+5)(s+1) = 0 \quad \begin{array}{l} \nearrow s_1 = \alpha_1 = -1 \\ \searrow s_2 = \alpha_2 = -5 \end{array}$

poles $\rightsquigarrow$ Enkel och reel $\Rightarrow$

$$y(t) = A e^{\alpha_1 t} + B e^{\alpha_2 t} \qquad \text{where}$$

$$A = \left[\frac{T(s)}{N'(s)}\right]_{s=-1} = \left[\frac{s+3}{2s+6}\right]_{s=-1} = \frac{-1+3}{-2+6} = \frac{1}{2}$$

$$B = \left[\frac{T(s)}{N'(s)}\right]_{s=-1} = \left[\frac{s+3}{2s+6}\right]_{s=-5} = \frac{-5+3}{-10+6} = \frac{1}{2}$$

so $y(t) = 0{,}5\, e^{-t} + 0{,}5\, e^{-5t}$ #