Litt om summasjonssymbolet Σ

Defn

$$\sum_{k=k_0}^{n} a_k \stackrel{\text{def}}{=} a_{k_0} + a_{(k_0+1)} + a_{(k_0+2)} + \dots + a_{(n-1)} + a_n \quad \blacksquare$$

Sats (Aritmetisk summe)

$$\sum_{k=1}^{n} k = 1+2+3+\dots+n-1+n = \frac{n(n+1)}{2}$$

B!

$$S \stackrel{\text{def}}{=} 1+2+3+\dots+n-1+n \implies$$

$$S = n+n-1+n-2+\dots+2+1$$

$$2S = n+1+n-1+2+n-2+3+\dots+n-1+2+n+1$$

$$= \underbrace{(n+1)+(n+1)+(n+1)+\dots+(n+1)+(n+1)}_{n \text{ termer}} =$$

$$= n(n+1)$$

$$\therefore S = \frac{n(n+1)}{2} \quad \blacksquare$$

Sats (Aritmetisk summe)

Annet bevis: $(k+1)^2 =$ kvadreringsformel $=$

$$= k^2+2k+1 \implies (k+1)^2-k^2 = 2k+1 \implies (n+1)^2-1^2 =$$

$$= (n+1)^2+n^2+(n-1)^2+\dots+2^2-(n^2+(n-1)^2+\dots+2^2+1^2) =$$

$$= \sum_{k=1}^{n}(k+1)^2 - \sum_{k=1}^{n} k^2 = \sum_{k=1}^{n}\left((k+1)^2-k^2\right) = \sum_{k=1}^{n}(2k+1) = 2\sum_{k=1}^{n} k + \sum_{k=1}^{n} 1 =$$

$$= 2\sum_{k=1}^{n} k + n \implies \sum_{k=1}^{n} k = \frac{(n+1)^2-1-n}{2} = \frac{n^2+2n+1-1-n}{2}$$

$$= \frac{n^2+n}{2} = \frac{n(n+1)}{2} \quad \blacksquare$$